Unit Overview
Introduction to Bar Models

Summary of learning goals

As part of the special topic '**The Bar Model Method**', this unit demonstrates how the bar model method is used to solve mathematical word problems involving whole numbers and fractions. The bar model method is primarily a problem-solving tool – a way of *organising information* from a word problem or other problem set in a context, and an efficient tool for *visualising relationships* between known and unknown quantities. As such, the intention of all these lessons is to give students a flexible tool that they can carry with them in all areas of the curriculum.

These lessons introduce two basic types of bar models – the part-whole model (Lessons 1 to 5) and the comparison model (Lessons 6 to 8). Each model is relevant to problems which involve all four arithmetic operations. Some problems involve fractions, so that the bars representing quantities sometimes represent absolute numbers (e.g. 6 eggs from a carton) and sometimes fractional parts (e.g. half a carton). This unit shows how to use a bar model as a pictorial tool linking given and unknown quantities in a word problem with the real situation and the complementary symbolic mathematical statements.

As this is an introduction to the method, the instructional content is accessible throughout the primary years. Student tasks are aimed at the upper primary level, but could be modified to use in earlier years if desired. A pedagogical approach is adopted which focuses on the bar model as a crucial pictorial connection between a real situation, and abstract mathematical statements.

Australian Curriculum: Mathematics (Years 4 to 6)

ACMNA076: Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder (Year 4).

ACMNA077: Investigate equivalent fractions used in contexts (Year 4).

ACMNA082: Solve word problems by using number sentences involving multiplication or division where there is no remainder (Year 4).

ACMNA100: Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies (Year 5).

ACMNA291: Use efficient mental and written strategies and apply appropriate digital technologies to solve problems (Year 5).

ACMNA103: Investigate strategies to solve problems involving addition of fractions with the same denominator (Year 5).

ACMNA126: Solve problems involving addition and subtraction of fractions with the same or related denominators (Year 6).

ACMNA127: Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (Year 6).

Summary of lessons

Who is This Unit For?

No background with bar models is assumed in these lessons. This unit is designed for students in Year 5 and below, who have a basic understanding of whole number operations and equivalent fractions. Little formal fraction calculation is required, because the bar models generally support calculation by intuitive means. Teachers can adjust the difficulty of the arithmetic involved by changing numbers in the examples.

The pair of lessons on Multiplication & Division (Lessons 3 and 8) and Fractions (Lessons 4 and 5) can be completed independently of the remaining lessons in this unit (e.g. in Year 6), to tailor to students. However, it is recommended that the remaining lessons be taught in order, to build students' competence.

Lesson 1: Addition of Whole Numbers

Students are introduced to the bar model method with the part-whole bar model through this lesson. Students learn how to construct the part-whole bar model and how it can help to represent addition contexts through worked examples with simple numbers and tasks with larger numbers. Three types of addition contexts are used: (1) one set of items made up of 2 parts; (2) the whole being formed from two distinct sets of items; and (3) another set of items "adding on" or "joining" to form a new whole.

Lesson 2: Subtraction of Whole Numbers

In this lesson, part-whole bar models are used to represent subtraction problems involving whole numbers. Students study various different subtraction situations (e.g. 'take away', compare) through five examples. Students then practice with further problems to consolidate this learning.

Lesson 3: Multiplication & Division

Students learn how the bar model can help represent multiplication and division contexts by studying examples and practicing with further tasks. The tasks encountered involve equal groups multiplication and partition and quotition variations if division problems. The examples use very simple numbers. Consolidation tasks have larger numbers and contain more mathematical information to sort through.

Lesson 4: Addition of Fractions

In this lesson, students learn how to use bar model as a tool to represent a variety of worded addition problems involving fractions, by studying worked examples and practising with further tasks. They encounter situations where the bars that make up a bar model may simultaneously represent an absolute number (e.g. $50) and a fraction of a specified quantity (quarter of a cost).

Lesson 5: Subtraction of Fractions

Students learn how the part-whole bar models can represent subtraction contexts involving fractions. Students study examples and practice with further tasks, individually or in groups. The bar models provide support for intuitive methods of solving these problems, which supports the development of fraction concepts and skills.

Lesson 6: Comparison Model for Addition

Students are introduced to a new type of bar model, the comparison model, through two very simple examples. Problems, for individual or group solving, involve quantities with known differences (e.g. one quantity is 10 more or less than another; one quantity is one fifth larger than another). Drawing the bar model supports an intuitive approach to the fraction calculations involved.

Lesson 7: Comparison Model for Subtraction

This lesson introduces the comparison model for subtraction problems. The examples use only whole numbers. In a comparison context, variation comes from changing which of the three relevant quantities (larger, smaller and difference) are known or to be found, and by expressing the comparison using different language (e.g. fewer than, increased to). Consolidation problems use larger numbers and contain more information.

Lesson 8: Comparison Model for Multiplication

Through this lesson, students learn how the comparison bar model can display the mathematical relationships in problems involving multiplication and division. The calculations only involve whole numbers. Examples demonstrate bar models for problems solved by multiplying and problems of two visually different types solved by division. Students then complete tasks, individually or in groups, to consolidate and extend their learning.

Reflection on this unit

Rationale

This unit aims to build students' capacity to use bar models as a flexible problem-solving tool.

It adopts several **spiral curriculum progression** features. The unit begins by introducing the part-whole model from Lessons 1 to 3 systematically visiting the four arithmetic operations. Subsequently, the model is revisited in Lessons 4 and 5, but in greater depth as it applies to fractions. Subsequently, Lessons 6 to 8 introduce the comparison model, mainly for whole numbers, to expand students' repertoire and develop flexibility. The gradual introduction of the bar model allows students to develop firm foundations before moving to more complex problems.

Mathematical content knowledge in every lesson is also sequenced carefully within the unit such that the complexity of use of the bar model is complemented by deeper content-based mathematical thinking as the unit progresses. There is systematic variation of the problem types to which the bar model is applied

reSolve Mathematics is Purposeful

The bar model method is offered as an effective strategy to help students reduce their cognitive load during problem solving. It is a tool to assist students in solving problems by organising information and visualising relationships.

All the lessons contain clear animated solutions of the problems so students can learn to use the model. Practice tasks are included to consolidate knowledge Furthermore, teachers can discuss different approaches to these problems, drawing upon the bar model as an intermediary visual tool for summarising mathematical relationships between key quantities.

reSolve Tasks are Challenging Yet Accessible

A tool for visualisation is especially valuable when students come from a variety of language backgrounds. The bar model method is a pillar of mathematics education in Singapore, as a practical aid to support students' problem solving. It is especially valued because there are multiple home languages spoken yet education is in English.

The tasks are increasingly challenging both within each lesson and across the unit. However, the bar models make the problems accessible. For example, dividing the length of a bar into parts to show both halves and thirds, supports the otherwise abstract procedure of finding a common denominator. Prompts in each lesson are designed to scaffold students' critical thinking about the relationships between the variables in each question and the best use of the two types of bar models being studied.

reSolve Classrooms Have a Knowledge Building Culture

The unit is carefully designed to encourages students to draw links between prior knowledge and new learning, specifically in terms of model drawing and mathematical content. Students can share and critique various ways of setting up bar models that make finding a solution easy. There are opportunities for building success through collaborative inquiry if students work together to unpack the word problem and present differing approaches to represent key quantities in various bar models.

Further Reading

Ministry of Education [MOE]. (2009). *The Singapore model method*. Singapore: EPB Pan Pacific.

Ng, C. H. J., & Lim, K. H. (2006). *A handbook for teachers in mathematics*. Singapore: Marshall Cavendish Education.

Polya, G. (1945). *How to solve it*. Garden City, New York: Doubleday.

Yeap, B. H. (2011). *Bar modelling - A problem-solving tool: From research to practice (an effective Singapore math strategy)*. Singapore: Marshall Cavendish Education.