PRIME FACTORISATION

Lesson 1: Factor Strings

Australian Curriculum: Mathematics – Year 7

ACMNA149: Investigate index notation and represent whole numbers as products of powers of prime numbers.

- Applying knowledge of factors to strategies for expressing whole numbers as products of powers of prime factors, such as repeated division by prime factors or creating factor trees.

Lesson abstract

This lesson introduces prime factorisation. Students search for factor strings using the context of a puzzle and then justify that the longest factor string for a number is made up of primes. This introduces *The Fundamental Theorem of Arithmetic:* that all whole numbers greater than 1 can be represented as a product of prime numbers in exactly one way. Students then create similar puzzles and participate in a short class challenge to look at why prime factorisation is used to encrypt information.

Mathematical purpose (for students)

To find strings of factors that make up a composite number and investigate how different strings are related to each other.

Mathematical purpose (for teachers)

This lesson introduces *The Fundamental Theorem of Arithmetic:* that all whole numbers greater than 1 can be represented as a product of prime numbers in exactly one way. It provides experience in writing numbers as products of prime numbers and in other ways, including with index notation.

Lesson Length 1 hour approximately

Vocabulary Encountered

- factor
- prime
- composite

Lesson Materials

- Student Sheet – Factor String Puzzle (1 per student)
- Teacher Sheet – Factor Strings for 480

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Factor String Puzzle

15	84	2	6	92	3	80	51
5	32	20	24	2	180	2	4
96	2	9	2	4	60	5	120
12	32	2	44	12	40	5	2
8	5	6	3	8	48	15	16
48	29	80	5	10	5	2	3
3	16	3	4	2	2	8	2
4	204	10	4	16	10	51	2

Students will search the puzzle above for strings of factors that multiply to give 480. The strings can connect numbers horizontally, vertically and diagonally. Students will look for the longest factor string possible.

Introduce the Inquiry

Hand out Student Sheet – Factor String Puzzle. Allow students time to explore the puzzle. This can be facilitated by asking questions such as:

- *What is the largest number you can make using a string of two factors?*
- *What is the smallest number you can make using a string of two factors?*
- *What are the smallest and largest numbers you can make using a string of three or four factors?*
- *Can you find a string of numbers that make 48?*
- *Can you find a string of numbers for 60? What about 120?*

Participate in the inquiry

Ask students to search for strings of numbers that multiply to give 480. Strings of varying lengths can be found within the puzzle.

Students connect and record the different factor strings that they find. Encourage them to consider an appropriate way to collect and organize their strings. Some of these methods can be shared with the class. Create a table on the board so that students can come and record the strings that they have found.

Enabling Prompt

- Ask students to look for factor strings for 240, 120, 60 or 48. As these are factors of 480, some of the factor strings for these numbers also appear in the puzzle. The prime factor string for all these numbers can be found in the prime factor string for 480. Ask students: How do the factor strings for these numbers help in finding factor strings for 480?

The following questions can be used in conversation with students as they work:

Is there a relationship between the strings for 480 that are 3 factors long and 4 factors long? Can you use a string of 3 factors to make a string of 5 for 480?

- Factorising numbers already in a string enables the length of the string to be increased. For example, a string that is 48 x 5 x 2 starting in row 5 of column 1 can be made into a 4 factor string by using 48 = 6 x 8. The 4 factor string is therefore 6 x 8 x 5 x 2, starting in row 4 of column 1.

Could we use a string of 4 factors to make a string of 3 factors?

- Reversing the process of the previous question's answer shows how shorter strings can be found. The marked 4 factor string of 5 x 2 x 16 x 3 can be made into a 3 factor string by multiplying the 3 x 5 to make the string 15 x 16 x 2.

What is interesting about the numbers that make up the longer strings?

- Students will notice that the numbers get smaller and are often repeated (e.g., there are multiple 2s). The most important point is that the number of prime numbers in the string increases.

What numbers in the grid cannot be used in a factor string for 480? Why not?

- Numbers that are not factors cannot be used. Prime factorisation can be used to find numbers that are factors of 480 and those that are not. For example, 46 has a prime factor of 23 which is not a prime factor of 480. 102 and 204 each have a prime factor of 17 which is not a factor of 480. 18 has two 3s as prime factors whereas 480 only has one 3.

Ask if anyone found a really long factor string. Ask if anyone found another one of that length.

Are these two strings different or are they the same?

- The prime factor string for 480 is 2 x 2 x 2 x 2 x 2 x 3 x 5 which can be written as 2^5 x 3 x 5. This can be found in two places in the puzzle. The numbers occur in different orders but they are the same prime factorisation.

15	84	2	6	92	3	80	46
5	32	20	24	2	180	2	4
96	2	9	2	4	60	5	120
12	32	2	44	12	40	5	2
8	5	6	3	8	48	15	16
48	102	80	5	10	5	2	3
3	16	3	4	2	2	8	2
4	204	10	4	16	10	18	2

Using these two factor strings students can be provided with the following challenges:

Is it possible to find a longer factor string than this one? Why, or why not?

Is it possible to find another string of length equal to this one? Why, or why not?

This introduces *The Fundamental Theorem of Arithmetic:* that all whole numbers greater than 1 can be represented as a product of prime numbers in exactly one way.

For example, 480 = 2 x 2 x 2 x 2 x 2 x 3 x 5 or 480 = 2^5 x 3 x 5

This explains why 1 is not considered prime. A whole number can be identified uniquely by its prime factorisation. If 1 were a prime number, this would not be true. Any string of primes could be extended with an unlimited number of 1s.

Enabling Prompt

Students who looked for factor strings for 48, 60, and/or 120 can also look for the longest string possible for these numbers. Using these smaller numbers, they are able to join the class discussion on why the longest string is made up of primes and why it is unique.

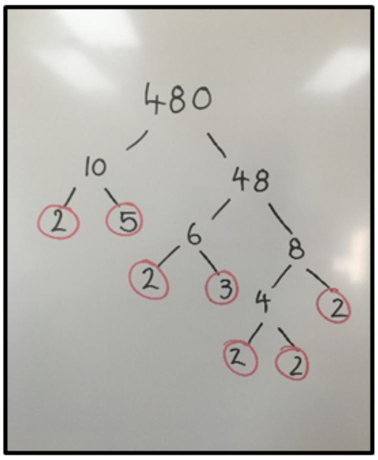
Extending Prompt

What are all the factor pairs for 480? What are all the factors strings that have 3 factors? How do you know you have found them all? What about factor strings that are 4, 5 or 6 numbers long?

Teacher Notes

Two methods for finding prime factors of a number:

Factor tree



Factor ladder

2	480	*480 ÷ 2 = 240*
3	240	*240 ÷ 3 = 80*
2	80	*80 ÷ 2 = 40*
2	40	*40 ÷ 2 = 20*
2	20	*20 ÷ 2 = 10*
2	10	*10 ÷ 2 = 5*
5	5	
	1	

The prime factors of 480 are 2, 3 and 5

480 = 2 x 2 x 2 x 2 x 2 x 3 x 5

480 = 2^5 x 3 x 5

The Maths 300 lesson 104 Factorgrams also provides a good way to display prime factors. Following the factors from the starting number to 1 in any order illustrates visually the Fundamental Theorem of Arithmetic.

Consolidating Tasks

Task 1

Ask students to make a *Factor String Puzzle* that has factor strings for a number of their choosing. The number needs to contain several factors. Encourage students to choose a number that has larger prime factors such as 7, 31 or 113. The puzzles can then be solved by other members of the class.

Task 2

Prime factorisation is used today in the encrypting of data, including our financial information in banks.

A possible way to look at this as a powerful tool is to have a race in the class. One half of the class is asked to multiply a secret pair of 2-digit primes, such as 23 x 89 = 2047. At the same time the other half of the class is given the product (eg 2047) and is asked to find the prime factorisation. This illustrates that it is much quicker to multiply primes than to find the prime factorisation for a number that contains two or more large prime numbers.

ABC Splash provides a clip that describes this method for encrypting confidential information:
http://splash.abc.net.au/home#!/media/154992/prime-number-keys

Teacher Sheet - Factor Strings for 480

1 Factor	2 Factors	3 Factors	4 Factors	5 Factors	6 Factors	7 Factors
480	2 x 240	**2 x 2 x 120**	**2 x 2 x 2 x 60**	2 x 2 x 2 x 2 x 30	2 x 2 x 2 x 2 x 2 x 15	**2 x 2 x 2 x 2 x 2 x 3 x 5**
	3 x 160	**2 x 3 x 80**	2 x 2 x 3 x 40	**2 x 2 x 2 x 3 x 20**	**2 x 2 x 2 x 2 x 3 x 10**	
	4 x 120	**2 x 4 x 60**	2 x 2 x 4 x 30	**2 x 2 x 2 x 4 x 15**	**2 x 2 x 2 x 2 x 5 x 6**	
	5 x 96	**2 x 5 x 48**	**2 x 2 x 5 x 24**	**2 x 2 x 2 x 5 x 12**	**2 x 2 x 2 x 3 x 4 x 5**	
	6 x 80	2 x 6 x 40	**2 x 2 x 6 x 20**	2 x 2 x 2 x 6 x 10		
	8 x 60	2 x 8 x 30	**2 x 2 x 8 x 15**	**2 x 2 x 3 x 4 x 10**		
	10 x 48	2 x 10 x 24	2 x 2 x 10 x 12	**2 x 2 x 3 x 5 x 8**		
	12 x 40	**2 x 12 x 20**	2 x 3 x 4 x 20	**2 x 2 x 4 x 5 x 6**		
	15 x 32	**2 x 15 x 16**	**2 x 3 x 5 x 16**			
	16 x 30	3 x 4 x 40	**2 x 3 x 8 x 10**			
	20 x 24	3 x 5 x 32	**2 x 4 x 4 x 16**			
		3 x 8 x 20	2 x 4 x 5 x 12			
		3 x 10 x 16	2 x 4 x 6 x 10			
		4 x 4 x 30	**2 x 6 x 8 x 5**			
		4 x 5 x 24	3 x 4 x 2 x 20			
		4 x 6 x 20	**3 x 4 x 4 x 10**			
		4 x 8 x 15	**3 x 4 x 5 x 8**			
		4 x 10 x 12	**4 x 4 x 5 x 6**			
		5 x 6 x 16				
		5 x 8 x 12				
		6 x 8 x 10				

Note: Not all these strings appear in the puzzle: only those in bold.

What factor does not appear in the above table?

- 1 does not appear as it could appear as many times as you like in any of the strings. This is the reason why 1 is not called a prime number or a composite number. It is useful to be able to say how many prime factors a number has.

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re(Solve) MATHS BY INQUIRY

Factor String Puzzle

Name: ______________

15	84	2	6	92	3	80	51
5	32	20	24	2	180	2	4
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3	16	3	4	2	2	8	2
4	204	10	4	16	10	51	2

Search the puzzle above for strings of factors that multiply to give 480. The strings can connect numbers horizontally, vertically and diagonally. Write them here. Two are done for you.

15 × 32 = 480

5 × 2 × 16 × 3 = 480

What is the longest factor string that you can find?

How do you know when you have found the longest factor string possible?

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