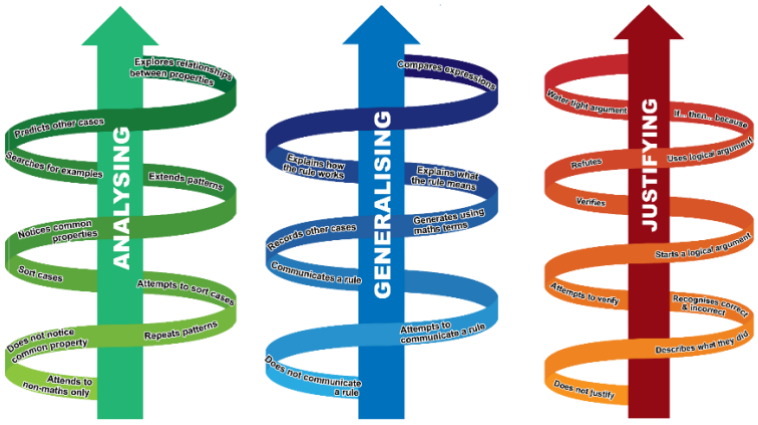
Overview: Exemplars

Summary of learning goals

This suite of resources has been developed to assist in the teaching of mathematical reasoning, one of the four proficiencies of the Australian Curriculum: Mathematics. By providing a scheme for assessing mathematical reasoning in their classrooms, the resources assist teachers to emphasise reasoning in any lesson and to recognise different levels of the reasoning actions, applicable across all primary year levels and all mathematics content. An important goal is to assist teachers to collect and analyse evidence of students' reasoning to report against this Australian Curriculum proficiency.

Eight exemplar tasks which are ready for classroom use are provided. Each exemplar showcases a stand-alone task which is designed to highlight mathematical reasoning. Each exemplar describes the aspects of mathematical reasoning which the task especially supports. Annotated work samples are provided to illustrate practical use of the Assessing Mathematical Reasoning Rubric and to give concrete examples of students' reasoning. The rubric is designed around three reasoning actions: Analysing, Generalising and Justifying. Further details and suggestions for professional learning are given in the Teachers' Guide. A whole school approach to learning about and using the assessment rubric is recommended.



Australian Curriculum: Mathematics

The assessment rubric is relevant to all mathematical topics. The exemplar tasks that illustrate the assessment process relate to the following content descriptions.

ACMNA053: Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems
ACMNA054: Recognise and explain the connection between addition and subtraction
ACMNA055: Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation
ACMNA071: Investigate and use the properties of odd and even numbers
ACMNA081: Explore and describe number patterns resulting from performing multiplication
ACMNA098: Identify and describe factors and multiples of whole numbers and use them to solve problems
ACMMG088: Compare and describe two dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies.

We value your feedback after these tasks via https://www.surveymonkey.com/r/RJC6FPC

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Summary of exemplars

Who are these Exemplars for?

The exemplar tasks and the associated assessment rubric is for use across the primary levels and into junior secondary school. Most of these tasks can be easily adapted for students across Years 3 to 6. The year levels specified below indicate the year level of the students who provided the sample work.

Exemplar: Is it True? (Year 3)

This task asks students to decide whether an addition is correct or not and to explain their reasons. The content focus is on place value and grouping, re-grouping and renaming for multi-digit addition. The reasoning foci of this task are explaining and justifying statements; exploring and noticing relationships (analysing); and forming conjectures and generalising. The task is easy to adapt to other content and year levels.

Exemplar: Number Towers (Year 3)

Number Towers gives students an opportunity to develop and test conjectures and form generalisations by reasoning mathematically about numerical structures with addition. The task promotes careful analysis of a mathematical structure. Students need to experiment systematically, keep track of results, and choose cases carefully to test the rule. The task can be adapted for older students by using fractions or decimals.

Exemplar: The Magic V (Year 4)

The Magic V task affords an opportunity to explain the reasons why a conjecture is true. Students begin by creating Magic Vs by trial and error, but come to see constant features. By manipulating numbers, analysing totals and recording their ideas, students will come to see why the properties of odd and even numbers are relevant. The main purpose is to develop students' capacity to analyse situations, to find reasons and develop logical arguments.

Exemplar: Matchsticks (Year 4)

Students explore making rows of squares from matchsticks. The patterns can be described pictorially, numerically and symbolically. Students learn to work systematically and keep a record of results that assist them to develop and test conjectures. As the students describe and explain patterns, they will begin to move from additive to multiplicative reasoning. By changing from rows of squares, the task is easy to adapt to other year levels.

Exemplar: Shapeshifter (Year 4)

Shapeshifter is based on a picture story book. Students find a rule for using a straight line to dissect a polygon to make a polygon with one more side. The content foci are naming shapes and their components. The reasoning foci are especially Analysing and Justifying – seeing and explaining when and why cutting off a corner increases the number of sides.

Exemplar: What Else Belongs? (Year 5)

In this task, students find common properties of three given numbers. They notice and describe properties of number; such as size, order, composition, place value, multiples, factors, even or odd. The students justify why some numbers have similar or dissimilar properties. There is a strong emphasis on Analysing – seeing what is the same and what is different. The task can very readily be adapted to other content and year levels.

Exemplar: Area and Perimeter (Year 6)

In this task, students respond to a conjecture related to a common misconception: that a rectangle with a larger perimeter will always have a larger area. Students will learn that it is sufficient to offer one counter example to refute a conjecture or general statement that makes a claim about all cases.

Exemplar: Painted Cube (Year 6)

The Painted Cube task is rich and complex, providing students with opportunities to explore a variety of patterns that can be described spatially, numerically and symbolically. There are good opportunities for using visualisation. Students learn to work systematically by keeping a clear record of results which will encourage them to develop and test conjectures and to ask themselves questions about further cases.

Reflection on this special topic

Rationale

Being able to reason mathematically is an important goal of the Australian Curriculum: Mathematics. This suite of resources gives teachers an understanding of the nature of mathematical reasoning and its assessment. Teachers can plan tasks which students will undertake over time to foster the development of mathematical reasoning. As students grow in this proficiency, they will progressively require less support and assume greater responsibility for their mathematical reasoning actions. Ongoing assessment for evidence of these actions will provide teachers with the necessary information to revise and/or design tasks that support the development of this key proficiency.

The suite provides several carefully structured resources to assist teachers to notice and assess mathematical reasoning. The exemplar tasks and completed rubrics for each of the work samples provide examples of teacher actions and prompts to encourage higher order mathematical thinking. The resources support teachers' assessment of reasoning through the different levels of mathematical reasoning outlined in the Assessing Mathematical Reasoning Rubric.

The exemplars are not intended as a developmental sequence, rather as illustrations of how reasoning may be embedded in lessons and how to assess reasoning. This development of reasoning is done, perhaps by students first looking at examples and counter-examples, then assembling evidence systematically, all the while looking for patterns and conjectures and seeking reasons. Teachers may also choose to have students write a record of their work and explain why their results are true, to develop the students' communication capability.

reSolve Mathematics is Purposeful

Mathematical reasoning underpins all mathematics. By attending to the assessment of mathematical reasoning teachers will become more aware of the necessity of embedding reasoning in all mathematics lessons thus supporting a rich interpretation and enactment of the Australian Curriculum: Mathematics. Being intentional about embedding and assessing reasoning in all mathematics lessons acknowledges that this mathematical proficiency is crucial to students' ability to connect mathematical ideas and to think mathematically, creatively and imaginatively.

reSolve Tasks are Challenging yet Accessible

The reasoning exemplar tasks provided for classrooms use are structured to maximise students' mathematical development and reasoning capability. The tasks challenge students to engage in sustained inquiry by prompting them to explore relationships between key ideas, thus meeting a range of student capabilities. The use of the Assessing Mathematical Reasoning Rubric provides evidence of students' progress which can inform feedback and subsequent teacher actions, such as, planning tasks that develop students' capacity for reasoning actions identified as needing further development.

reSolve Classrooms have a Knowledge Building Culture

Students explore a variety of perspectives through listening to each other's reasoning in small groups or whole class discussions. The collaborative inquiry nature of the tasks lends itself to the communication of reasoning to others, which is a natural way to test the strength of arguments. To assess students' reasoning, the teacher intentionally builds a culture of communication of reasoning through oral, text, gesture, or visual representations.

Further Reading

The Teachers' Guide (*ST5_Reasoning_TeachersGuide.docx*) gives a full description of the Assessing Mathematical Reasoning Rubric, along with a discussion of the three key reasoning actions of Analysing, Generalising and Justifying. It also provides resources for leading professional learning to introduce the assessment of reasoning to teachers. Many other references for further reading are also provided.