re(Solve) MATHS BY INQUIRY

Prisms and Pyramids

Y7

Summary of learning goals

- This sequence provides an accessible context for students to use simple algebra. Students build their skills in algebra by developing algebraic rules for the numbers of faces, edges and vertices in prisms and pyramids. They make deductions about unknown prisms and pyramids from these rules (which links to equation solving) and then use their algebraic expressions to show that Euler's formula works for all prisms and pyramids. Students also build their spatial skills through construction of pyramids and prisms and learn about a new class of polyhedra: antiprisms.

Australian Curriculum: Mathematics (Year 7)

ACMNA175: Introduce the concept of variables as a way of representing numbers using letters.

ACMNA176: Create algebraic expressions and evaluate them by substituting a given value for each variable.

ACMNA177: Extend and apply the laws and properties of arithmetic to algebraic terms and expressions.

ACMMG161: Draw different views of prisms and solids formed from combinations of prisms.

Summary of lessons

Who is this sequence for?

- These lessons are designed for students who are beginning to learn algebra. Students will need to understand that pronumerals stand for numbers and know the most basic conventions of algebra. The emphasis is on developing algebraic relationships through visualisation, rather than through looking at patterns in tables. For the Euler's formula task, students must also be able to collect like terms. For example, they will need to be confident that $2b \neq b + 2$ and be able to simplify expressions such as $(b + 2) - 2b$. The lesson can involve solving very simple equations.

Lesson 1: Faces, Edges and Vertices

Students determine the numbers of faces, edges and vertices of prisms and pyramids, in terms of the number of sides of the base shape. They use these results to show that Euler's formula holds for all prisms and pyramids.

Lesson 2: Antiprisms

Students learn about a new class of polyhedra called antiprisms. They build on Lesson 1 to determine the numbers of faces, edges and vertices of an antiprism when given the number of sides in the base shape.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Reflection on this sequence

Rationale

A focus on the properties of familiar three-dimensional (3D) objects has been selected as the vehicle for describing relationships with algebra. Students visualise and work with concrete objects, algebraically expressing relationships between the number of sides in the base shape and the numbers of faces, edges and vertices of the 3D object. This provides meaning and purpose for the algebra. Application of students' algebraic rules to Euler's formula could be a first opportunity for them to prove a result using algebra.

reSolve mathematics is purposeful

- These lessons emphasise the links between algebra and the physical world; in this case, 3D objects. Students develop skills in reasoning and communication as they make and justify generalisations, verbally and algebraically. The geometric situation illustrates the meaning of the algebra, and the algebra can describe the geometry and make predictions about it. Students develop a sense of why algebra is useful.

reSolve tasks are inclusive and challenging

- Using constructed prisms and pyramids as a vehicle for developing algebraic understanding provides students with an accessible entry point to generalisation. There is an opportunity to link formal equation-solving processes with intuitive methods.
- Students are challenged to manipulate algebraic variables to find unknowns, and to produce two short algebraic proofs.

reSolve classrooms have a knowledge-building culture

- The sequence is a guided investigation during which students can work together to discover patterns and make deductions. Each task builds on the previous task to help students move towards a greater understanding of what algebra can do.

Acknowledgements

All images of antiprisms used in Lesson 2 are reproduced from the Florida Center for Instructional Technology *ClipArt ETC* Collection; see https://etc.usf.edu/clipart/.