Unit Overview: Expanded Square Designs

Summary of learning goals

This unit incorporates content in measurement and geometry to deepen students' understanding of and confidence working with informal area and symmetrical/asymmetrical designs. They design an 'expanded square' where approximately half the area of the original square is flipped to the outside. The lessons provide opportunities for students to devise and use methods to informally measure area including developing the awareness that areas sometimes need to be combined before the total area can be mathematically calculated and compared. As students refine their design, they record the mathematical thinking they will use as visual and numerical evidence to convince others that approximately half the area has been flipped. Both reflection (flip) and rotational symmetry (turn) are included informally. This continues students' informal exposure before rotational and reflection symmetry is formalised in Year 5.

Australian Curriculum: Mathematics (Year 4)

ACMMG091: Create symmetrical pictures and shapes with and without digital technologies.
ACMMG087: Compare the areas of regular and irregular shapes by informal means.

- Comparing areas using metric units, such as counting the number of square centimetres required to cover two areas by overlaying the areas with a grid of centimetre squares.

ACMMG088: Compare and describe two dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies.

- Identifying common two- dimensional shapes that are part of a composite shape by re-creating it from these shapes.

ACMNA077: Investigate equivalent fractions used in contexts

Summary of lessons

Who Is This Unit For?

This unit is for students who are building an understanding that areas can be compared using informal means and metric units. Students will be familiar with the effects of flipping two dimensional shapes and turning shapes with a quarter turn or a half turn, and will have some knowledge of fractions (¼, ½ and ¾, and equivalent). Some prior experience with composite shapes is useful but not a pre-requisite.

Lesson 1: Discover

Students are introduced to expanded squares (based on the Eastern art of Notan) and explore their characteristics. They construct their first expanded square and discuss its features, including symmetry.

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Australian Academy of Science

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Lesson 2: Devise

Students use fractions to describe the amount of white space in some partially coloured grid squares, and then in some expanded square examples. They consider what fraction of white space is best for visual balance. Students begin to design an expanded square that has approximately half the original square flipped to the outside.

Lesson 3: Develop

Students produce mathematical evidence to convince others that approximately half the area of their original square has been flipped to the outside. They find areas by counting grid squares, or covering with 1 cm cubes, and perhaps rearranging shapes. They seek constructive feedback on the method they used to determine the fraction of the area that has been flipped and on the mathematical evidence they recorded.

Lesson 4: Defend

Students adjust their designs (or describe possible adjustments) and ensure their evidence is detailed and organised clearly so it can be displayed for others to critique. During the feedback session students provide constructive feedback on the strengths and weaknesses of the mathematical evidence and designs. They reflect on feedback given on their display and on their learning throughout the inquiry.

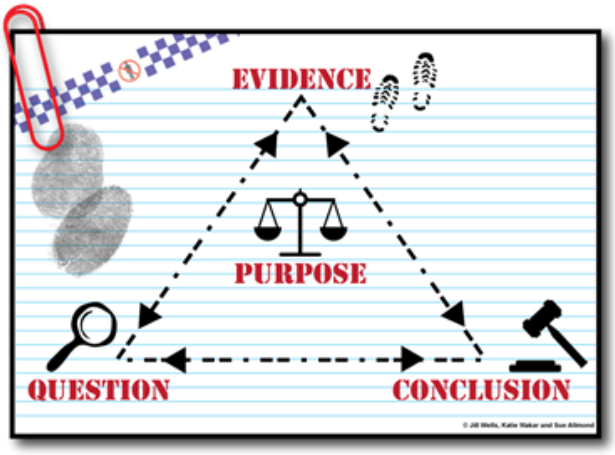
Reflection on this sequence

Rationale

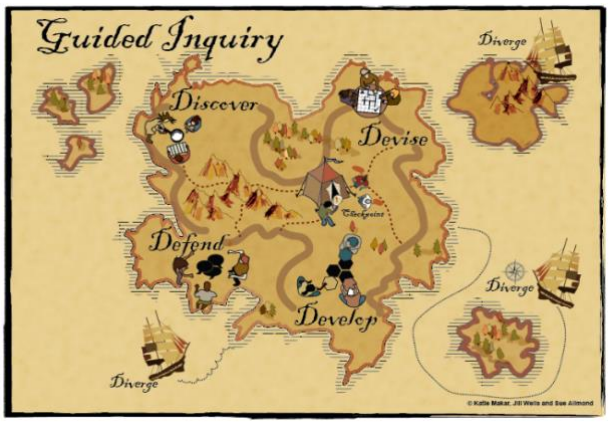
Students in Year 4 are introduced to measurement of area by informal means using metric units enabling them to make more accurate area comparisons. This unit provides a meaningful context for students to use a problem-solving approach to connect area concepts and fractional understanding. Students often equate ½ as partitioning into two equal parts created by folding or drawing a line to divide the shape into two equal parts and do not always make the connection that ½ of an area can also be made up of several different spaces combined. Creating an expanded square design requires students to flip multiple shapes from all sides of the square, all of which need to be accounted for when calculating the total fraction of the area flipped. Students also estimate the fraction of the area: students typically have more difficulty estimating fractions than calculating exact fractions (Van De Walle, Karp and Bay-Williams, 2014). Students need to be able to visualise what fractions look like, so it is important that they provide visual evidence as well as numerical evidence to justify the adjustments required to make the flipped area as close as possible to a half.

Throughout the inquiry, students use evidence they have gathered to support, justify and convince their peers that their solution answers the inquiry question. Lessons stress the need to gather mathematical evidence and the importance of explicitly connecting the inquiry question, the evidence and the conclusion, as shown in the Evidence Triangle (below, left). The unit is structured around the 4D Guided Inquiry model (below, right), which guides the teacher to support and scaffold students through each phase.

Further information is given in the *Mathematical Inquiry into Authentic Problems Teachers' Guide*.



Evidence Triangle



4D Guided Inquiry map

reSolve Mathematics is Purposeful

reSolve relies on the Mathematical Proficiencies of the Australian Curriculum: Mathematics to apply mathematical ideas and practices to everyday problems in authentic ways. This creates connections to deepen students' understanding.

Understanding: Students identify all cut out areas need to be combined when determining the fraction of the square that is flipped to the outside.

Fluency: To create interesting shapes, students combine common two-dimensional shapes. They create symmetrical and asymmetrical expanded designs. By counting whole and part squares they calculate the areas of two-dimensional shapes.

Problem Solving: Students use a problem-solving approach, applying fractional thinking and their knowledge of reflections and informal area concepts.

Reasoning: Students provide sufficient, appropriate visual and numerical evidence to convince themselves and others that their expanded square meets the given criteria. They analyse the reasoning of others, seeking clarification where required and explain their thinking when challenging ideas, methods or mathematics used.

reSolve Tasks are Challenging Yet Accessible

The context of the unit allows students at multiple levels of performance to access the key ideas to solve the problem. For example, students working below grade level can be guided towards choosing a symmetrical design using common two-dimensional shapes such as a rectangle or a square. This will allow them to calculate the area using whole squares and simplify the adjustments to the design in the Defend phase. Calculators may be used to calculate areas and areas can be physically 'measured' by counting unit squares. Students working above grade level may be challenged to create an asymmetrical design that uses a variety of different sized, composite shapes. Area calculations will engage them in making decisions about how to deal with the different sized part squares in their design and justifying these decisions. The visual and tactile nature of the work increases its accessibility at all levels. As this unit requires students to produce their own designs and evidence, regular sharing at Checkpoints and teacher prompts provide the support and feedback students require to improve their ideas and evidence and to embrace setbacks as challenges that can be overcome.

reSolve Classrooms Have a Knowledge Building Culture

The inquiry addresses a knowledge building culture as it engages all students in pervasive knowledge building, requiring students to plan a pathway to answer the Inquiry question and, during Checkpoints, improve on their ideas by negotiating between their own ideas and understandings and those of others. Expecting all students to be active listeners and contributors who share and value diverse ideas, build on others' ideas, seek clarification where required, and question or challenge ideas respectfully, ensures all students contribute towards the advancement of knowledge in the classroom and provides opportunities for them to build, reconceptualise, recreate and extend mathematical concepts. Providing opportunities for more than one iteration of student designs and regular sharing of progress assists students to work with messiness and to embrace and work through challenges as they compile and organise the evidence which will support and justify their solution.

The expectation that everyone completes written feedback on the strengths and weaknesses of displayed evidence and designs in the Defend phase provides each student with an opportunity to demonstrate their ability to critique the thinking and reasoning of others constructively.

Acknowledgements

Van de Walle, J., Karp, K. S., & Bay-Williams, J. M. (2014). *Elementary and Middle School Mathematics: Teaching Developmentally* (Eighth International Edition). Essex: Pearson.