Pricing for Profit
Lesson 3: Better models with spreadsheets

Australian Curriculum: Mathematics (Years 8 - 10)

ACMNA189: Solve problems involving profit and loss, with and without digital technologies (Year 8)
ACMNA296: Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (Year 9)
ACMNA 294: Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software.
ACSIS169 and ACSIS203: Analyse patterns and trends in data, including describing relationships between variables and identifying inconsistencies

- using spreadsheets to present data in tables and graphical forms and to carry out mathematical analyses on data
- exploring relationships between variables using spreadsheets, databases, tables, charts, graphs and statistics (Years 9 and 10)

Lesson abstract

Students refine their models and implement them as spreadsheets. First, they make a spreadsheet where sales decrease as price increases for any linear decreasing pattern, and observe the effect of assumptions such as how many people buy at a given price. Then they include production costs in the model, and optionally make increasingly automated spreadsheets. There are good opportunities to use algebra.

Mathematical purpose (for students)

A spreadsheet is a very useful tool to make predictions from a mathematical model.

Mathematical purpose (for teachers)

This lesson has three threads. The main modelling thread is to create models of increasing complexity, including more factors in the model, and using the models give insight into the real situation. The main technology thread is to demonstrate using a spreadsheet to implement a model. Skills with spreadsheets can be extended through making the calculations required by the model more automated. Students also see how a spreadsheet can make model more useful by allowing users to input their own parameters. Mathematically, there are good opportunities for using algebra, that can be adjusted according to the capabilities of the class. Students can use formulas for straight line graphs and their products, and translate standard algebra notation to spreadsheet formula notation.

Lesson Length 50 minutes approximately

Vocabulary Encountered
- parameters

Lesson Materials
- Slide show *ST7_Pricing_3a.pptx*
- Spreadsheet access (essential) (1 computer per pair)
- *ST7_Pricing_3b_Models.xls* (for demonstration or distribute)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

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Why make a spreadsheet model?

Aim of lesson

Introduce the aim in this lesson - to provide Jack and Megan with a flexible tool to set their prices.

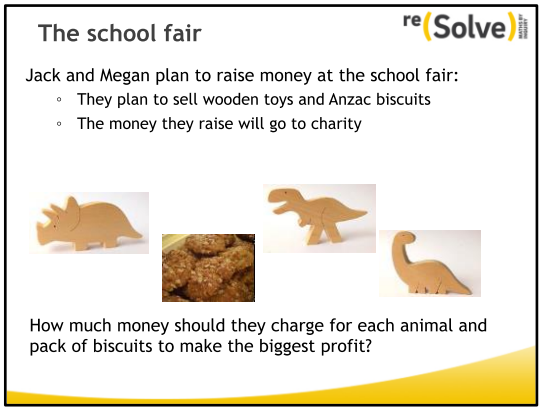
Up to now, the models have been easily able to be calculated by hand with selected data. Students now create spreadsheets to automate the calculations for Jack and Megan.

Later in the lesson, they can improve the models in other ways, as time allows.

There is considerable scope here for students to stretch their individual mathematical and technological skills. Some will be stretched by creating a simple spreadsheet for a small variation on the model developed in the previous lesson; others can create a highly automated spreadsheet taking account of several extra factors.

How a spreadsheet can implement a general model

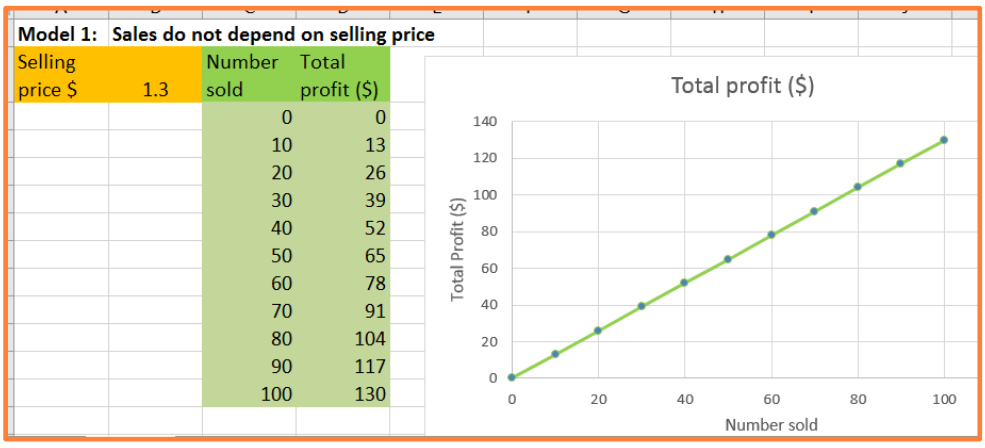
Show the slide The school fair. Remind students that when they developed their previous models, they guessed how many people were likely to buy a product at various prices. Suggest that if they could make models for some different "guesstimates", implemented with a spreadsheet, then this would be a more useful tool for Jack and Megan.



Illustrate the idea with the first model created in this unit, Model 1 in the spreadsheet *ST7_Pricing_3b_Models.xlsx*. This spreadsheet comes with four worksheets, each one implementing a different model. The spreadsheet can be kept only for demonstration or distributed to all computers for students to investigate and modify.

Show how the model does the calculations for any selling price, and any numbers of items sold.

Ensure students know how this worksheet is operating e.g. show formulas and manual entries and discuss the graph (scatterplot chart). Note how these axes are automatically rescaling when the selling price or numbers sold change. The apparent slope of the graph varies with rescaling.



Model 1: Sales do not depend on selling price

Selling price $	1.3	Number sold	Total profit ($)
		0	0
		10	13
		20	26
		30	39
		40	52
		50	65
		60	78
		70	91
		80	104
		90	117
		100	130

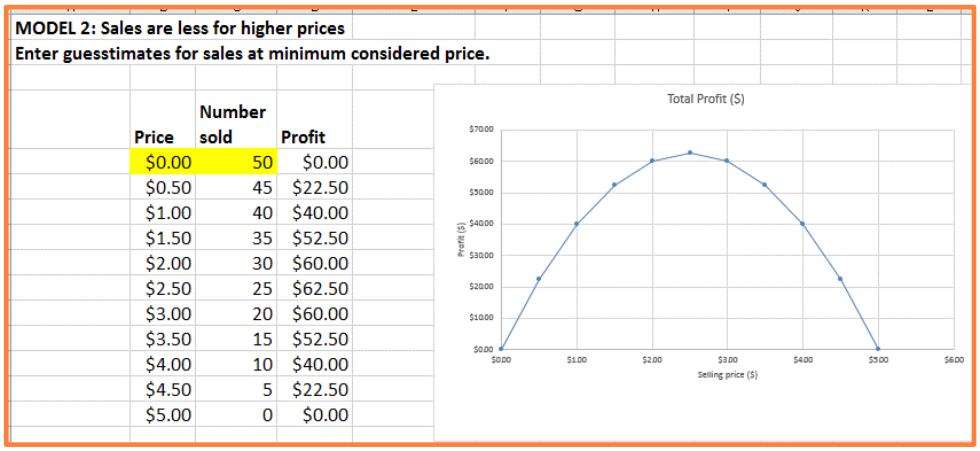
Screenshot of Model 1.

The selling price is entered in dollars.
The numbers sold have been entered manually.
*The Total Profit is calculated automatically (e.g. the formula in cell D3 is =C3*B2).*

Examine Model 2 on the second worksheet of *ST7_Pricing_3b_Models.xlsx* or slide Automating Model 2. Ask students to investigate what this spreadsheet enables Jack and Megan to calculate.

Consider:

- What number cells can be changed by the user? (ANS: Cell C5 is the only required input)
- What numbers are calculated automatically? What formulas are used, and what do they mean?
- How have the numbers of sales at maximum and minimum prices been used? (ANS: the constant decrease of 5 sales has been calculated manually from them, and then used in the formulas.)
- As appropriate ask students to rewrite the formulas in words or in conventional algebraic notation, choosing simple informative letters for each of the variables.



Screenshot of Model 2. This model is based on the guesstimate that there will be 50 sales if the items are (nearly) free. The formulas for price are iterative, adding 50 cents to the price and subtracting 5 from sales (e.g. C8 = C7 - 5).

Model 2 allows the user to input potential sales for the minimum considered price (yellow cells), and the spreadsheet works out the expected profit. This model assumes the raw materials are free and there are no costs associated with making the products.

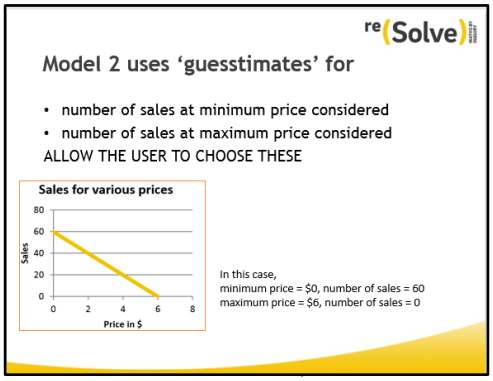
Generalising Model 2

Show slide Generalising Model 2.

Remind students that when they developed their model in the last lesson, they guessed how many people were likely to buy a product at various prices. Suggest that it would be useful to have a spreadsheet to do the calculations for different "guesstimates".

To make their models previously, students chose values for the parameters:

- Number of sales at minimum price considered
- Number of sales at maximum price considered.



(Example: In the graph shown, minimum price considered is 0 (or in reality very close to zero) and the sales then are 60, and the maximum price considered is $6 which was regarded as the price beyond which the sales would be zero. In this case, these points are the intercepts of the graph with the axes.)

Show worksheet 'Model2' of the spreadsheet file ST7_Pricing_3b_Models.xls and discuss its limitations to show students what they may vary. Some students will make formulas that they have to adjust manually; others will automate.

How to fill in the sales for prices between the maximum and minimum

Now focus on making the model more general by having students make a formula for the constant decrease in the formula for the number of sales, using the two 'data points':

- Number of sales at minimum price considered (in the example 50 sales for selling price $0)
- Number of sales at maximum price considered (in the example, 0 sales for selling price $5)

Formulas can be in words or algebraic symbols or spreadsheet algebra. They can be instructions for manually calculating the successive drop in sales to enter in the spreadsheet formulas, or (better) the formulas can be automated in the spreadsheet. Students should write clear instructions for using this spreadsheet.

Examples

In words:

(the maximum number sold minus the minimum number sold) divided by (the price for maximum sales minus the price for minimum sales) = change in number of sales per dollar increase in price. (Note the sign of this number!)

(E.g. with the numbers in the spreadsheet image above, (50-0)/(0-5) = -10. The drop in sales is 10 per dollar, so 5 per half dollar.)

In algebraic notation:

Let *S* be the maximum number sold, *s* the minimum number sold, *P* the price for maximum sales and *p* the price for minimum sales. Then the change in number of sales per dollar increase in price is $\frac{S-s}{P-p}$.

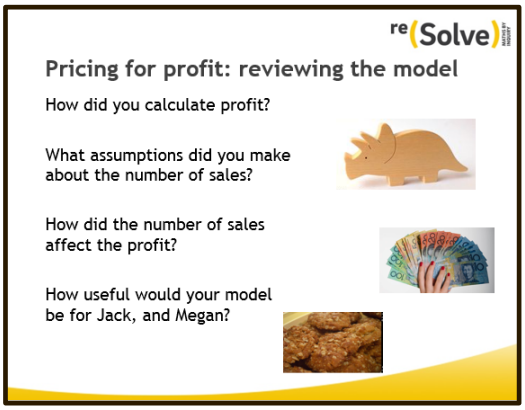
Even more usefully we can find an expression for the number sold (probably only used in later spreadsheets):
Let *y* be the number sold for a selling price *x*. The formula for *y* is found from $\frac{y-s}{x-p} = \frac{S-s}{P-p}$.

Understanding the spreadsheets and Model 2

Review the work, perhaps using slide Pricing for profit: reviewing the model.

Ask some students to share what they have done and answer questions such as:

- What variables did you use? (ANS: Price, Sales, Profit)
- What data is to be entered (ANS: minimum price and number of sales, manually calculated drop per dollar)
- What formula did you use to calculate the number of sales at a given price?
- How did you calculate the profit? (ANS: Price multiplied by Sales)
- What do you observe about the predicted change of profit when the guesstimates for sales change?



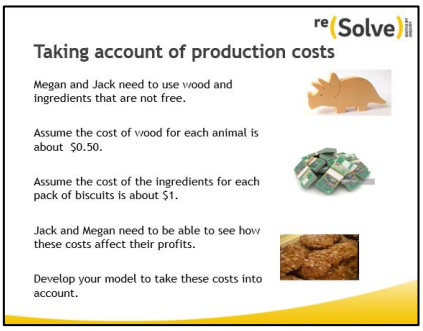
Assist as required with software difficulties, such as:

- Tables not extending far enough
- Having graphs that are too small or too big
- Dealing with negative sales (just ignore?)
- Dealing with negative profit (important interpretation point)

Aim to reach a point where all students have a secure understanding of all the formulas used in spreadsheet and their algebraic form. Students should be aware of how these are related to the assumptions used in their model.

Making Model 3, including production costs

Now students develop an even more sophisticated model that includes production costs, implemented in a spreadsheet. A sample is given on the worksheet 'Model 3' of ST7_Pricing_3b_Models.xlxs.



Show slide Taking account of production costs.

To begin concretely, make the following assumptions:

- the cost of wood for each animal is about $0.50
- the cost of ingredients for each a pack of biscuits is about $1.

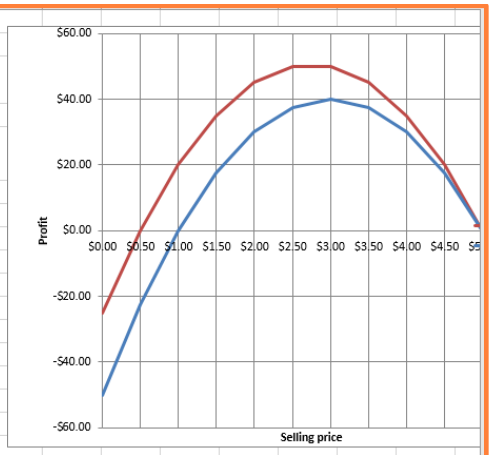
However, soon suggest that different costs should be able to be entered.

Students work together to modify their spreadsheets (or make a new one) for the new model. Worksheet 'Model 3' contains an example (also slide provided).

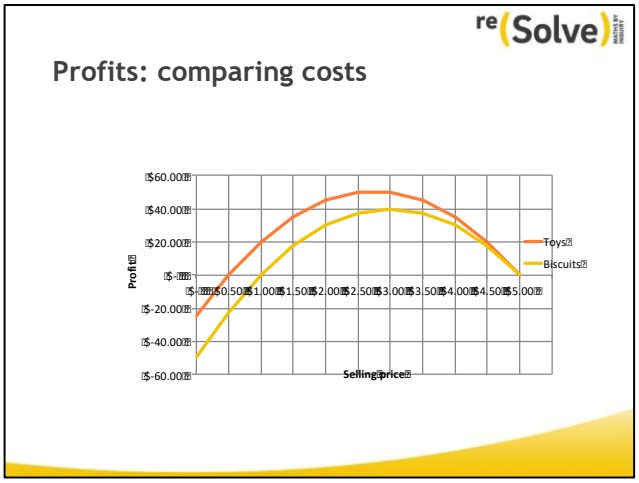
Screenshot of Model 3. Costs per item are entered manually. Total costs, income, and profit are calculated by formula. The guesstimates for sale-price can be altered separately for toys and biscuits.

MODEL 3: Sales are less for higher prices and production costs per item included

Wooden Toys						Anzac Biscuits					
wood for one toy	$0.50					ingredients for 1 pack	$1.00				
	Price	Number sold	Income	Cost	Profit		Price	Number sold	Income	Cost	Profit
	$0.00	50	$0.00	$25.00	-$25.00		$0.00	50	$0.00	$50.00	-$50.00
	$0.50	45	$22.50	$22.50	$0.00		$0.50	45	$22.50	$45.00	-$22.50
	$1.00	40	$40.00	$20.00	$20.00		$1.00	40	$40.00	$40.00	$0.00
	$1.50	35	$52.50	$17.50	$35.00		$1.50	35	$52.50	$35.00	$17.50
	$2.00	30	$60.00	$15.00	$45.00		$2.00	30	$60.00	$30.00	$30.00
	$2.50	25	$62.50	$12.50	$50.00		$2.50	25	$62.50	$25.00	$37.50
	$3.00	20	$60.00	$10.00	$50.00		$3.00	20	$60.00	$20.00	$40.00
	$3.50	15	$52.50	$7.50	$45.00		$3.50	15	$52.50	$15.00	$37.50
	$4.00	10	$40.00	$5.00	$35.00		$4.00	10	$40.00	$10.00	$30.00
	$4.50	5	$22.50	$2.50	$20.00		$4.50	5	$22.50	$5.00	$17.50
	$5.00	0	$0.00	$0.00	$0.00		$5.00	0	$0.00	$0.00	$0.00



Have some students share their work, and address technical difficulties as required. Also ask students to describe what is learned from the model. For example, the screenshot of 'Model 3' shows the common-sense insight that a difference in ingredient costs is more significant when more items are sold. The slide Profits: comparing costs has a larger graph to display for discussion.



Using these graphs, Megan and Jack can see that, with these assumptions, they will make most profit if they sell the wooden animals for between $2:50 and $3:00 and Anzac biscuits for $3:00.

Model 4: A more automated model (optional)

Some students can now automate the process of calculating the number of items sold from the price, by entering the guesstimates. The worksheet 'Model 4' provides an example. Some students can make their own spreadsheet, others can work to understand this one.

Students could calculate the decrease in number sold per dollar of increased price (automatically now, rather than manually), and use it in iterative formulas as in earlier spreadsheets. OR

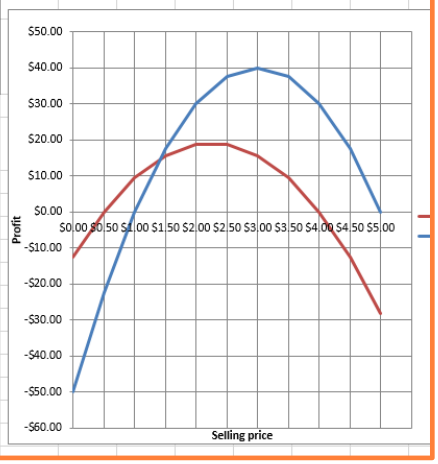
They could use the equation of the straight line between two points (see algebra above) to make a formula. This has been done in Model 4. The formula for cell C9, for example, is =D3 + (B9-D2)*(D3-F3)/(D2-F2).

Screenshot for Model 4.
Data is entered in the coloured cells. Automated processes calculate the number sold from the nominated prices.

MODEL 4: Sales are less for higher prices, production costs per item included, price-sales relationship changes

Wooden Toys		Min Price ($)	0	Max Price	4	Anzac Biscuits		Min Price ($)	0	Max Price	5
Cost of wood for one toy	$0.50	Sales at min price	25	Sales at max price	0	Cost of ingredients for 1 pack	$1.00	Sales at min price	50	Sales at max price	0

Price	Number sold	Income	Cost	Profit	Price	Number sold	Income	Cost	Profit
$0.00	25	$0.00	$12.50	-$12.50	$0.00	50	$0.00	$50.00	-$50.00
$0.50	21.875	$10.94	$10.94	$0.00	$0.50	45	$22.50	$45.00	-$22.50
$1.00	18.75	$18.75	$9.38	$9.38	$1.00	40	$40.00	$40.00	$0.00
$1.50	15.625	$23.44	$7.81	$15.63	$1.50	35	$52.50	$35.00	$17.50
$2.00	12.5	$25.00	$6.25	$18.75	$2.00	30	$60.00	$30.00	$30.00
$2.50	9.375	$23.44	$4.69	$18.75	$2.50	25	$62.50	$25.00	$37.50
$3.00	6.25	$18.75	$3.13	$15.63	$3.00	20	$60.00	$20.00	$40.00
$3.50	3.125	$10.94	$1.56	$9.38	$3.50	15	$52.50	$15.00	$37.50
$4.00	0	$0.00	$0.00	$0.00	$4.00	10	$40.00	$10.00	$30.00
$4.50	-3.125	-$14.06	-$1.56	-$12.50	$4.50	5	$22.50	$5.00	$17.50
$5.00	-6.25	-$31.25	-$3.13	-$28.13	$5.00	0	$0.00	$0.00	$0.00



Extensions

Students may like to consider other ways of improving their spreadsheets to make them more helpful. For example, they may include fixed costs in the model (e.g. for making a sign, renting a stall) in addition to the per-item costs.

As time permits, have students demonstrate their spreadsheets, discussing their models and the conclusions they draw from them.

Reflecting on the models

Show the slide Mathematical modelling.

Ask students to consider the questions:

- How realistic/unrealistic is your model now?
- What parts of the modelling process have you been involved with in this lesson?

Emphasise to students that they have again worked through the modelling cycle to develop a more sophisticated model.

Tell students that in the next lesson they will work in their pairs on their reports. They will need to explain exactly how their preferred model works, including an explanation of how they have modelled the costings and include instructions for using the spreadsheet.