re(Solve) MATHS BY INQUIRY

MATHEMATICAL MODELLING

Pricing for Profit

Lesson 5: Formulating models

Australian Curriculum: Mathematics (Years 7 - 9)

ACMNA174: Investigate and calculate 'best buys', with and without digital technologies (Year 7).

ACMNA176: Create algebraic expressions and evaluate them by substituting a given value for each variable (Year 7)

ACMNA180: Investigate, interpret and analyse graphs from authentic data. (Year 7)

ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. (Year 9)

Lesson abstract

Students focus on how to formulate models. First, students identify variables that are important in making a choice of what coffee to buy. Second, students select relationships between variables that might best model the likely spread of a bushfire. Third, they generate relationships to model various situations, graphically and algebraically.

Mathematical purpose (for students)

Assumptions about data and relationships underpin mathematical models.

Mathematical purpose (for teachers)

This lesson stands aside from the main problem of this unit to focus on the critical formulation phase of modelling, which forms the basis for all other stages. The lesson can be used at any convenient point. Students consider how problems in several contexts may be represented mathematically to solve a problem. They brainstorm possible contributory factors and consider the relationships between them, monitoring how their understanding of the real situation is being represented mathematically. They notice how each emerging model is affected by the contributory factors included, and how it might be made more sophisticated by including more factors. The lesson is organised around three subskills: identifying variables, selecting relationships and generating relationships.

Lesson Length 50 minutes approximately

Vocabulary Encountered

- formulation
- assumption

Lesson Materials

- Slide show: ST7_Pricing_5a.pptx
- Student Sheet 1 – Bushfire (1 per student)
- Student Sheet 2 – Relationships (1 per student)
- Video (optional) http://www.nova.org.au/video/bushfire-modelling
- Spreadsheet access and ST7_Pricing_5b_Bath.xls (optional)

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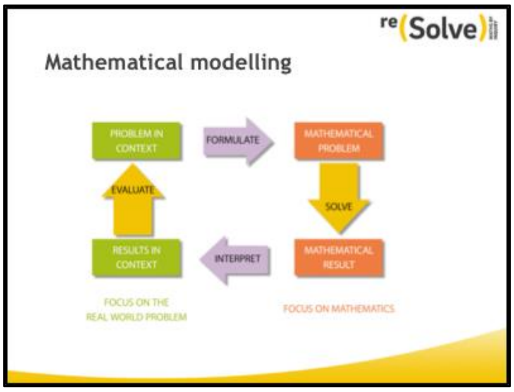
Lesson structure

- Teacher introduction: Focus on formulating in mathematical modelling (10 minutes)
- Teacher introduction/collaborative group work: 3 exercises each 10 minutes (30 minutes)
- Whole class discussion: Summarising key points of lesson (5 minutes)

Focus on formulating

Show the whole class slide Mathematical modelling.



Explain to students that we are going to consider the model formulation stage in detail. That is, when we develop a mathematical model we have to

- choose which factors to include as variables in our model, and
- identify the relationships between them.

This is the **formulation phase** and the product is our **mathematical model**.

Ask students to report some examples of doing this in other lessons.

Expected student responses

We assumed that all of the selling price was profit. We ignored the cost of making and selling the items.

Explain to students that during the rest of the lesson they will work in three different contexts to think about two aspects of setting up mathematical models:

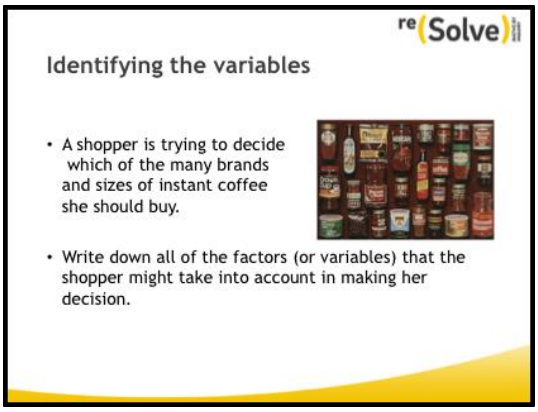
- identifying the important variables, and
- finding the relationships between them.

They will consider how the structure of the real situation affects thinking about the structure of the model.

Identifying the Variables

Organize the class into groups of 2 to 4 students.

Show the slide Identifying the variables.



Present the following situation.

"A shopper is trying to decide which of the many brands and sizes of instant coffee she should buy."

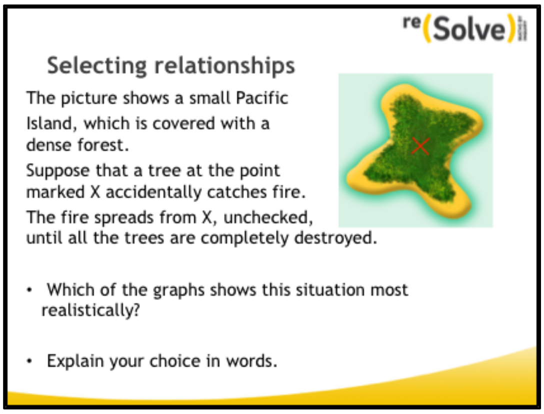
The groups now write down all the factors (or variables) that the shopper might take into account in making her decision.

Expected student responses

- Number of cups of coffee made from the jar
- Price per unit weight of coffee
- Preferences (e.g. taste, brand. strength, type - granulated, powder)
- Maximum total price that shopper wants to pay
- Size of jar to suit requirements (e.g. to make 30 cups for an event; can use up within 3 months)
- Non-coffee advantages (bonuses, containers, etc.)

In class discussion, group students' factors into several major categories (e.g. preferences, cost, non-coffee advantages). Draw out that although many of the factors above are important when selecting which pack of food to buy, we can only handle certain of them mathematically. If the shopper is only looking for the cheapest coffee, then we might decide on either the total cost of the jar, or alternatively finding the price per unit weight (dividing the price by the weight of coffee in the jar). However, more complex models are possible – including setting a maximum dollar outlay, for example, giving a money value to a bonus or the container.

Selecting Relationships



Show students the slide Selecting relationships.

Explain the need to select amongst several possible relationships between variables, the one that is most usefully realistic.

Give a copy of the Student Sheet 1 - Bush Fire to each student.

Ask students to work in their groups to decide which graph shown best describes the situation of the bushfire described on the slide.

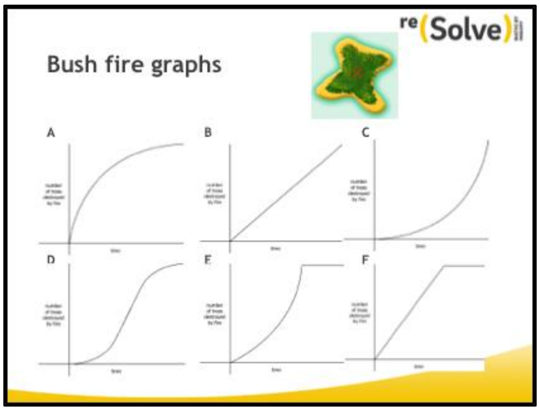
Enabling prompts

- How will the first spread? Which trees with burn first, and later?
- Describe in words to your partner what will happen as the trees in the forest start to burn.
- Describe in words to your partner what each of the graphs shown predicts about the number of trees that are catching on fire.

Expected student responses

- Graph D: shows the fire spreading more slowly at the start then it takes hold, with the number of trees on fire increasing rapidly and reaching a limit as all the trees are burnt.
- Graph A: shows the fire spreading faster at the start then as the first takes hold. This is not likely.
- Graph B: makes no allowance for the gradual increase in the rate at which trees will catch fire as the fire spreads.
- Graph C: this shows that the fire spreads faster as time goes on – this graph might be right if it indicated that the fire cannot spread indefinitely.
- Graph E: this would suit a circular island, the inlets will tend 'smooth out' the extinction of the fire.
- Graph F: (see graphs B, E)

In discussion with the class, draw out their thinking about the structure of what is likely to happen as the trees burn and the structure of the graphs as representations of this.



Use the slide Bushfire graphs to aid your discussions.

Highlight that as a mathematical model of the situation, the graph must capture the happening as best as possible.

Also discuss that students might know mathematical functions that can be used to describe some of the graphs but not others and that is sometimes why we work with a simpler model. If we want answers to real world problems, we have to build it on mathematics that we can do.

Extending prompts

- Draw a graph to show the change if there was a strong wind blowing from the north for the whole time. (Not all the trees will burn; the fire will spread very quickly towards the south, and slowly east and west.)
- Draw a graph to show the change if a strong wind blowing from the north started when the fire had burned a third of the island. (Not all the trees will burn, the trees towards the south will catch fire more quickly)
- Draw a graph to show the change if heavy rain started when the fire had burned a third of the island.
- Draw a graph to show the number of trees burned over time if the fire started at a corner of the island.

Predicting the spread and intensity of bushfires is very important in Australia, to save lives and property and to decide on control burns. The video http://www.nova.org.au/video/bushfire-modelling provides a 2 minute introduction.

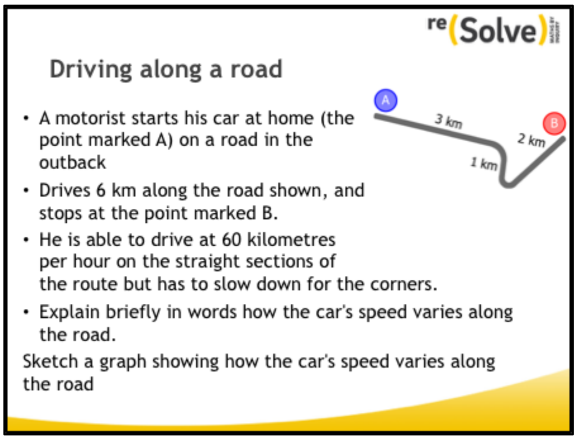
Generating Relationships

Driving: A graph as a model

Show students the slide Driving along a road and hand out Student Sheet 2 - Relationships.

Highlight that when solving problems, we need to specify the relationships between variables. A good way to think about the relationships is to describe them in words and to draw graphs to explain your ideas.

Ask students to work in their groups to consider the situation described on the slide, describing the speed of the car in words and by drawing a graph.



Expected student responses

- At point A, the car is standing still.
- The car's speed increases rapidly to 60 kph.
- The speed remains constant at 60 kph until the 3 km mark.
- The car slows down to about 30 kph as it turns a corner.
- The speed increases again to about 60 kph until about the 4 km mark.
- The car brakes suddenly as it turns a very sharp corner and its speed becomes about 15 kph.
- The speed rapidly increases to 60 kph for the rest of the journey.
- When the distance travelled from point A is about 6 km, the car stops.

Explain that here they have been developing their own graph as a model from a description of a situation. Many people will express in words what was happening before drawing the graph. This is often helpful as it helps you make sense of the real situation before trying to be precise in making the mathematical model you want.

Share some successful student graphs with the whole class and highlight how they have the same underlying structure of where the car slows down and where it speeds up. Discuss the differences between them.

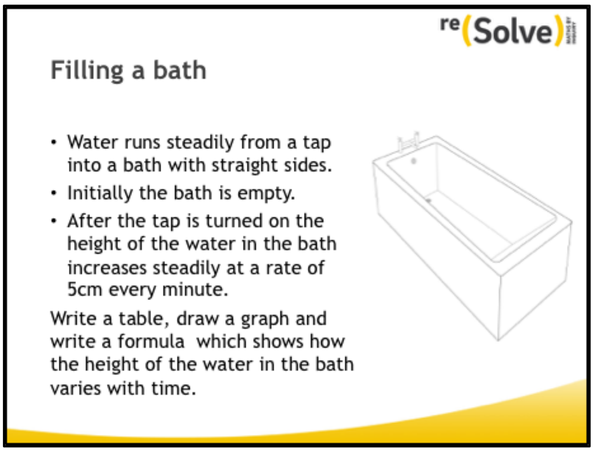
Filling the bath: Writing a model algebraically

This situation of filling a bath is expressed in words, and students are asked to express it in a table, graphically and algebraically.

Show the slide Filling a bath

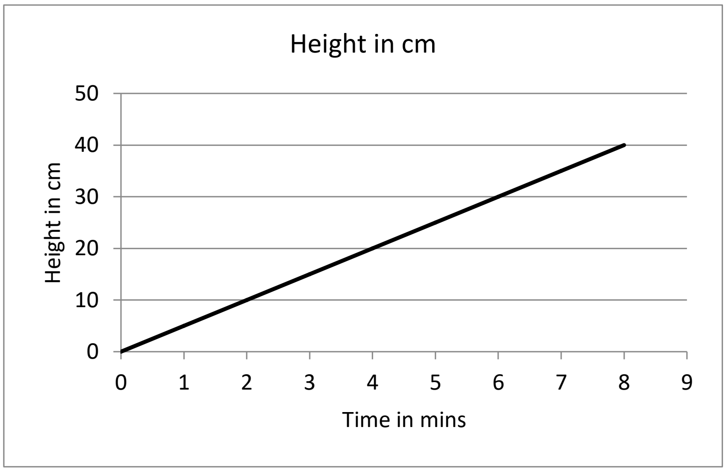
Students generate a formula that models how the height of water in the bath varies with time. If students can't do this algebraically perhaps they can express it in words.

Making a spreadsheet is a good idea. The spreadsheet ST7_Pricing_5b_Bath.xls shows how it can be done.



Expected student responses

Time in mins	Height in cm
0	0
1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40



Discuss with the whole class how this situation can be modelled using a formula. The height is a direct proportion relationship with time, and when plotted as a graph gives a straight line passing through the origin. The graph of the straight line is therefore of the form $y = mx$. Point out how the straight line and the formula each have mathematical aspects that reflect the constant rate of flow (that is, the constant gradient of the line and the single power of x in the formula reflect that the water rises by the same height for each unit of time.)

Extension:

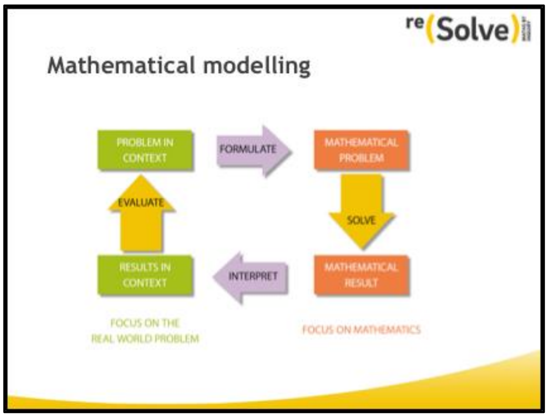
What would be the relationship if the bath was a different shape (e.g. a hemisphere, a cylinder, gently curved)?

Summarise key points on formulating

Show students the slide Mathematical modelling.

Discuss with students that in this lesson we have considered the formulating stage of mathematical modelling in detail. Draw on the different tasks to illustrate how formulating depends on:

- Understanding the real problem in some detail
- Considering all the possible factors that might need to be taken into account, and identifying these as variables.
- Deciding which of these factors (variables) we might hold constant and which we will allow to change.
- Identifying mathematical relationships between the variables.
- Deciding how we might represent relationships: tables, graphically, in words or using algebra.



Bushfire

Name: ______________

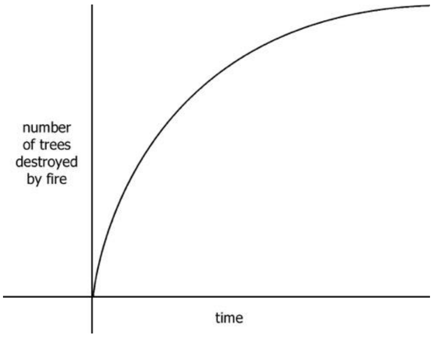
The picture shows a small Pacific Island, which is covered with a dense forest.

Suppose that a tree at the point marked X accidentally catches fire. The fire spreads from X, unchecked, until all the trees are completely destroyed.

- Which of the graphs shows most realistically the number of trees that have been destroyed over time?
- Explain what your chosen graph indicates about the situation, and give reasons for your choice.

If you don't think any of these graphs are realistic, draw your own version and explain it fully.

A.



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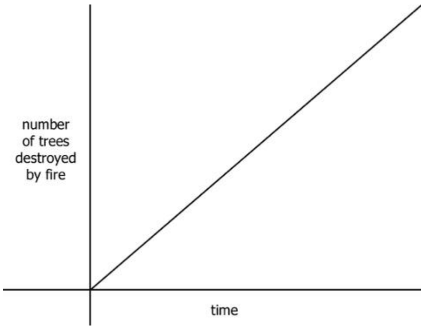

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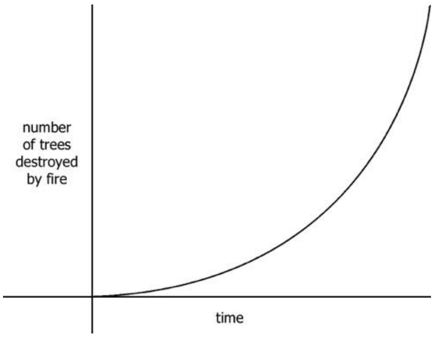
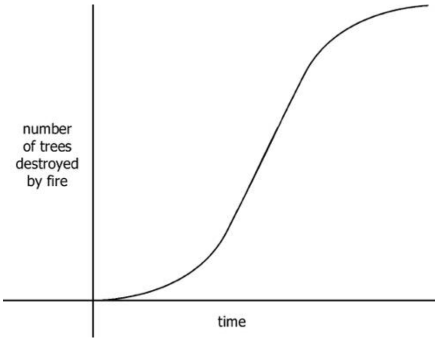
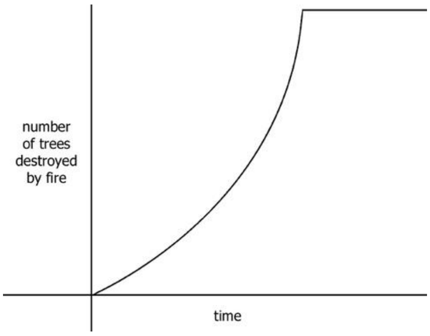


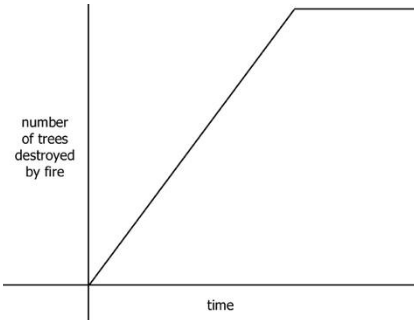
D.



E.



F.



Extensions: Sketch an appropriate graph if there is a strong wind from the north.

Sketch an appropriate graph if the fire started at one corner.

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Relationships

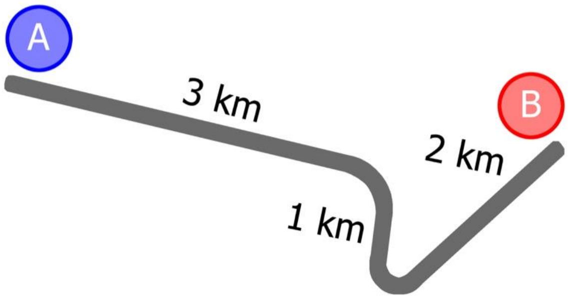
Name: ______________

Driving along a road

A motorist starts his car at the point marked A on a country road, drives 6 km along the road shown, and stops at the point marked B.

He can drive at 60 kph on the straight sections of this route but has to slow down for the corners.

- Explain briefly in words how the car's speed varies as the car drives along the road.
- Sketch a graph showing how the car's speed varies as the car drives along the road.

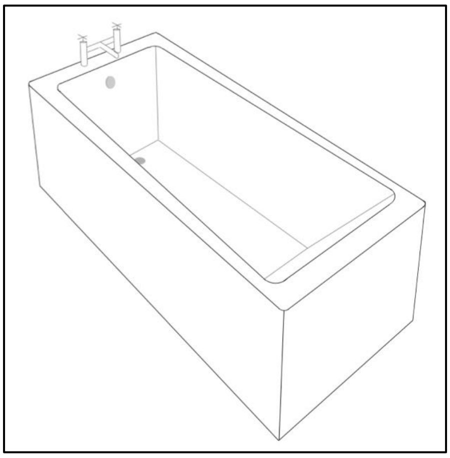


Bath water

Here is a further situation in which you need to consider the relationships between variables.

Water runs steadily from a tap into a bath with straight sides. Initially the bath is empty. After the tap is turned on, the height of the water in the bath increases steadily at a rate of 5cm every minute.

Make a table, draw a graph and write an equation which shows how the height of the water in the bath varies with time.



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