re(Solve) MATHS BY INQUIRY

MATHEMATICAL INQUIRY INTO AUTHENTIC PROBLEMS

Unit Overview: 10 000 Centicubes

Inquiry Question: What is the best container to hold 10 000 centicubes?

Summary of learning goals

This unit integrates content in number and measurement to deepen students' understanding and confidence working with larger numbers. Students work flexibly with numbers up to 10 000 as they determine suitable dimensions for a container that can hold 10 000 centicubes. Students have to convince others that their container is the best. This provides them with a purpose to record their mathematical calculations, represent their base with concrete materials and grid paper and construct a 3D model of their container to use as evidence to justify their solution. If desired, the vocabulary of area and volume can be used throughout, along with the formal units of square centimetre and cubic centimetre, but the unit can also be preliminary to these concepts, working only with numbers of centicubes.

Australian Curriculum: Mathematics (Year 4)

ACMNA072: Recognise, represent and order numbers to at least tens of thousands.

ACMNA073: Apply place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems.

ACMNA076: Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and division where there is no remainder.

ACMMG290: Compare objects using familiar metric units of area and volume.

Summary of lessons

Who Is This Unit For?

This unit is for students who are building an understanding that numbers can be partitioned and combined flexibly to assist calculations and solve problems. Students need fluency with multiplication facts, at least for 2, 5 and 10. Some experience is recommended with using uniform informal units to compare shapes based on volume and capacity, recognising 3D objects using their obvious features, and using a ruler to draw two dimensional shapes and measure lengths. Later lessons provide opportunities for students to begin to develop awareness that it is possible to calculate volume from the length and width of a rectangular base, and the number of 'layers' making up the height of a prism; therefore, no prior experience with calculating volume is necessary.

Lesson 1: Discover

The challenge is to make the best container to hold 10 000 centicubes. Students review mathematical vocabulary, and possible shapes, then predict the size of a suitable prism by looking at just one centicube as a guide. They use the idea of stacking layers of equal size to suggest some container dimensions arising from multiplicative partitioning of 10 000. They record using number sentences and pictures.

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Lesson 2: Devise

Students discuss what is meant by “best” container for an educational supplier to use to package 10 000 centicubes. They work in small groups to create a plan to make a suitable container (including giving all dimensions) before presenting their ideas for feedback.

Lesson 3: Develop

Groups agree on their ‘best’ container to use and determine the dimensions for each face. Plans for the best containers are swapped to provide feedback on whether sufficient mathematical evidence has been recorded to enable the container to be constructed easily. The models of the containers are then constructed.

Lesson 4: Defend

Groups prepare and present their justified solution to the inquiry question. Students examine the reasoning of other groups, and use calculators and rules to validate the solutions. Later, they act on feedback on their own presentations. Groups compare their container with one constructed by another group and record the similarities and differences. As an extension, students consider what reasonable mathematical adjustments might be made to the container if neat packing of cubes is not assumed.

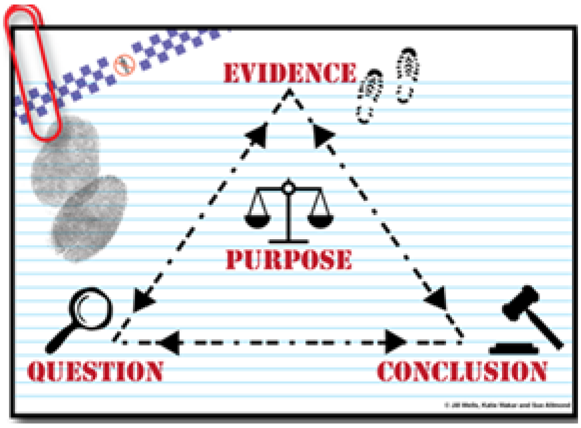
Reflection on this sequence

Rationale

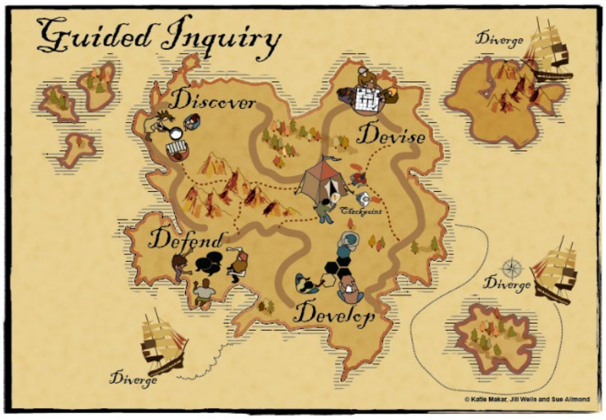
Applying mathematical ideas and practices to everyday problems in authentic ways creates connections to deepen students’ understanding. This unit provides the context of making a box for students to develop multiplicative thinking through the exploration of properties of numbers and through developing strategies for multiplication such as doubling and halving and extended fact families (e.g. 5 x 10 = 50, 5 x 100 = 500, 5 x 1000 = 5000). This experience with decomposing numbers multiplicatively adds to students’ computational flexibility (Van de Walle, Karp & Bay-Williams, 2014). Students also develop appreciation of the magnitude of large numbers. This unit also provides a meaningful context for students to develop an awareness of the attributes of area and of volume through a focus on the number of centicubes on the base of a prism, and the height of the prism when built with equal sized ‘layers’.

Throughout the inquiry, students use evidence they have gathered to support, justify and convince their peers that their solution answers the inquiry question. Lessons stress the need to gather mathematical evidence and the importance of explicitly connecting the inquiry question, the evidence and the conclusion, as shown in the Evidence Triangle (below, left). The unit is structured around the 4D Guided Inquiry model (below, right), which guides the teacher to support and scaffold students through each phase.

Further information is given in the *Mathematical Inquiry into Authentic Problems Teachers’ Guide*.



The Evidence Triangle



4D Guided Inquiry Map

reSolve Mathematics is Purposeful

Understanding: Students partition 10 000 multiplicatively, to connect number representations. Through repeated decomposition of the same number, students can see patterns and thus informally use both the commutative law (e.g. 5 x 10 = 10 x 5) and the associative law (e.g. (5 x 2) x 10 = 5 x (2 x 10)). They make informal connections between length, area and volume and identify the commonalities and differences of three-dimensional shapes.

Reasoning: Students describe the thinking used to determine multiplicative partitions. They provide sufficient, appropriate evidence to convince others that they have created the best container to hold 10 000 centicubes. Students analyse the reasoning of others, seeking clarification where required and explaining their thinking when challenging ideas or the mathematics used. Students use reasoning from mathematics in concert with reasoning from real world considerations of suitable shapes.

reSolve Tasks are Challenging Yet Accessible

This inquiry provides challenges to students as they work collaboratively to make decisions. The context of the unit allows students at multiple levels of performance to access the key ideas to solve the problem. For example, students working below grade level can choose simple factors of 10 000 (e.g. 10 x 10 x 100), or make a container for fewer centicubes (e.g. 1000), and be supported by MABs to physically construct a container. Students working above grade level may work with more complex factors of 10 000, or using non-factors may need to interpret decimals in context. The visual and tactile nature of the work increases its accessibility at all levels.

Regular class sharing assists groups to embrace setbacks as challenges that can be overcome. The expectation that all group members will contribute, provides each student with an opportunity to demonstrate their ability to reason, justify and develop conceptual understanding.

reSolve Classrooms Have a Knowledge Building Culture

The inquiry addresses a knowledge building culture through requiring students to take on many roles usually undertaken by the teacher. In their groups, students collaboratively plan the pathway they will use to answer the inquiry question and negotiate between their own ideas and understandings and those of others. Teaching students to provide feedback is a central part of this unit. During the Devise Phase, students acknowledge that all ideas, including those that are unformed and un-evidenced, can be improved on through the giving and receiving of constructive feedback. Expecting all students to be active listeners and contributors who share ideas, build on others' ideas, seek clarification where required, and question or challenge ideas respectfully, ensures all students contribute towards the advancement of knowledge in the classroom and provides opportunities for them to build, reconceptualise, recreate and extend mathematical concepts.

References

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2014). *Elementary and middle school mathematics. Teaching developmentally* (8th int'l ed.). Essex UK: Pearson.