Unit Overview: Cornering

Summary of learning goals

This unit provides insight into the complex situation of long vehicles turning corners and going around roundabouts. It is useful for learning about road safety and for learner drivers. The motion of turning vehicles is difficult to visualise and prone to misconceptions, yet it makes an excellent context for developing students' modelling skills. Students start from a very simple paper scale model and gradually build their understanding with help from a practical experiment and several pre-made dynamic geometry models. At each stage, they find new questions to ask and consider how a different or improved model could provide the answers. The central question is to find the space (size and shape) that is required for turning vehicles. There is a continual need to move back and forth between understanding of the real situation and interpretation of the mathematical constraints and results.

At the end of the unit, students give a presentation about road safety, and produce a report for local council on road design. Communicating successfully by being aware of different audience's needs and prior knowledge is a key learning goal. The scope of the presentation and of the report will be defined by each group of students as they pursue questions of interest to an appropriate depth.

Australian Curriculum: Mathematics (Year 8-10A)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area. (Year 8)

ACMMG222: Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles. (Year 9)

ACMMG224: Apply trigonometry to solve right-angled triangle problems. (Year 9)

- Selecting and accurately using the correct trigonometric ratio to find unknown sides (adjacent, opposite and hypotenuse) and angles in right-angled triangles.

ACMMG272: Prove and apply angle and chord properties of circles. (Year 10A)

Summary of lessons

Who is this Unit for?

This unit is for students who have had some practice with mathematical modelling and are reasonably familiar with its aims and methods. Although most of the models are provided, students will still need to decide how to use the models to find and interpret results relevant to the real world context, sometimes in complex ways.

Students need to know Pythagoras' theorem and recognise parts of a circle such as the radius and tangents. Knowledge of trigonometric ratios is used for one model, but an alternative is provided. Familiarity with dynamic geometry software (GeoGebra) is useful but not essential.

Students with an interest in mechanics and innovation will have many opportunities to go deeper into the context. Advanced GeoGebra users may wish to modify the models or build their own, and for this they need to be aware of some angle and chord properties of circles e.g. *the angle in a semicircle is 90°*.

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

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Lesson 1: Sharing the Road

Students discuss the meaning of the "Do not overtake turning vehicle" sign found on some trucks and buses, and recall instances when a truck or bus has had difficulty getting around a corner or roundabout. They use a simple paper scale model to visualise what happens when long vehicles turn corners, and discuss their findings, and the limitations of the model. The need for a model with steerable wheels is highlighted.

Lesson 2: A Better Model

Students use a bicycle or scooter to find the space covered when it turns a corner. They walk the bicycle in an arc, discovering that the rear wheel travels along an arc of smaller radius than the front wheel. They draw a diagram of the experiment (from above) to understand the geometry of the situation, and identify the centre of rotation. Students apply Pythagoras' theorem to confirm that their diagram matches the results of the experiment.

Lesson 3: Software Models

Students use dynamic geometry models that match their bicycle experiments. They observe what happens to the rear wheel path as they vary the wheelbase or the tightness of the corner. Students then use the models to predict how much the rear wheels will cut in for a variety of real vehicles. The final products and human factors in vehicle safety are discussed.

Lesson 4: Modelling Four Wheels

Students use new software models, first to investigate the effect of front and rear overhangs on space required for turning, and then including variable width. They investigate the effect of changing these variables on the space required for a vehicle to turn, including the lane usage. Students use the models to gather data for their final products, and also sharpen their focus and scope. Clear communication of mathematical ideas is discussed.

Lesson 5: Presentations and Parking

After time to complete their work, students make their short road safety presentations, and give each other feedback. An optional activity uses the model in a new way to design a car park. The class reflects on the modelling process throughout the unit.

Reflection on this sequence

Rationale

This unit demonstrates mathematical modelling in a mechanical context, with a focus on road safety and road planning. The main insight of the unit is that when turning, the rear wheels of a long vehicle do not follow the path of the front wheels, but 'cut the corner' or 'cut in'. This has implications for the safety of pedestrians, cyclists and drivers, and as such is relevant to students' everyday lives.

reSolve Mathematics is Purposeful

Through mathematical modelling, mathematics provides an important way to understand the world and make decisions: it is a purposeful activity. Students frame questions about the behaviour of vehicles, and use their models to investigate how to be safe around long vehicles and how to design intersections to have enough space. The final presentations will give useful advice on road safety, and can be used to inform other students. There is particular relevance to young people about to learn to drive.

reSolve Tasks are Inclusive and Challenging

The work in this unit can be undertaken at various levels. Each new model adds complexity, with a few main insights to be gained. Some students will base their presentation and report on the simplest of these insights, whilst other students will investigate further, formulating and testing different scenarios.

reSolve Classrooms Have a Knowledge Building Culture

Students work collaboratively through iterations of the modelling cycle, building on their understanding with each successive model, as they relate its insights to the real world situation.