re(Solve) MATHS BY INQUIRY

MECHANICAL LINKAGES and DEDUCTIVE GEOMETRY

Unit Overview: Proof

Summary of learning goals

As part of the Special Topic **Mechanical Linkages and Deductive Geometry** this unit looks at geometry involved in the everyday tools and objects. The lessons aim to show students that there is mathematics in the world around them. Five of these linkages were invented by famous mathematicians, to solve important problems of their time. With the aim of physical models and computer simulation, the lessons move from a view of geometry as a study static diagrams to encompass movement.

The main goal is to give students experience of deductive reasoning and a reason to engage in it, and to assist them to move towards developing their own proofs. There is an opportunity to use geometric facts from all year levels.

Australian Curriculum: Mathematics (Year 10)

ACMMG243: Formulate proofs involving congruent triangles and angle properties.

- Applying an understanding of relationships to deduce properties of geometric figures (for example the base angles of an isosceles triangle are equal).

ACMMG244: Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes.

- Performing a sequence of steps to determine an unknown angle giving a justification in moving from one step to the next.
- communicating a proof using a sequence of logically connected statements.

ACMMG272: Prove and apply angle and chord properties of circles.

- Performing a sequence of steps to determine an unknown angle or length in a diagram involving a circle, or circles, giving a justification in moving from one step to the next.

Summary of lessons

Who is this unit for?

The unit is for students who are learning about proof and how to develop logical sequences of reasoning with some degree of rigour. Some lessons involve circle geometry; all draw on content from earlier year levels. The lessons stand alone, and teachers may select any combination. The lessons are approximately ordered by increasing challenge, with the later lessons providing challenges for strong students undertaking Australian Curriculum Mathematics 10A.

Lesson 1: Chebycheff's Linkage

Students investigate Chebycheff's linkage and determine whether it produces approximate or exact linear motion from circular motion. They make a physical model, observe the motion of points and try to explain them, and then observe a computer simulation. Visually, the motion appears as a very convincing straight line, so students are misled until measurements expose the small variation from linear motion. This establishes awareness of the need for proof in other geometry contexts. Students can also use Pythagoras' theorem to establish some lengths.

Lesson 2: Scott Russell Linkage

Students investigate the design and operation of a car jack based on the Scott Russell linkage. As the horizontal screw is turned, the car attachment point moves vertically (perpendicular to the screw). Students make a physical model and operate a computer simulation. Students then use the geometry of the linkage to find if the motion is really vertical. This involves reasoning about angles in three connected triangles.

Lesson 3: Pascal's Angle Machine

Students investigate the design and operation of an angle trisector invented by mathematician Blaise Pascal. The lesson has been deliberately named *Pascal's angle machine* rather than *Pascal's angle trisector*, so that students first explore the angle machine to find out what its purpose is, and then how it would be used. They use physical models and computer simulation to explore. Students then use the geometry of isosceles triangles and exterior angles to prove why the machine works.

Lesson 4: Consul

Students construct a physical model of Consul, a toy calculator, and use a computer simulation of Consul to explore how the geometric design enables the product of numbers between 2 and 12 to be displayed. Students may then develop a proof based on a sequence of deductive reasoning.

Lesson 5: Sylvester's Pantograph

Students construct a physical model of Sylvester's pantograph; a drawing instrument designed for copying drawings. They explore how the copied image compares with the original drawing and use a computer simulation of the pantograph to explore how the geometric design of the pantograph allows the pantograph to work. Students then develop a geometric proof based on a sequence of deductive reasoning in which they use their knowledge of rhombus properties and congruent triangles.

Lesson 6: Peaucellier's Linkage

Peaucellier's linkage converts circular motion to linear motion. Students explore how the linkage moves by constructing a physical model and using a computer simulation of the linkage. They discover how the geometric design of the linkage allows it to produce exact linear motion.

Reflection on this sequence

Rationale

When devising these tasks in deductive reasoning in geometry, the guiding principles have been:

- Providing rich visual imagery, both static and dynamic.
- Providing an opportunity for students to use the language of geometry.
- Providing meaningful context that can motivate argumentation and conjecturing.
- Highlighting the need for deductive reasoning as an answer to the question 'why'.
- Providing links with other STEM subjects, in particular, engineering and technology.
- Providing links with history through historical inventions.

A strong feature of these lessons is that students are able to see and manipulate the linkages. Students operate the real linkage whenever possible, they make physical models out of plastic strips or light card, and they use pre-prepared dynamic geometry computer simulations.

The tactile experience of operating the actual tool or a physical model of the linkage provides an instant sense of satisfaction and gives insight into the way in which the linkage moves. The provided dynamic geometry computer simulations show the geometry more clearly, and enable accurate measurements to be made. Students can observe what stays the same and what varies as the dynamic geometry models are operated.

reSolve Mathematics is Purposeful

Lessons in this unit show mathematics as both a way of modelling the real world and as an abstract discipline. The linkages are used in practical everyday tools, and analysis of the tools leads to conjecture, argumentation and proof. There are clear links to STEM, including engineering.

reSolve Tasks are Challenging Yet Accessible

For many students, motivation for this unit will come from interest in the real world contexts. Using the physical and computer models assists students to visualise the motion and they can use them to develop and test conjectures. The expectations for the deductive reasoning can be moderated by the teacher, and structured approaches to support are offered.

reSolve Classrooms Have a Knowledge Building Culture

Students work together to build models, make and test conjectures and develop and test argumentation.