INTRODUCTION

re(Solve) MATHS BY INQUIRY

Sums and Differences of Squares

Y10

Summary of learning goals

- Students build their skills in using the special algebraic results relating to perfect squares and differences of squares. They use algebraic reasoning to justify arithmetic results and make connections to visualisations that involve areas.

Australian Curriculum: Mathematics (Year 10)

ACMNA233: Expand binomial products and factorise monic quadratic expressions using a variety of strategies.

- Using the identities for perfect squares and the difference of squares to factorise quadratic expressions.

Summary of lessons

Who is this sequence for?

- This set of three lessons is designed to deepen and consolidate students' understanding of the algebraic identities $a^2 - b^2 = (a + b)(a - b)$ and $(a + b)^2 = a^2 + 2ab + b^2$. It is assumed that students have already encountered these identities in their initial learning of the expansion and factorisation of binomial products. The lessons may be done in any order.

Lesson 1: Quarter Squares

A historical calculation method is used to show students an alternative method for multiplying two-digit numbers. After exploring and becoming familiar with the method, students use algebraic skills, in particular the binomial expansion of perfect squares, to justify why the method always works. Several sustaining activities, including a visual method for showing the identity used in the multiplication, are included.

Lesson 2: Filling Corners

Students use a visual method for transforming a rectangle into a square by dissecting and rearranging the rectangle, then filling in the missing corner with a square. This introduces a reasonably efficient method for finding pairs of factors and, hence, testing for primality, and the method is trialled using a spreadsheet and Python code. Students use the algebra of the difference of two squares to show why the method works. Some related activities sustain the learning.

Lesson 3: Algebraic Allsorts

Students engage in a range of activities, including visualisation, methods for rapid calculation, and solving word problems that rely on the difference of two squares or the binomial expansion of perfect squares. Several of these activities rely on atypical problem-solving wherein students look at problems holistically (e.g. using unknown values, but *without solving for the unknown values*).

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Australian Academy of Science

Reflection on this sequence

Rationale

Approaching algebra as generalised arithmetic shows students the power of algebra in abstracting number. This focus on algebra as generalised arithmetic is typically under-represented in high school mathematics in favour of more time spent on functions and equations. The first two lessons of this sequence are deliberately designed to have historical and cultural connections in order to provide real-world foundations for this abstract approach.

reSolve mathematics is purposeful

- By providing students with historical and visual interpretations of the same algebraic concepts, students are encouraged to form connections between mathematics as an abstract discipline, as an endeavour that has evolved over time, and as a way of modelling real-world problems.
- This sequence also utilises technology: students construct spreadsheets to test hypotheses and can code their own algorithms (or work with precoded programs) to explore ways of solving the set tasks.

reSolve tasks are inclusive and challenging

- Each task in this sequence begins with an initial shared experience that orients students to the task, allowing all students to come into the lesson on an equal level. The challenge of this sequence rests in requiring students to use previously established algebraic knowledge in new and unusual applications and contexts: atypical visual representations, historical methods originally designed for very particular circumstances, and written problems that explicitly avoid routine approaches.

reSolve classrooms have a knowledge-building culture

- The tasks in this sequence require students to investigate individually and then pool their findings in order to draw conclusions that can be generalised. Rather than setting problems to be solved, each task provides a resource or strategy that students apply to their own questions. Students set their own parameters for their investigation and then justify their findings.