CIRCLE AREA

Lesson 3: Round Peg in a Square Hole

Australian Curriculum: Mathematics (Year 8)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Lesson abstract

The lesson consists of a single investigation: which is the better fit - a round peg in a square hole or a square peg in a round hole? The investigation here is somewhat structured to support students, but could be presented in a more open way if desired. Students work together on understanding the problem and formulating the problem mathematically and then solving it. The task may be used for assessment.

Mathematical purpose (for students)

An opportunity to improve problem solving skills and use circle formulas to answer an intriguing question.

Mathematical purpose (for teachers)

This lesson poses a problem. The processes involved in its solution include: drawing 2-D diagrams of a 3-D situation; deciding how to model (and hence compare) 'fit' mathematically; devising a strategy to solve the problem; and either generalising from numerical cases or using algebra. Solutions use area formulas and also revisit the geometric arguments of earlier lessons. Problem solving involves uncertainty, which can generate stress for students. This lesson offers support to help students to work through this.

Lesson Length — 45-60 minutes approximately

Vocabulary Encountered

- sector
- generalise

Lesson Materials

- scientific calculator
- Student Sheet – Round Peg in a Square Hole (1 per student)
- Teacher Sheet – Round Peg in a Square Hole

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Getting Started

Today's challenge is about a round peg in a square hole. The phrase 'a round peg in a square hole' is usually meant metaphorically, but in this lesson it is used literally.

Students should discuss where someone might feel like 'a round peg in a square hole'. What does the phrase mean?

Expected Student Response

Responses are likely to be stories about situations where the students feel that they are in the wrong place, or doing an activity for which they are unsuitable or do not fit in with the people around them.

Dictionary.com (http://www.dictionary.com) defines a square peg in a round hole as "a misfit, especially a person unsuited for a position or activity".

The Task

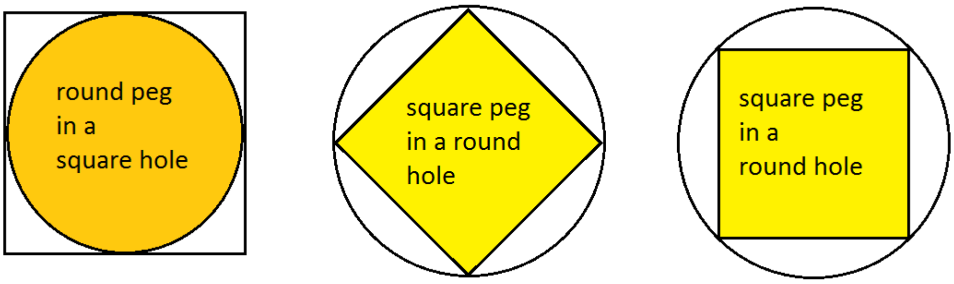
Suggested Strategy

The problem requires careful thinking, and is suitable for working in small, mixed ability groups. Students will benefit from multiple viewpoints and from justifying their thinking to others. Student Sheet – Round Peg in a Square Hole has instructions for students (also in the slide show).

Teacher Notes

1. Students decide on and draw the best diagrams for a well-fitting 'round peg in a square hole' and 'square peg in a round hole'. Check each group's diagrams before the next step.

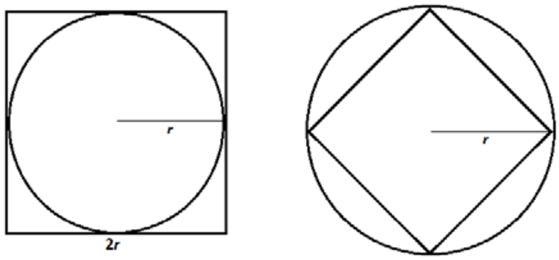
   - Diagrams should have the inside circle or square touching the outside shape at four points.
   - The orientation of the square in the round hole could vary.
   - Students need to move from the 3-D situation to seeing that the cross section is what is relevant.



2. Students discuss what 'best fit' means, and decide what quantities will be needed in order to do the comparison.

   - Deciding on the criterion for 'best fit' is the critical 'mathematical modelling' step of this problem and will need discussion. What really is the right quantity to compare? One criterion could be that the best fit is touching more of the sides of the hole or a tighter fit. In this lesson, we look for the one that fills up the hole the most completely. This can be measured by the proportion of the area of the hole that is blocked by the peg. If the hole was the neck of a bottle, the peg that covered the most area would slow the flow of liquid the most.
   - Students will need the side length of each square and the radius of each circle. Note that for each of the holes, the side length of the square is determined by the chosen radius of the circle, or vice versa.

- The least difficult calculations come from selecting lengths for the radii of the circle (not the side length of the square). This is more likely to be noticed if the diagonal of the square is shown as the horizontal diameter of the circle, as shown below.

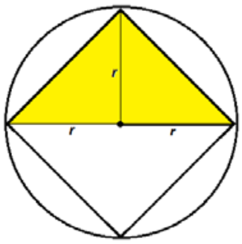


3. Students choose a value for the radius of the circle or the side length of the square in each diagram to begin.

   - It is good practice to look at a specific case before attempting to generalise. It is sensible to choose simple numbers first (e.g. both circles have radius 1 cm).

4. Students calculate the quantities that they have decided to compare.

   - One method of calculating the area of the inner square uses twice the area of the triangle coloured yellow below. This method was used in lesson 1. It avoids the need for Pythagoras' Theorem.



5. Students discuss how they will judge which fit is better.

   - Possible ways include:
     - Expressing the area of the inner shape as a percentage or fraction of the area of the outer shape for both 'round peg in a square hole' and 'square peg in a round hole'.
     - Calculating the area of unfilled space in each hole as a percentage or fraction of the whole area.
   - It is likely that students will want to directly compare the areas rather than using a percentage or fraction of the hole. This will make it difficult for students to decide which fit is better, especially when additional starting values are tested.

6. Students decide which is the better fit, given their chosen starting values.

   - Expect either answer depending on comparison method and values chosen.
   - Teacher Sheet – Round Peg in a Square Hole has an algebraic solution showing that a round peg in a square hole is a better fit.

7. Students investigate if this arrangement fits better all the time or only sometimes. Some students will try holes of other sizes, while others may use algebra to find a general result.

   - Most students will work with numerical examples, choosing the size of the holes or pegs. Encourage these students to observe the general result, and move to algebra to demonstrate it.
   - The key solution step is to decide to measure 'fit' as the percentage of the area of the hole that is filled.
   - Students working numerically may be unsure what pairs of sizes of round and square holes to compare. In fact, the percentage of area covered does not depend on the size of the holes, so students' choices do not matter.

Round Peg in a Square Hole

Name: _______________

1. Decide on and draw good diagrams for a well-fitting 'round peg in a square hole' and a 'square peg in a round hole'. Before you go on to the next part ask your teacher to check your diagrams.

2. Discuss what 'best fit' means, and decide what quantities you will need in order to do the comparison.

3. Choose a value for the radius of the circle or the side length of the square in each diagram to begin the investigation.

4. Calculate the quantities that you need to compare.

5. How will you judge which fit is better?

6. Decide which is the better fit, given your starting values.

7. Investigate if this arrangement fits better all the time or only sometimes. You could:
   - Try other starting values.
   - Use pronumerals for the size of the holes and use algebra to find expressions for the quantities that you need to compare.

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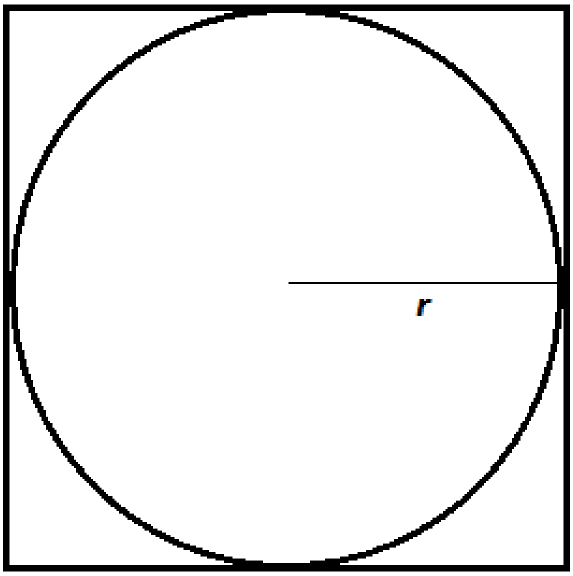
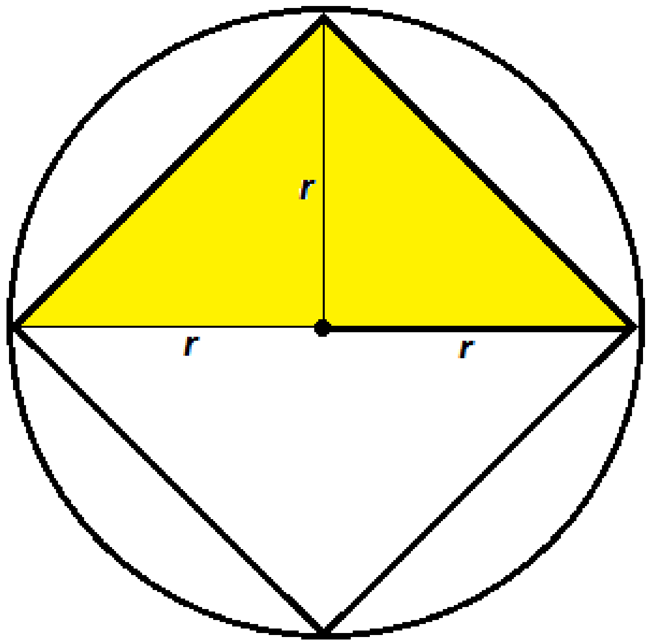
Teacher Sheet - Round Peg in a Square Hole





For the round peg in a square hole:

Hole

$A_{square} = 2r \times 2r$

$= 4r^2$

Peg

$A_{circle} = \pi r^2$

Fraction of the hole filled by the peg

$= \frac{\pi r^2}{4r^2}$

$= \frac{\pi}{4}$

$\cong 0.79$

Approximately 79% of the hole is filled by the peg.

Note this does not depend on the size of the hole.

For the square peg in a round hole:

Hole

$A_{circle} = \pi r^2$

Peg

$A_{square} = 2 \times A_{triangle}$

$= 2 \times (\frac{1}{2} \times 2r \times r)$

$= 2r^2$

Fraction of the hole filled by the peg

$= \frac{2r^2}{\pi r^2}$

$= \frac{2}{\pi}$

$\cong 0.64$

Approximately 64% of the hole is filled by the peg.

Note this does not depend on the size of the hole.

CONCLUSION: a round peg in a square hole is a better fit than a square peg in a round hole.

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