Unit Overview: Quadratic Functions

Summary of learning goals

As part of the Special Topic **Bringing the Real World into Algebra** this unit looks at linking the different representations of quadratic functions, i.e. quadratics as seen in the real world, represented by a function rule, represented by a graph or represented by a table of values. Special emphasis is given to the value of considering the function rule arranged in "turning point" form.

All units in the Special Topic: Bringing the Real World in to Algebra are guided by the principles of Realistic Mathematics Education (RME). As a consequence, this unit:

- Uses realistic situations to develop mathematics.
- Places less emphasis on memorising and more on describing patterns and creating function rules.
- Places more emphasis on sense making.
- Uses "guided reinvention".

Students will have the opportunity to consider real world objects and actions that can be described using quadratic functions. Through technology assisted exploration these examples will become models for understanding transformations, reflections and stretches to parabolas. Students will learn to identify, and where appropriate interpret, critical points (turning point, x-intercepts, y-intercepts) from the graph and from the function rule.

Australian Curriculum: Mathematics (Year 10)

ACMNA239: Explore the connection between algebraic and graphical representation in relation to simple quadratics … using digital technology as appropriate.

- Sketching graphs of parabolas.
- Applying transformations, reflections and stretches to parabolas.

Summary of lessons

Who is this unit for?

Year 10: enhancing a standard approach to teaching quadratics or for revision. The lessons could be inserted as the various forms of the function rule are encountered. Alternatively, the lessons could be used to re-sequence the teaching of quadratics by starting with "turning point form" and, through guided discovery, allow the students to build schematic links between the various representations. Since the lessons may introduce or consolidate students' conceptual links between various representations of quadratic functions these lessons could also be used for revision at Year 10 or revisiting the topic in a fresh and engaging manner at Year 11.

Lesson 1: Fitting Curves

Students are introduced to GeoGebra (or alternative software), by fitting lines to a digital image. This is extended to curve fitting using the turning point form for quadratic functions. Students make systematic adjustments to the function rule to fit the curve to an image of water from a hose or the path of a ball.

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Lesson 2: Trajectories

Students take a set (burst) of photos of the path of a ball, and fit a parabola to points from the images by systematically altering the parameters of the quadratic function. The usefulness of the turning point form (highest point reached) and of the factorised form (distance travelled) are explored in this context.

Lesson 3: Linking Different Forms

Students compare different symbolic forms for the same quadratic function using GeoGebra, and then by hand sketching. Further practice using text book type examples is suggested for fluency.

Reflection on this sequence

Rationale

In teaching or revising quadratic functions by starting in the real world we aim to link visual, graphical, symbolic and numerical representations of functions in a way that will help students build strong and memorable conceptual schema. We aim to guide them through activities that give purpose to the various conventional symbolic arrangements of a quadratic function rule. Using digital images to link to the real world can engage students, especially if they have some ownership of the photos. Then, with the aid of technology, the tasks of exploring links between the parameters in a function rule and transformations (reflection, dilation, translations) of a graph can be made accessible to students across a wide range of mathematical ability.

Quadratics have typically been taught by starting with $y=x^2$ and progressively introducing $y=ax^2+bx+c$ (perhaps noting the impact of "a" on dilation or reflections and "c" as the y-intercept) then $y=(x+a)(x+b)$ or $y=(x-a)(x-b)$ (noting zeros or x-intercepts) and finally "completing the square" method to obtain "turning point form" $y=a(x-g)^2+h$,by which stage many students have switched off. Starting with $y=x^2$ and moving immediately to reflection, translation and dilation of the parabola through strategic choice of the parameters in $y=a(x-g)^2+h$ to achieve curve fitting gives purpose to this form of the function rule and, with the support of technology as necessary, all students can manipulate the rule to move to the expanded or factorised form.

The range of real world applications for quadratics is limited. There are some examples in architecture and design, such as the classic McDonalds arches and some bridges that may be artificially modelled by a quadratic curve. Trajectories of footballs, basket balls, cricket balls, javelins, discus, cannon balls, missiles, water from a hose or fountain etc. follow the path of a quadratic for a scientific reason. This can be extended to modelling BMX or skateboard jumping, or juggling. The side profile of a satellite dish or radio telescope will be parabolic and you can usually see a suitable image on the Australian National Telescope website or by looking at one of their webcams (http://www.atnf.csiro.au/).

If we can help our students understand and engage with the algebra and graphs of quadratic functions through even a few real examples then it is a small step to help them see that the same mathematical principles might be applied to use algebra to model far more complicated real world shapes and paths.

reSolve Mathematics is Purposeful

Students commonly wonder why the curriculum requires them to consider different representations of quadratics and especially why they should bother with more than one arrangement of the function rule. This lesson series on quadratics aims to provide experiences for students that help them to recognise the value of alternative representations of quadratic functions and in particular the role of each equivalent symbolic rules in providing different, useful information for describing real world phenomena.

While some aspect of the tasks are artificial, experience has shown that students enjoy the challenge of curve fitting and so are motivated to pay attention to the impact of varying each parameter of the quadratic function rule.

reSolve Tasks are Challenging Yet Accessible

reSolve contests a view that some students can "do" mathematics and others cannot. By working with appropriate software all students are able to participate in the activities of curve fitting and learn through the experience of guided experimentation. Activities can be scaffolded to different degrees for different students.

Accessibility to the concepts encapsulated by quadratic functions arises from the use of the real world situation to develop a firm mental model. Transition from relying fully on the real world situation, to using the somewhat more abstract graphs, then algebra occurs at a pace set by the student. The use of several real world situations throughout the sequence provides an opportunity to reinforce the new ideas, as well as extend them.

reSolve Classrooms Have a Knowledge Building Culture

This unit builds knowledge through active exploration of different representations of quadratics and in particular requires students to work to achieve the best models they can by varying the parameters of the quadratic function rule. The process of working through the process of strategic trial and error requires students to keep trying and to reflect, at each stage, on the impact each change they make to the algebraic rule has on the graphic representation and, in some examples, the real world interpretation of that result. The use of software means that errors may be quickly corrected or even erased. There need be no embarrassment!

Further Reading

Read more about RME: http://e-library.math4teaching.com/what-is-realistic-mathematics-education

Acknowledgements

Sections of this unit draw on examples from the RITEMATHS project. These lessons can still be accessed from:

https://extranet.education.unimelb.edu.au/DSME/RITEMATHS/