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FRIEZE PATTERNS: Sequence Overview

Summary of learning goals

This sequence of three lessons is an exploration of symmetry and transformation in the context of friezes (repeating strip patterns). Friezes are widely used as decorative borders on furnishings, furniture, fabrics and architecture. They are seen in fencing and railings, and even on tyre treads. Using the formal terminology of transformations (reflection, translation and rotation), students explore the effects of transformations on friezes and identify and describe line symmetry and rotational symmetry (including the centre of rotation and axes of symmetry). Students move beyond looking at examples of symmetry to using symmetry as a way to classify designs, thereby building up a more abstract notion of pattern in Mathematics.

Australian Curriculum: Mathematics (Year 5)

ACMMG114: Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries.

- Identifying and describing the line and rotational symmetry of a range of two-dimensional shapes, by manually cutting, folding and turning shapes and by using digital technologies.
- Identifying the effects of transformations by manually flipping, sliding and turning two-dimensional shapes and by using digital technologies.

Summary of lessons

Who is this Sequence for?

Students beginning this sequence will have had prior experience of transforming shapes by flipping, reflecting in a mirror or rotating them. They will understand the concept of symmetry, and will be able to identify the line of symmetry in a shape such as a butterfly. At a simple level, these lessons provide substantial manipulative experience of geometric transformations, but they may also provide substantial challenges requiring strong visualisation and careful geometric analysis.

Lesson 1: Footprints in the Sand

Students physically recreate the movements used to make patterns of footprints left in sand. Through this, they revisit the ideas of flip, slide, turn and mirror image and symmetry and learn more precise language. They then explore the symmetries in the footprint patterns and record them in a table.

Lesson 2: Frieze Patterns

This task introduces friezes with examples from furnishings in Parliament House, decorative house railings and tyres. Students identify the different symmetries that are present in the frieze patterns by turning, flipping and rotating images or using a mirror. Students then group frieze patterns based on their symmetries using a table. They consolidate their learning by looking at what would happen if one panel in a wrought iron 'lace' railing was installed the wrong way around. Some students may classify tyre treads. A class display of friezes is begun.

Lesson 3: Creating Frieze Patterns

Students create their own frieze designs with chosen symmetries. Students work together to apply transformations to a design element which creates a cell. This cell is then repeatedly translated a fixed distance to create a frieze. A think/pair/share activity is suggested to collaboratively build knowledge. Together they make an attractive class display of friezes made in class or observed elsewhere.

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Reflection on this Sequence

Rationale

The first lesson introduces the symmetries with simple footprints designs, and the second lesson re-examines these ideas with more complex design elements. In the third lesson students create their own friezes with desired symmetries, following a procedure described with the language of transformations that they have learned. A confident understanding of the effect of these transformations is thereby built up over the sequence.

The sequence begins the work on transformations with students analysing how to make patterns from footprints, giving a physical meaning to rotation (turn or spin), translation (step or jump) and reflection (swapping left and right feet). The work then moves to designs on paper, with manipulation of overlays and possible use of mirrors providing assistance to build visualisation of the effect of transformations and different types of symmetry. Some students will be easily able to visualise a transformed image after these experiences; all students will know what to use to help them find it. Students' fluency with these transformations and symmetry is developed across the lesson sequence.

Throughout this sequence, a wide range of examples of friezes are encountered, from footprints in the sand, to the furnishings of Parliament House, wrought iron lace on terrace houses, and then tyre treads. In addition, students are encouraged to find friezes in their own environments and bring sketches or pictures to the class display.

reSolve Mathematics is Purposeful

Geometry is all around us, and these lessons on the symmetries in friezes provide multiple opportunities for students to observe how geometry is evident in the world around us. Geometry also has close links to art and this sequence provides opportunities for demonstrating both mathematical and artistic creativity.

Whereas in earlier years, geometry in the environment is about the shapes that can be seen, this sequence moves on to highlight the mathematical relationships between shapes that make up the pattern.

Students' fluency with transformations is developed as they classify patterns and create their own frieze designs.

reSolve Tasks are Challenging Yet Accessible

Access is provided for all students by beginning to study transformations related to simple physical movements (footprints patterns) and by providing manipulable transparent overlays (and possibly mirrors) to support visualisation.

The types of symmetries of friezes vary in complexity, so while some students may focus on patterns with basic symmetry, students who require further challenge are extended through complex movements and complex designs. The opportunity to create their own frieze patterns allows students to engage in the activity at a variety of levels of complexity.

The attractive designs involved should appeal to students with a range of interests relating to art and design. Several of the world's cultures are known for their frieze designs.

reSolve Classrooms Have a Knowledge Building Culture

Students work collaboratively to sort and classify the frieze patterns. In their small groups and/or pairs, students are encouraged to build consensus on the symmetry of designs through active exploration, mathematical reasoning and clear communication.

Acknowledgements

- Old Parliament House for granting permission to photograph frieze patterns within the building.
- Isabella Tripet for videos and photographs of friezes.
- Tyre Images - Bridgestone Tyres.
- Wrought Iron Frieze images - Central Foundry (Sydney), Hindmarsh Fencing (Adelaide), Melbourne Lacework (Melbourne).

We value your feedback after this lesson via http://tiny.cc/lesson-feedback

FRIEZE PATTERNS
Lesson 1: Footprints in the Sand

Australian Curriculum: Mathematics – Year 5

ACMMG114: Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries.

- Identifying and describing the line and rotational symmetry of a range of two-dimensional shapes, by manually cutting, folding and turning shapes and by using digital technologies.
- Identifying the effects of transformations by manually flipping, sliding and turning two-dimensional shapes and by using digital technologies.

Lesson abstract

Students physically recreate the movements used to make patterns of footprints left in sand. Through this, they revisit the ideas of flip, slide, turn and mirror image and symmetry and learn more precise language. They then explore the symmetries in the footprint patterns and record them in a table.

Mathematical purpose (for students)

Shapes, and patterns made from shapes, can have several different types of symmetry.

Mathematical purpose (for teachers)

This lesson reviews flip, slide and turn transformations introduced in earlier years by looking at how patterns of footprints can be made. It introduces the language of rotation, reflection and translation. In addition to line symmetry (associated with reflection) introduced in earlier years, students encounter rotational symmetry (half turn only) and translation symmetry. There is potential to discuss centre of rotation and axis of symmetry.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- transformation
- translation
- rotation
- reflection
- half turn
- glide reflection
- symmetry
- horizontal reflection
- vertical reflection
- axis

Lesson Materials

- footprint patterns (1a Footprint Pattern Strips powerpoint) 1 per pair of students printed on paper and on a transparent overlay, or printed on paper with tracing paper also provided)
- slide show 1b Looking at Symmetries powerpoint
- Student Sheet 1 – Footprints in the Sand (1 per student)
- Teacher Sheet 1 – Footprints in the Sand
- Student Sheet 2 – Footprint Symmetries (1 per student)
- Teacher Sheet 2 – Footprint Symmetries

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Making Footprints in the Sand

Show students one of the footprint patterns from 1a Footprint Pattern Strips powerpoint. Explain that some people were playing on the beach leaving interesting patterns made from footprints in the sand. The patterns went as far as we could see.

Ask students to think how the people made the footprint patterns.

Students try to produce all seven of the footprint patterns provided by actually performing the movements. The footprint patterns are also on Student Sheet 1 – Footprints in the Sand. Students can record how the patterns can be made in the space provided in the table.

During the activity, describe features of the patterns using the informal language of flips, slides and turns and mirror image. Introduce the formal language of translations, rotations and reflections. In describing the transformations, point out the axes of reflection and the centres of rotation.

NOTE: These lessons use 'horizontal reflection' to mean a reflection with the axis along the direction of travel of the footprints. 'Vertical reflection' means a reflection with the axis across the direction of travel.

Searching for Symmetries

The main inquiry is to find the symmetries of each of the footprint patterns. The patterns have been chosen because they each have a different form of symmetry. The physical movements students used to generate the pattern will help to highlight these symmetries. Teacher Sheet 1 – Footprints in the Sand has answers.

The slide show 1b Looking at Symmetries powerpoint shows the five different symmetries that a long strip can have and how they can be found using a transparent overlay. As the students explore the symmetry of the different footprint patterns they need to look for the symmetry in whole strip. This is demonstrated in the video where one strip or part thereof, is overlayed over another strip to show different forms of symmetry.

Students work to identify symmetries in the footprint pattern strips. Student Sheet 2 – Footprint Symmetries can be used to record and organise the results. (Answers on Teacher Sheet 2 – Footprint Symmetries)

Two approaches to facilitate the exploration are:

- Provide students with 2 copies of 1a Footprint Pattern Strips powerpoint, one copied onto paper and the other copied on clear plastic film or an OHP transparency. The clear plastic strip of footprints can then be manipulated to explore the symmetry, as in the slideshow.
- Provide the students with a copy of 1a Footprint Pattern Strips powerpoint and some tracing paper or non-greased baking paper. Working in pairs, students can trace over the strips and then manipulate the tracing paper to check for symmetries.

Show students that flipping the transparency or tracing paper produces the same image as reflecting in a mirror (so the transformation is called a reflection).

Extending Prompt

- Make several other footprint patterns with the same symmetry as one of the footprint strips. For example, Footprints 1 has translational symmetry only. Another footprint pattern like this is hop-hop-jump repeated. Jumping with heels together and feet pointed outwards will have the same symmetry as Footprints 4.

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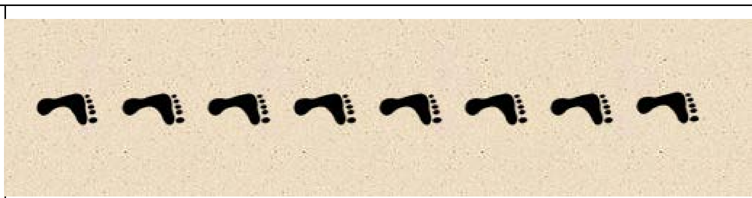
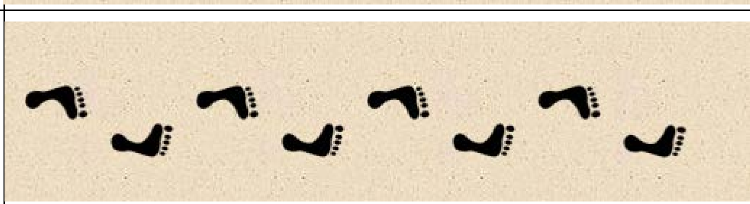
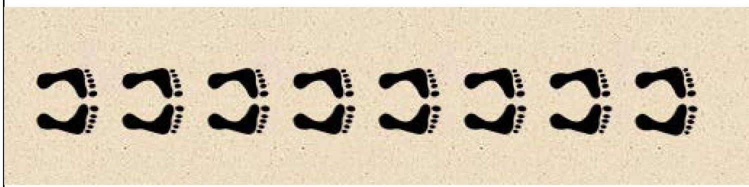
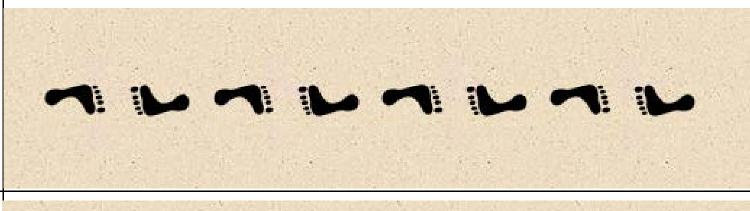
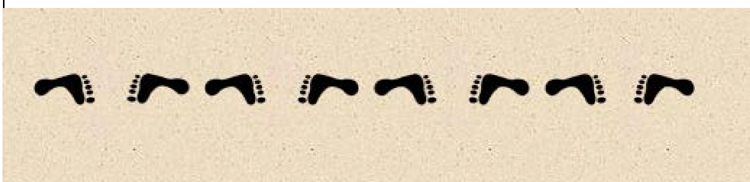
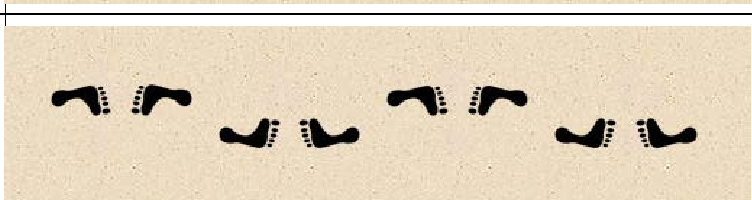
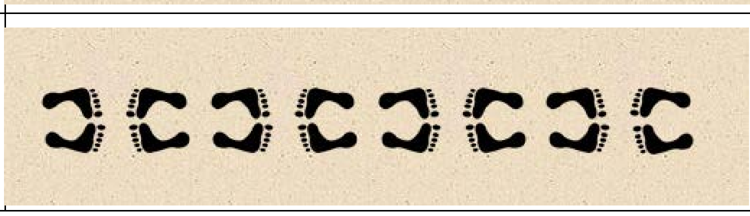
Footprints in the Sand

Name: ______________

Some people were playing on the beach leaving interesting patterns made from footprints in the sand. The patterns went as far as we could see.

How do you think they made these footprint patterns?

What forms of symmetry can you find in each footprint pattern?

Footprints 1		
Footprints 2		
Footprints 3		
Footprints 4		
Footprints 5		
Footprints 6		
Footprints 7		

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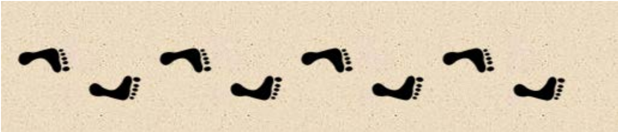
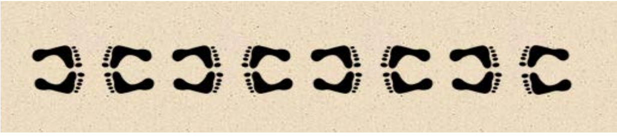
Teacher Sheet - Footprints in the Sand

	Footprint Patterns	Movements* & Symmetry
Footprints 1		**1 person HOP** *Translation symmetry only*
Footprints 2		**1 person STEP** *Translation and glide reflection symmetry*
Footprints 3		**1 person JUMP** *Translation and horizontal reflection symmetry*
Footprints 4		**1 person SPIN HOP** *Translation and half turn rotation symmetry*
Footprints 5		**2 person MIRROR HOP** *Translation and vertical reflection symmetry*
Footprints 6		**2 person MIRROR STEP** *Translation, vertical reflection, glide reflection and half turn symmetry*
Footprints 7		**2 person MIRROR JUMP** *Translation, horizontal reflection, vertical reflection and half turn rotation symmetry*

* Other movements may create the same symmetries

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Footprint Symmetries

Name: ______________

Explore the different footprint patterns. Place a tick in the columns to identify which symmetries can be found in each footprint strip.

	SYMMETRIES				
	Translation	**Horizontal Reflection**	**Vertical Reflection**	**Half Turn Rotation**	**Glide Reflection**
Footprints 1					
Footprints 2					
Footprints 3					
Footprints 4					
Footprints 5					
Footprints 6					
Footprints 7					

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Teacher Sheet - Footprint Symmetries

	SYMMETRIES				
	Translation	Horizontal Reflection	Vertical Reflection	Half Turn Rotation	Glide Reflection
Footprints 1	✓				
Footprints 2	✓				✓
Footprints 3	✓	✓			
Footprints 4	✓			✓	
Footprints 5	✓		✓		
Footprints 6	✓		✓	✓	✓
Footprints 7	✓	✓	✓	✓	

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re(Solve) MATHS BY INQUIRY

FRIEZE PATTERNS

Lesson 2: Frieze Patterns

Australian Curriculum: Mathematics (Year 5)

ACMMG114: Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries.

- Identifying and describing the line and rotational symmetry of a range of two-dimensional shapes, by manually cutting, folding and turning shapes and by using digital technologies.
- Identifying the effects of transformations by manually flipping, sliding and turning two-dimensional shapes and by using digital technologies.

Lesson abstract

This task introduces friezes with examples from furnishings in Parliament House, decorative house railings and tyres. Students identify the different symmetries that are present in the frieze patterns by turning, flipping and rotating images or using a mirror. Students then group frieze patterns based on their symmetries using a table. They consolidate their learning by looking at what would happen if one panel in a wrought iron 'lace' railing were installed the wrong way around. Some students may classify tyre treads. A class display of friezes is begun.

Mathematical purpose (for students)

We can investigate the symmetry of shapes and patterns by flipping, sliding and turning them or reflecting them.

Mathematical purpose (for teachers)

This lesson extends the learning from lesson 1 by introducing the idea of a frieze: a pattern created by repeated translation. Students group friezes according to the symmetry in the pattern, and find examples of all seven possible symmetry combinations. Through physical manipulation of the images, students will increase their capacity to visualise the effects of rotation and reflection transformations. The consolidation task on wrought iron lace links everyday language (e.g. back-to-front) with the language of transformations.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- frieze
- transformation
- half turn
- glide reflection
- symmetry
- horizontal reflection
- vertical reflection

Lesson Materials

- slide show 2a Introducing Frieze Patterns
- slide show 1b Looking at Symmetries powerpoint from previous lesson
- frieze strips from slide show 2b Frieze Patterns To Classify powerpoint (enough for students to use in small groups, copied onto paper and transparencies)
- Student Sheet 1 – Frieze Pattern Symmetries (1 per student)
- Student Sheet 2 – Wrought Iron Friezes (1 per student)
- slide show 2c Frieze Patterns Tyre Treads powerpoint (optional)

Frieze Patterns

Friezes are decorative strips on which a design is endlessly repeated at a constant distance apart. All friezes have translation symmetry; they are made by continuously translating a 'cell' by a fixed distance both forwards and backwards. Frieze patterns can also be symmetric in other ways: the whole strip can look the same after it is reflected in a horizontal axis, or reflected in a vertical axis, or when the whole strip is rotated by half a turn, depending on the cell pattern that has been used. Friezes can also have glide reflection symmetry.

The slide show 2a Introducing Frieze Patterns establishes a definition and shows examples of friezes used to decorate Parliament House. As they watch, ask students to identify the cells that are repeated to make the friezes. Be sure to find the smallest part that is being translated to make each frieze.

It may be necessary to re-watch slide show 1b Looking at Symmetries powerpoint from the previous lesson.

Seven different groups of frieze patterns will emerge when the symmetries in the frieze patterns have been identified. Mathematicians have shown that these are the only possible combinations of symmetry for frieze patterns. The seven groups of friezes are:

- Friezes with translation symmetry only.
- Friezes with translation and glide reflection symmetry.
- Friezes with translation and horizontal reflection symmetry.
- Friezes with translation and half turn rotation symmetry.
- Friezes with translation and vertical reflection symmetry.
- Friezes with translation, vertical reflection, glide reflection and half turn symmetry.
- Friezes with translation, horizontal reflection, vertical reflection and half turn rotation symmetry.

Conducting the Inquiry

Provide students with a selection of frieze patterns and ask them to identify the symmetry in each frieze design.

The slide show 2b Frieze Patterns To Classify powerpoint provides a selection of frieze patterns demonstrating the different forms of symmetry. Copy the images onto paper and transparencies and cut these into strips for use in the classroom. The slide show might also be used for discussion later.

Two possible approaches are:

- Look at each frieze pattern individually and have them identify the symmetries in the frieze.
- Start with the symmetry and pose the questions: *Which of your frieze patterns have…*
  - *…translational symmetry?*
  - *…horizontal reflection symmetry?*
  - *…vertical reflection symmetry?*
  - *…half turn rotation symmetry?*
  - *…glide reflection symmetry?*

The second option may be easier for students.

Student Sheet 1 – Frieze Pattern Symmetries has space to record the symmetries of eight frieze patterns.

Have students share the friezes that they have classified with others. When each pair has agreed on the symmetry for a pattern strip, the frieze can be added to a class display of frieze patterns grouped according to their symmetry. This can be added to over time. Eventually there should be seven groups. Students can also bring in photos or drawing of simple frieze designs that are in their homes and on clothes (e.g. hair ribbons) or that they see out and about.

Enabling Prompt

- Provide students with friezes that have vertical reflection and/or horizontal reflection symmetry only.

Extending Prompts

- Ask students to identify all the centres of rotation and all the vertical and horizontal axes of symmetry.
- Is it possible to have all five forms of symmetry in the one frieze pattern?

Extension Problem

Explore and identify the symmetry of tyre tread patterns. These are just like the other friezes, but wrapped around the tyre. These images have been supplied in the separate slide show 2c Frieze Patterns Tyre Treads powerpoint

Consolidating Task - Wrought Iron Friezes

Show the students the four different wrought iron frieze panels on [Student Sheet 2 – Wrought Iron Friezes]. The task for students is to decide whether the completed iron lace railings would look right if a panel was installed the wrong way around.

Copying the four frieze panels onto a clear transparency allows students to see the effect of the transformations. A slide for printing has been provided at the end of slide show [2b Frieze Patterns To Classify powerpoint].

The mark in the top left-hand corner should make it easy to track each transformation. An example is shown in the image below. Assume that the panels are made so that they look good from both the back and the front.

Wrought Iron Panels

	This panel can be put in upside down, back-to-front or both. The frieze has half turn rotation, vertical reflection and horizontal reflection symmetry.
	This panel can only be put in this way. The frieze has glide reflection symmetry.
	This panel can be put in back-to-front. The frieze has vertical reflection symmetry.
	This panel can be put in back-to-front. The frieze has vertical reflection symmetry.

Frieze Pattern Symmetries

Name: ______________

Place a tick in the columns to show which symmetries can be found in your frieze patterns.

	SYMMETRIES				
	Translation Symmetry	**Horizontal Reflection**	**Vertical Reflection**	**Rotational (Half-turn)**	**Glide Reflection**
Frieze Pattern Number					
Frieze Pattern Number					
Frieze Pattern Number					
Frieze Pattern Number					
Frieze Pattern Number					
Frieze Pattern Number					
Frieze Pattern Number					
Frieze Pattern Number					

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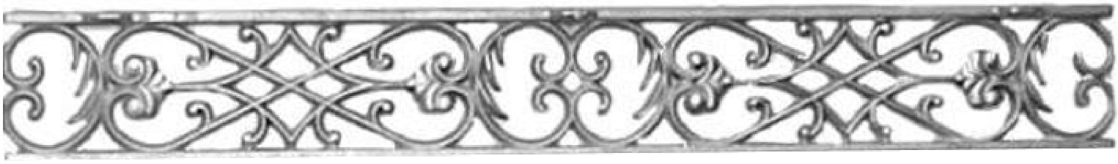
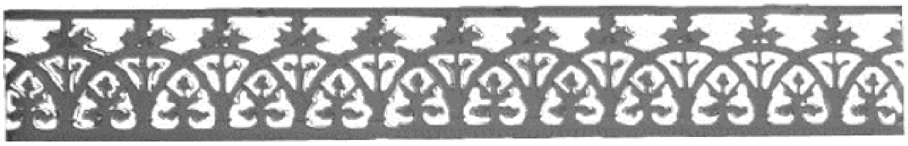
Wrought Iron Friezes

Name: ______________

Wrought iron 'lace' is used on buildings to make railings, fences and decorative borders. The borders, fences and railings are made by joining multiple panels together.

These are some panels for wrought iron railings:





Would the railing look correct if one panel was put in upside down?

Would the railing look correct if one panel was put in back-to-front?

Would the railing look correct if one panel was put in both back-to-front and upside down?

What does this tell us about the symmetries of each frieze?

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FRIEZE PATTERNS

Lesson 3: Creating Frieze Patterns

Australian Curriculum: Mathematics (Year 5)

ACMMG114: Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries.

- Identifying and describing the line and rotational symmetry of a range of two-dimensional shapes, by manually cutting, folding and turning shapes and by using digital technologies.
- Identifying the effects of transformations by manually flipping, sliding and turning two-dimensional shapes and by using digital technologies.

Lesson abstract

Students create their own frieze designs with chosen symmetries. Students work together to apply transformations to a design element which creates a cell. This cell is then repeatedly translated a fixed distance to create a frieze. A think/pair/share activity is suggested to collaboratively build knowledge. Together they make an attractive class display of friezes made in class or observed elsewhere.

Mathematical purpose (for students)

Use your knowledge of transformations to create and describe your own friezes.

Mathematical purpose (for teachers)

In this lesson students reflect and rotate and translate basic shapes purposefully to create friezes with desired types of symmetry. They will analyse the patterns and may identify centres of rotation and axes of symmetry.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- centre of rotation
- axis of symmetry

Lesson Materials

- slide show 3a Creating Frieze Patterns powerpoint
- a selection of coloured paper and/or pens to make the friezes
- picture hole punches, shapes to trace around, cardboard to make shapes to trace
- computer drawing package (optional)

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Creating Frieze Patterns

In this lesson, students create their own frieze patterns with different symmetries.

Friezes can be made freely, by taking any shape and translating it. However, the slide show 3a Creating Frieze Patterns powerpoint gives instructions on how to create frieze patterns with any desired symmetry. It shows how a design element is chosen and transformed to make a symmetric cell, then translated to make the frieze.

Large picture hole punches or shapes that can be traced around work well as the design element. Students could also make their own simple design element and cut it out of card so it can be traced around.

Stamps, stickers or other printed shapes are good, but have limitations as they cannot be reflected on paper. This can be an interesting part of the inquiry for students. If they use these resources, ask them to explain which symmetries are or are not possible in the cell and also in the frieze as a whole.

Computer software may be used. In Word and PowerPoint, shapes can be easily rotated using the rotate feature in the picture tools. Reflections are also possible, but the names are confusing. 'Flip Vertically' reflects the image in a horizontal axis so it performs what we call a horizontal reflection. Similarly,'flip horizontally' gives a vertical reflection.

Conducting the Inquiry

Use a modified version of *Think, Pair, Share* to create and analyse frieze designs:

Think

Students create at least two frieze patterns with different symmetries. For each frieze strip that they make, students can record the following:

- The design element.
- The cell and how it was created using the different transformations.
- The symmetries of the frieze pattern.

Enabling Prompt

- Can you create a frieze pattern that has one form of symmetry?
- Now can you create a frieze pattern that has two forms of symmetry?

Extending Prompt

- Identify all the centres of rotation and axes of symmetry in each of your friezes.
- Can you create all seven different frieze patterns?

Pair

Students create pairs and share their frieze designs and how they were made. The pairs come to consensus that the transformations have been used correctly to create the cell and that the symmetries intended are indeed present in the friezes. The pair can then work together to create a frieze pattern with different symmetry.

Share

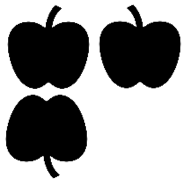
The pairs of students then swap their friezes with a different pair in the class. Each pair needs to decide what symmetry is present in the friezes they have been given. Once they have correctly identified the different symmetries they can be added to the appropriate group in the class frieze display begun in the earlier lessons.

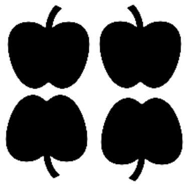
For the display, group the friezes according to their symmetries. Finish by looking at each of the seven groups.

Expected Student Response 1

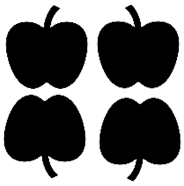
This is the design element.

Horizontal reflection | Vertical reflection | Half Turn

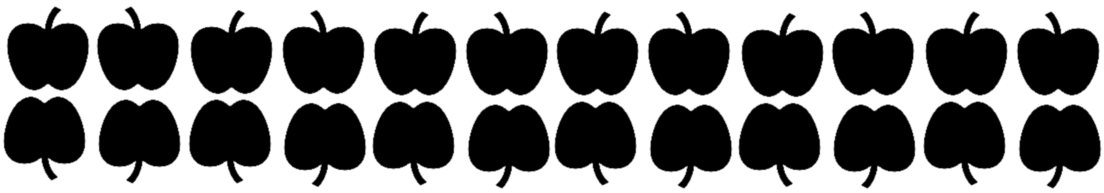




This is the cell for the frieze design.



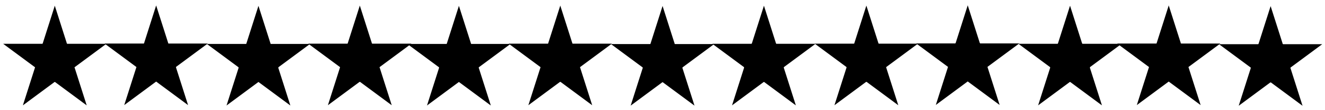
This frieze design has translational, horizontal reflection, vertical reflection and half turn symmetry.



Expected Student Response 2

I made two friezes with different symmetry using a five-pointed star.

This frieze has translation and vertical reflection symmetry.



This frieze has translation and horizontal reflection symmetry.