re(Solve) MATHS BY INQUIRY

# reSolve Bakery

Y5

## Summary of learning goals

- This sequence gives students a range of strategies for multidigit multiplication, highlighting strategies based on the distributive property and the associative property.
- The lessons develop the array as a tool for multiplication. Students move in a careful developmental sequence from an array with all items perceived, to a grid array, then on to an open array and area model for multiplication.

### Australian Curriculum: Mathematics (Year 5)

**ACMNA100:** Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies.

**ACMMG109:** Calculate perimeter and area of rectangles using familiar metric units.

**ACMNA098:** Identify and describe factors and multiples of whole numbers and use them to solve problems.

## Summary of lessons

### Who is this sequence for?

- These lessons are designed for students with prior knowledge of multiplication.
- The focus on understanding multiplication and its properties through arrays is intended both to support learning of new multiplication skills and to deepen students' appreciation of rules that they have learned.

### Lesson 1: How Many Cupcakes?

Students calculate the number of cupcakes baked each day in a bakery. The cupcakes are cooked in $6 \times 4$ trays. Students solve the problem using their own strategies and illustrate the strategies using the array. As strategies are shared, students reflect on the efficiency and ease of strategies, as well as their suitability for the context.

### Lesson 2: The Cupcake Order

This lesson generalises repeated doubling strategies. Students find the number of cupcakes in 40 trays by multiplying by the factors of 40. This is a practical use of the associative property.

### Lesson 3: Cupcake Boxes

This lesson uses arrays to build an understanding of the distributive property and the way in which it relates to the multiplication of two-digit numbers. During an investigation comparing the areas of boxes for cupcakes, students observe the convenience of partitioning the array into tens and units, then using the distributive property to multiply. Students use open arrays and an area model for multiplication, and engage with equal group and rate problems.

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# Reflection on this sequence

## Rationale

This sequence of lessons strengthens prior learning about multiplication and prepares students to select and apply appropriate strategies for multidigit multiplication. An overarching goal is to develop the array as a tool for reasoning about multiplication. With this tool they will be able to deepen their understanding of earlier content. For example, they can explain the links between the number facts in the 6 times and the 3 times tables, and generalise these links. Looking forward, students can link partitioning principles to the multiplication of fractions and algebraic expressions.

### reSolve mathematics is purposeful

- Students' flexibility with multiplication is developed.
- Students are required to explain their reasoning about multiplication, using an array.

### reSolve tasks are inclusive and challenging

- Work samples are provided for teachers so that they can identify the level of students' understanding and assist students to move forward.
- The nature of the array model is carefully sequenced, moving from an array with all items perceived to the abstraction of a grid (which still represents all items), and on to the more abstract open arrays and area model for multiplication.

### reSolve classrooms have a knowledge-building culture

- In this sequence there are many opportunities for students to learn from each other. In discussions, students will see work samples presented by other students and hear their reasoning.
- The teacher is encouraged to actively orchestrate discussion and to highlight connections between solution strategies, explore the efficiency of some strategies over others and allow opportunities for students to ask questions.

LESSON 1

# reSolve Bakery

## How Many Cupcakes?

Y5

## About this lesson

Students calculate the number of cupcakes baked each day in a bakery. The cupcakes are cooked in 6 × 4 trays. Students solve the problem using their own strategies and illustrate the strategies using the array. As strategies are shared, students reflect on the efficiency and ease of strategies, as well as their suitability for the context.

## Australian Curriculum: Mathematics (Year 5)

**ACMNA100:** Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies.

## Mathematical purpose

- To build students' flexibility with computation and representation, highlighting the importance of connecting symbols and arrays in mathematics.
- Highlights the associative and distributive properties of multiplication.

## Learning intention

- To use arrays to help us multiply.

### Time

A lesson of approximately 1 hour.
Start with the array number string warm-up presented in reSolve PDF and PowerPoint *1a Bakery Warm-Up*

### Resources

- reSolve PDF and PowerPoint *1a Bakery Warm-Up*
- reSolve PowerPoint *1b reSolve Bakery*
- reSolve PDF *1c Cupcake Array*
- reSolve PDF *1d Busy Day Array*

### Vocabulary

- array
- compensate
- multiplication
- partition
- place value

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## Introducing the inquiry

**Resources:** Introduce the context of a cupcake bakery, using reSolve PowerPoint *1b reSolve Bakery*.

> *Charlie is a baker who has his own cupcake shop. It is a small shop but very popular! Each day he bakes fresh cupcakes to be sold. The cupcakes are baked in a tin that looks like this.*
>
> *How many cupcakes can be baked at one time in this tin? How do you know?*

Allow different responses and different ways of seeing the 24 spaces in the cupcake tray (slide 2).

Show the picture of the cupcake array (slide 3). $8 \times 24$ can be seen in two ways in the picture. There are eight trays with 24 cupcakes each. You can also see eight rows of cupcakes with 24 cupcakes along each row of the array.

> *Each day, eight different flavours of cupcakes are made: one full tray of cupcakes for each flavour. There is one tray of chocolate, one of vanilla and one of red velvet. There is one tray each of strawberry, choc-raspberry and lemon meringue. There is even a tray of rocky road and one of salted caramel.*
>
> *Charlie bakes eight trays of different flavoured cupcakes each day. I would like to know how many cupcakes Charlie bakes each day.*

Do not take answers from students at this point.

Have students solve the problem and present their solution as a poster that shows how they solved the problem.

**Resources:** Students may choose to use a copy of the array (included as reSolve PDF *1c Cupcake Array*) to help explain how their strategy works.

### Enabling prompt:

- Make the problem smaller: consider how many cupcakes are on two trays of 24, then perhaps three, four and eight trays.

### Extending prompt:

- **Resources:** Provide students with copies of reSolve PDF *1d Busy Day Array*. **Ask:** *On a very busy day, Charlie baked 12 trays of cupcakes. How many cupcakes is that altogether?* Students also create a poster of their working.

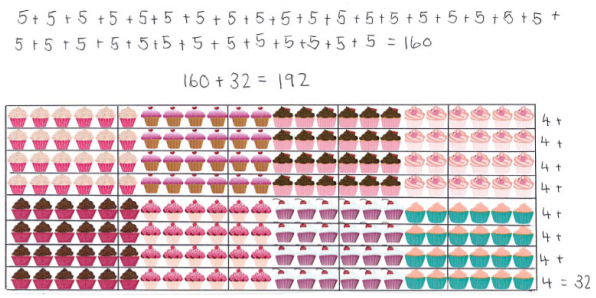
## Noticing students' thinking

Calculating the answer to this problem is well within the capacity of many Year 5 students. The real challenge for students arises when using the array to present an explanation of how a strategy works.

The goal at this point is to not direct students to use particular strategies but to allow students to 'invent' their own way of solving the problem. The strategies used by students will be diverse. They will reveal a lot about their understanding of multiplication and their fluency.

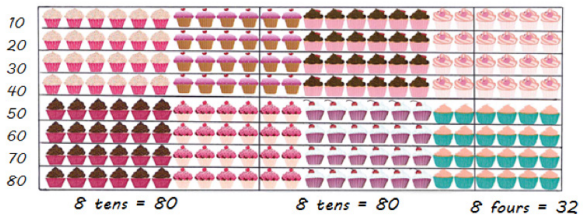
**Counting**

Counting is the most basic strategy.




Link skip counting with repeated addition and then with multiplication.

- *What would be the most efficient way for you to count the cupcakes?*



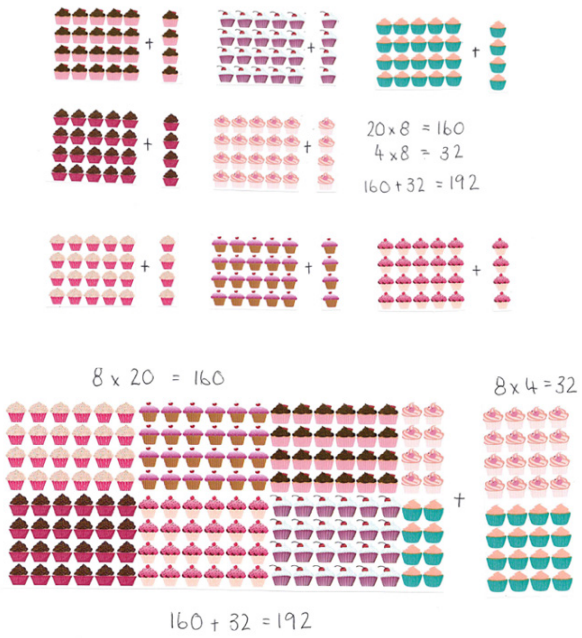

Students using tens can consider the number of tens in each section to reinforce the connection between counting in tens and multiples of 10.

- *In what way might tens help you count?*

**Distributive strategies**

Partitioning the array into smaller sections and then multiplying to find the number of cupcakes in each section applies the distributive property: $a \times (b + c) = (a \times b) + (a \times c)$

$$8 \times (24) = (8 \times 20) + (8 \times 4)$$



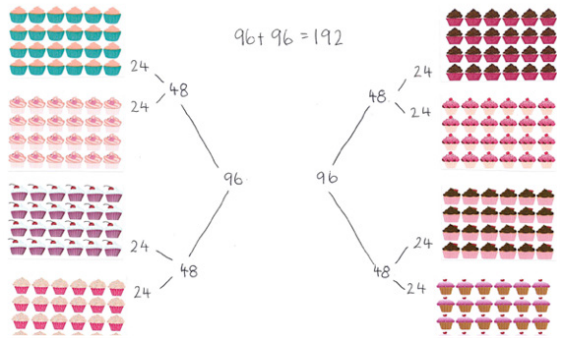


**Compensation**

Making a 'friendlier' number of rows or columns by extending the array and later subtracting is another use of the distributive property of multiplication.

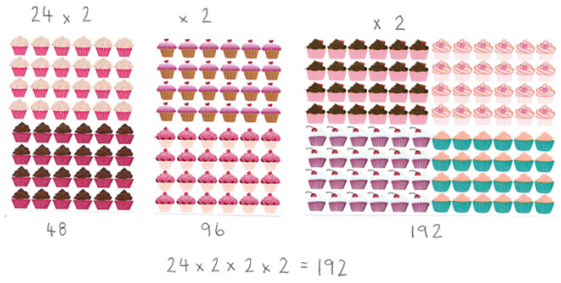
**Doubling strategies**

The following strategies both use repeated doubling. The left-hand sample has doubled using addition (24 to 48 to 96 to 192), which is evident in the use of the array. The right-hand sample uses doubling of the same numbers by multiplication, again represented clearly by the array.




**Repeated doubling using addition**

Repeated doubling using addition is an effective strategy because there are eight groups. It would not be as effective with other numbers of groups.




**Repeated doubling using multiplication**

Repeated doubling using multiplication uses the associative property. To multiply by 8, we can multiply by 2, then 2, then 2 again. The associative property can be used with factors of numbers. As another example, to multiply by 15, I can multiply by 5 and then by 3.

$a \times (b \times c) = (a \times b) \times c$

## Reflection

Select some students to present their work to the class. Key points for discussion include:

- In what ways are the various strategies similar and different? Highlight the connections between strategies.
- How efficient are the different strategies?
- How well would each strategy work with a different set of numbers; for example:
  - Doubling works well with 8 because it is a special number (i.e. $8 = 2 \times 2 \times 2$).
  - Multidigit numbers can be easily partitioned based on its place-value parts (e.g. $24 = 20 + 4$).

LESSON 2

re(Solve) MATHS BY INQUIRY

# reSolve Bakery

## The Cupcake Order

Y5

## About this lesson

This lesson generalises repeated doubling strategies. Students find the number of cupcakes in 40 trays by multiplying by the factors of 40. This is a practical use of the associative property.

## Australian Curriculum: Mathematics (Year 5)

**ACMNA100:** Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies.

## Mathematical purpose

- Multiplying by a number can be done by multiplying by its factors; for example, multiplying by 40 is the same as multiplying by 4 and then by 10. This principle is illustrated by arrays (now in grid form) and revealed in two inquiries: one calculating $24 \times 40$ in various ways and the next calculating $32 \times 25$.
- There are applications to making mental multiplication easy (e.g. the easy way to multiply by 25).

## Learning intention

- Splitting numbers into their factors can help with multiplying.

### Time

A lesson of approximately 1 hour.

### Vocabulary

- associative property
- factors
- multiples

### Resources

- students' work samples from reSolve Bakery Lesson 1: How Many Cupcakes?
- reSolve PowerPoint *2a reSolve Bakery*
- grid paper (at least $40 \times 30$ small squares) (one page per student)
- calculators for checking work and consolidation activity

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## Teacher background information

This lesson looks at the associative property of multiplication: $(a \times b) \times c = a \times (b \times c)$.

In the context of this lesson, we have 24 cupcakes on a tray, four trays of each flavour with 10 different flavours:

$$24 \times 40 = 24 \times (4 \times 10) = (24 \times 4) \times 10 = 96 \times 10$$

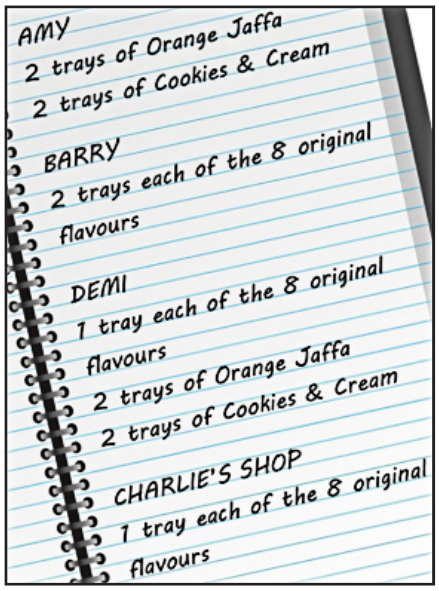
The associative property says that I can either multiply 24 directly by 40 or I can multiply separately by its factors (in this case, 4 then 10). This is a very important understanding. It builds students' fluency with multiplication.

When students come to learn the multiplication algorithm, an understanding of the associative property helps them realise why they 'put down a zero' when multiplying numbers in the tens column. The associative property says that to multiply by a number, we can multiply by its factors in turn. To multiply 24 by 60, we only have to think about multiplying by 6, and then 'put down the zero' to multiply by 10. To illustrate, 24 is multiplied by 63 using an algorithm.

## A cupcake order

**Resources:** Show slide 2 of reSolve PowerPoint *2a reSolve Bakery*, depicting the eight different flavours of cupcakes that Charlie bakes each day.

*Charlie's cupcake shop might only be small but he takes a lot of orders. His cakes are used for school fundraisers and they are also a favourite at birthday celebrations. Today is a big day, as there are a lot of cupcakes to bake!*




Show slide 3.

*Amy has put in a special order. She would like Charlie to bake two special flavours for a very special birthday celebration: orange jaffa and cookies & cream. She would like two trays of each.*

*Barry has ordered two trays of each of the eight original flavours for his school's fete.*

*Demi has ordered cupcakes to serve after a show in the town hall. She has ordered one tray of each of the eight original flavours. She likes the sound of orange jaffa and cookies & cream and so she has ordered two trays of each of these as well.*

*Charlie must also make an extra tray of the eight original flavours to be sold in his shop.*

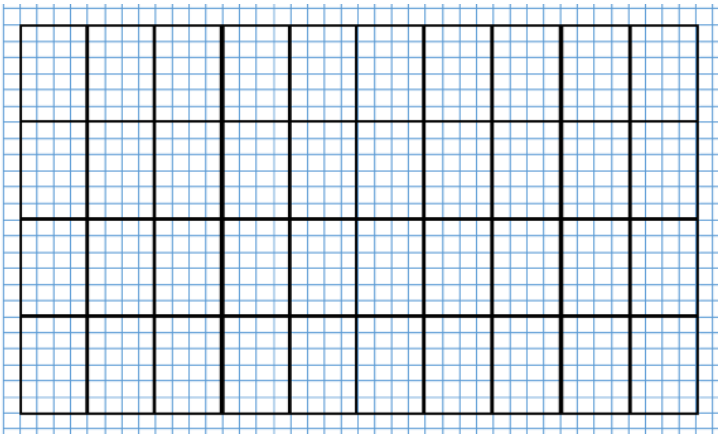
*In total, that is four trays of each flavour! Four trays of 10 flavours.*

*That is 40 trays of cupcakes! 40 trays of 24 cupcakes.*

# How many cupcakes?

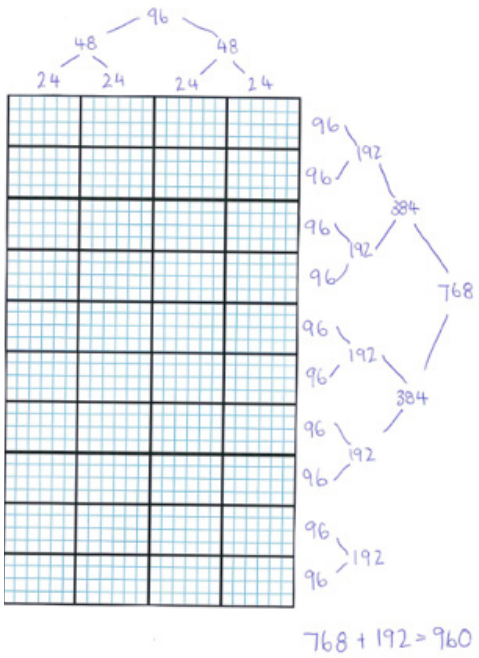
Show the students the problem represented on grid paper (slide 4). Each large rectangle represents one tray of cupcakes. Each small grid square in that rectangle represents an individual cupcake.

Have students solve the problem and include an explanation for how they worked out the answer. Support the investigation as students are working. Take note of the strategies used and confer with students to support and challenge their thinking.



## Noticing students' strategies

**Doubling strategies**

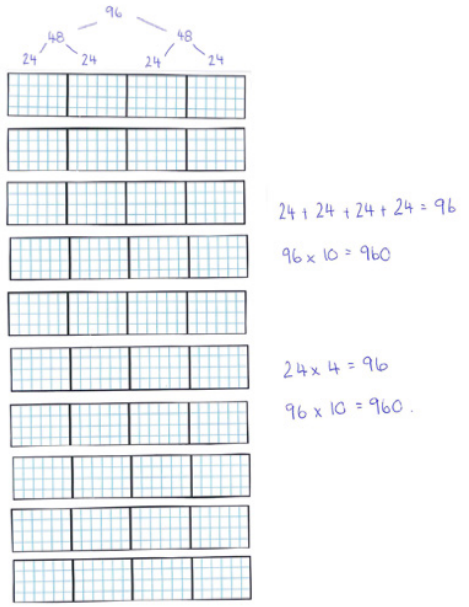



Use doubling to find that there are 96 cupcakes in each row of trays.

What is the difference between doubling in this question and doubling in Lesson 1?

- 8 as the multiplier in Lesson 1 meant 24 could be doubled 3 times. In this lesson, doubling does not work as neatly.

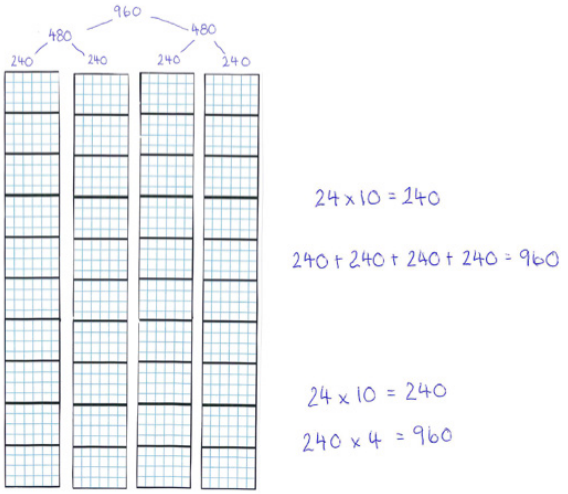
**Multiplying by factors**




Finding the number of cupcakes on 40 trays by multiplying by 10 first and then multiplying by 4.

This is the associative property:

$$24 \times 40 = 24 \times (10 \times 4) = (24 \times 10) \times 4$$




Finding the number of cupcakes for each flavour by multiplying by 4 and then multiplying by 10 flavours.

This is the associative property:

$$24 \times 40 = 24 \times 4 \text{ tens}$$

$$24 \times (4 \text{ tens}) = (24 \times 4) \text{ tens}$$

$$24 \times (4 \times 10) = (24 \times 4) \times 10$$

## Reflection

Discuss the following two strategies: $(24 \times 4) \times 10$ and $(24 \times 10) \times 4$. If any groups have used these as their solution strategies, have them present to the class. The class reflection will use the work sample $(24 \times 4) \times 10$. If this strategy was not used by any students, a work sample has been provided in slide 5.

Record $24 \times 40 = 24 \times (4 \times 10) = (24 \times 4) \times 10 = 96 \times 10$ on the board.

**Ask:** *Is it okay to multiply by 40 by multiplying by its factors like this? Will it change the answer? Does $24 \times 40$ really equal $96 \times 10$?*

Discuss these questions as a class. Introduce the associative property to the students: To multiply by a number, you can multiply by each of its factors in turn. This also works with division.

# Further activities

## Activity 1

Calculate how much Amy, Barry and Demi should be charged for their orders. This provides another context in which to use the associative property.

Boxes of 12 cupcakes at $25 a box:

- 8 boxes for Amy
- 24 boxes for Demi
- 32 boxes for Barry.

## Activity 2

Imagine that you must use a calculator to multiply and divide some numbers, but the 6 and the 5 buttons and the + and − buttons are all broken. Write down the buttons you would press instead to multiply the following numbers.

| | | | |
|---|---|---|---|
| $26 \times 5$ | $4 \times 66$ | $6 \times 17$ | $6 \times 180$ |
| $16 \times 35$ | $36 \times 15$ | $25 \times 18$ | $55 \times 66$ |

LESSON 3

re(Solve) MATHS BY INQUIRY

# reSolve Bakery

## Cupcake Boxes

Y5

## About this lesson

This lesson uses arrays to build an understanding of the distributive property and the way in which it relates to the multiplication of two-digit numbers. During an investigation comparing the areas of boxes for cupcakes, students observe the convenience of partitioning the array into tens and units, then using the distributive property to multiply. Students use open arrays and an area model for multiplication, and engage with equal group and rate problems.

## Australian Curriculum: Mathematics (Year 5)

**ACMNA100:** Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies.

- Applying the distributive law and using arrays to model multiplication and explain calculation strategies.

**ACMMG109:** Calculate perimeter and area of rectangles using familiar metric units.

**ACMNA098:** Identify and describe factors and multiples of whole numbers and use them to solve problems.

## Mathematical purpose

- Students learn to use the array structure and the distributive property to multiply two-digit numbers.

## Learning intention

- To learn how to multiply two-digit numbers using the distributive property.

### Time

A lesson of approximately 1 hour.

### Vocabulary

- distributive property
- partition
- product

### Resources

- reSolve PowerPoint *3a Cupcake Trays*
- student work samples from reSolve Bakery Lesson 1: How Many Cupcakes?
- grid paper with at least 30 × 50 squares (1–2 pages per student)
- butcher's paper (optional) for class table of tray areas

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## Cupcake boxes

**Resources:** Show slide 2 of reSolve PowerPoint *3a Cupcake Trays.*

*Charlie has a box that holds 12 cupcakes and he has a box that holds 10. Inside each box is a flat cardboard tray. The tray fits snugly in the boxes and has circles cut out of it so the cupcakes have a place to sit safely.*

*Charlie is assembling boxes for 10 and 12 cupcakes. He puts the cardboard tray into the box for 10 cupcakes. He notices that the packaging says one side of the tray is 22 cm and the other side is 28 cm. He looks at the cardboard tray for 12 cupcakes. It measures 25 cm on both sides.*

Show slide 3. Students might notice that the sets of given side lengths both add to 50 cm.

*Both sets of side lengths add to 50! Charlie is surprised. Does this mean that both cardboard trays would be the same size? Surely the tray that holds 12 cupcakes would have a bigger area than the tray that holds 10 cupcakes?*

## Calculating the size of trays

Allow students time to work out if the cardboard trays are the same size. Have grid paper available if students would like to use it. Encourage students to explain how they worked out the answer, using a visual representation and symbolic recording.

### Noticing students' working

Students who do not use a grid will often make the mistake of using only two of the partial products. For example, they will calculate $22 \times 28$ as $(20 \times 20) + (2 \times 8)$. Using the array as a visual representation clearly shows that there are four partial products to add together. Encourage them to use the visual representation to see all the parts that need to be added together.

## Reflection

Discuss the finding that the area of the square tray is greater, even though the perimeters of the two cardboard trays are the same. Then select students to present to the class different strategies for finding the areas. Focus on the distributive strategy of partitioning both numbers using place value.

As a class, write a general statement about the distributive property and the way in which it is used in multiplication. The statement can be displayed in the room. For example: **We can multiply two numbers by expressing one, or both, as a sum. Each part of the sum needs to be multiplied by the other number or each part of its sum.**

# Further activities

## Activity 1

**Pose the question:** *Imagine that all the cardboard trays have side lengths that are whole numbers of centimetres and add to 50, like Charlie's two trays. We already know that the area can be 625 $cm^2$ or 616 $cm^2$. What other areas might these trays have?*

This question offers students the opportunity to apply their learning on the distributive and associative properties to multiply two-digit numbers. Encourage the students to record their results in a table, so they can make the following generalisation: **When multiplying pairs of numbers that add to the same total, their product will be largest when the two numbers are equal in value.**