Pricing for Profit
Lesson 2: Improving the Model

Australian Curriculum: Mathematics (Years 7 and 8)

ACMNA174: Investigate and calculate 'best buys', with and without digital technologies (Year 7)
ACMNA194: Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (Year 7)
ACMNA189: Solve problems involving profit and loss, with and without digital technologies (Year 8)
ACMNA296: Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (Year 8)

Lesson abstract

Students refine their model from Lesson 1 by addressing one major unrealistic assumption they made: that the number of sales is independent of selling price. They consider possible relationships between selling price and sales and derive (quadratic) relationships between price and profit.

Mathematical purpose (for students)

A model can be made more realistic by improving some of the assumptions behind it.

Mathematical purpose (for teachers)

Students revisit the modelling cycle and develop understanding that repeating the cycle after reconsidering the assumptions can lead to a more realistic model. In making the new model, there is the opportunity to set up equations (in words or symbols depending on algebraic skill), to make tables of numbers that show a linear relationship (so noting that a constant change in the independent variable requires a constant change in the other), draw a graph from a table. All these ideas can be expressed with algebra. Students with appropriate algebraic knowledge may observe that the product of two linear functions is quadratic, and use their knowledge in the analysis.

Lesson Length 45 minutes approximately

Vocabulary Encountered
- parameters

Lesson Materials
- Slide show: ST7_Pricing_2a.pptx
- Student Sheet 1 --Pricing for Profit 2 (1 per student)
- Spreadsheet access (desirable)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

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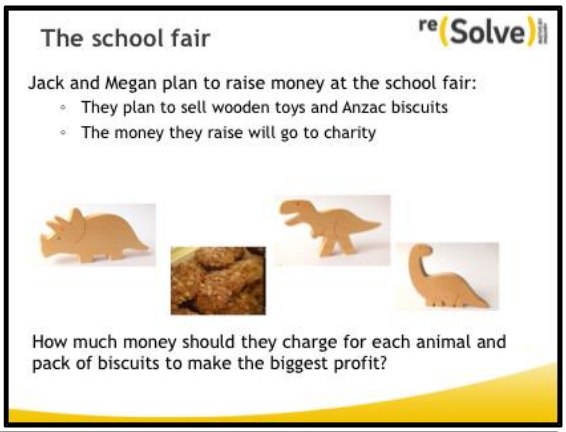
How will sales vary with selling price? (15 mins)

Thinking about the real situation



Throughout this lesson, use Student Sheet 1 – Pricing for Profit 2.

In this introductory phase, stimulate the need to consider refining the mathematical model developed in the previous lesson.

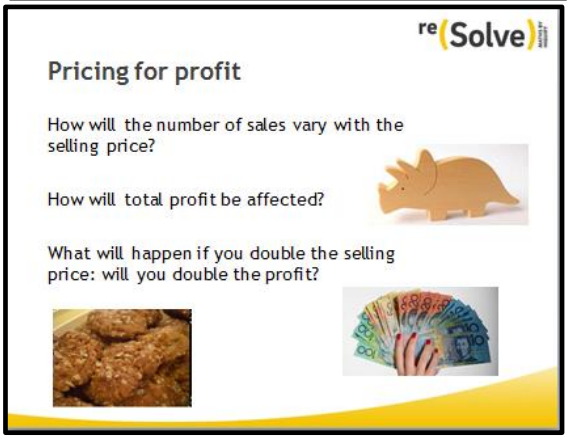
Show students the slide The school fair and ask whether they think Jack and Megan should aim to sell 1000 animals and biscuits at $100 each? It should be clear that this is unrealistic.

Show the slide Pricing for profit and ask students to consider in pairs the questions on the slide and student sheet.

Expected Student Responses

- As the selling price increases, the number of sales is likely to decrease.
- People might not buy the toys or biscuits if they are too expensive.
- If you double the selling price, the sales may decrease, so the profit would not double.

Gather students' thinking and draw out the idea that by increasing the price, sales are likely to fall. In this lesson they modify their models to take this into account. Explain that they could also develop their model to take account of the production costs of the wooden animals and biscuits, but this can be left until later.



Thinking of possible relationships

Consider possible relationships between the selling cost of a product and the number of sales. Point out that in modelling, it is likely to be best to choose relationships to be as simple as possible. There is a balance between manageability and realism.

Students should work in their small groups (2-4 students) to sketch what they think is a realistic graph of number of sales against selling price. Explain that at this stage, the numbers do not matter – only the general shape.

Enabling Prompt

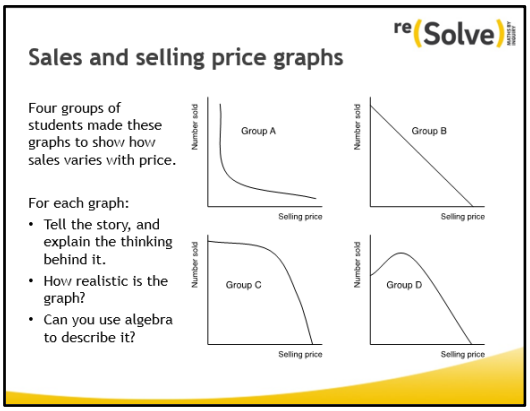
Imagine you are considering buying some small toys or biscuits for a present. What would happen to the number you would buy as the selling price increases. Are other people likely to react in this way?

Choosing a relationship

This section has three main purposes:

- To select a relationship between selling price and number of sales for later use.
- To highlight that there is not one correct representation of the situation, and there are many possible variables to take into account.
- To make the point that in developing models, it is better to choose a representation that relies on mathematics that you can manage easily.

Show several different graphs to the class to stimulate discussion about how profit is likely to depend on sales. Use those on the slide Sales and selling price graphs or replace by your own students' graphs.



Ask students: *(questions are on the student sheet)*

- Tell the story and explain the thinking behind it.
- How realistic is the graph?
- Can you use algebra to describe it?

Expected Student Response

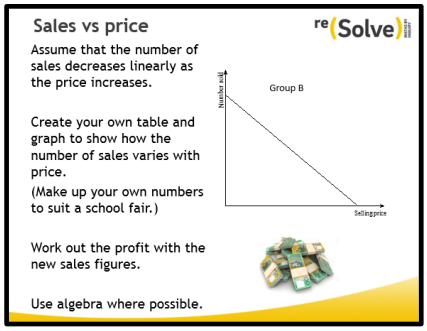
- Graphs A and B are the most realistic because as the price increases the sales go down.
- Graph D is unrealistic as in its early part when the price goes up the number of sales goes up.

Review the responses, noting that Group B's graph is the easiest to describe algebraically, or even in a table.

Explain that although students may think that Graph C, for example, is a more realistic representation than Group B's graph, for the purposes of modelling it could be better to work with Group B's graph because the maths is easier, and it retains the essential property: sales going down as price goes up. It can be expressed simply in words and algebra. This is what is done for the rest of this lesson.

How will profit vary with selling price? (25 mins)

In this, the main part of the lesson, students should work in their small groups to refine their original model, taking account that the number of sales is likely to decrease as price increases. Show students the slide Sales vs price



Ask students to:

- Draw a table and graph to show how number of sales varies with price.
- Draw a table and graph that shows how profit varies with price.

Students should be strongly advised to assume the sales against price relationship that Group B took.

Students will have to decide on their own numbers – explain that the numbers are not critical for this exercise, but they should make 'guesstimates' by imagining what might happen if they had a fair at their own school.

[If your students are uncomfortable about making up numbers, you could collect some data from them about how many think they would buy the animals and biscuits if they cost $1, $2 and so on. But all that will take time.]

Spreadsheets could be used – this will put the students in a good position for the next lesson.

Encourage students to work with tables, graphs and algebra. The work can all be done numerically, if required. Starting with the tables is probably the easiest.

Sample algebra (Students choose their own constants):

$$\text{Profit} = \text{number of sales} \times \text{profit per item}$$
$$\text{P} = \text{p} \times \text{n}$$
$$\text{Number of sales} = \text{Constant} - \text{constant} \times \text{price per item} = \text{c} - \text{mp}$$
$$Profit = p.(c - mp)$$

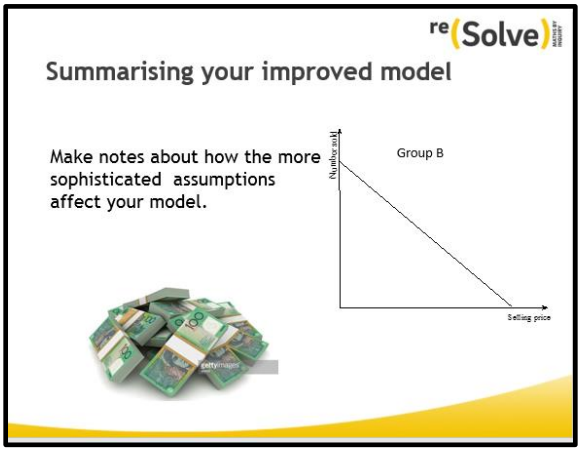
Expected Student Responses

Price in $	Sales
0	60
1	50
2	40
3	30
4	20
5	10
6	0

For how profit varies with price:

Price in $	Sales	Profit
0	60	0
1	50	50
2	40	80
3	30	90
4	20	80
5	10	50
6	0	0

Show students the slide Summarising your improved model. Ask groups to write a few notes about what they have found out and be prepared to report to the class.

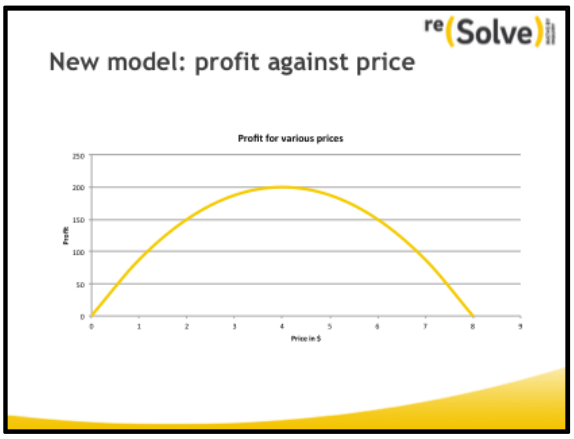


Examining the new model

Show one of the students' graphs, or alternatively, the slide New model: profit against price.

Ask students:

- Describe the shape? What do we call this shape of graph?
  [A parabola. It is like the graph of a quadratic function.]
- Which functions were used to construct it?
  [Price and Profit]
- Does the shape of this graph surprise you? Why?
  (Initially, students may have expected the Profit to increase as the Price increases.)
- Interpret the graph of profit against selling price in real world terms.
- When is the profit predicted to be zero – what causes this.



Expected Student Response

Profit is maximized at a selling price (about $3.00 using the assumed values used here). Having a very high selling price reduces the number of products sold – eventually the model predicts none! The profit is predicted to be zero, either when the selling price is zero, or no products are sold.

Reflection (5 mins)

In this final section of the lesson:

- compare and contrast the initial simple model with the improved model,
- consider the work in terms of the modelling cycle.

Remind the class of the discussion from a previous lesson, which used the simple model to predict that, for example, if you double the price you double the profit.

Emphasise that, with their new more sophisticated model, this is now not the case by asking students:

- How will the number of sales vary with selling price?
  [As the selling price increases, the number of sales decreases]
- How will the total profit be affected?
  (At first, as the selling price increases, the total profit increases, then profit decreases as the price becomes too high]
- What will happen if you double the selling price: will you double the profit?
  (No. If the price is doubled, the number of sales will be less, so the profit will be less than double.
- How useful would your model be for Jack and Megan?
  (They will need to think carefully about the 'best' selling price for their products: perhaps do a small survey.)
- What compromises have we made to develop this model?
  (The linear relationship between the price and number of sales may not be very true to life.)

Highlight that:

- There may be an optimum selling price
  (or range of selling prices) where price and number of sales lead to the greatest profit.
- When modelling, it is usually best to consider the relationship to be as simple as possible to reflect the realty. There is a balance between manageability and realism.

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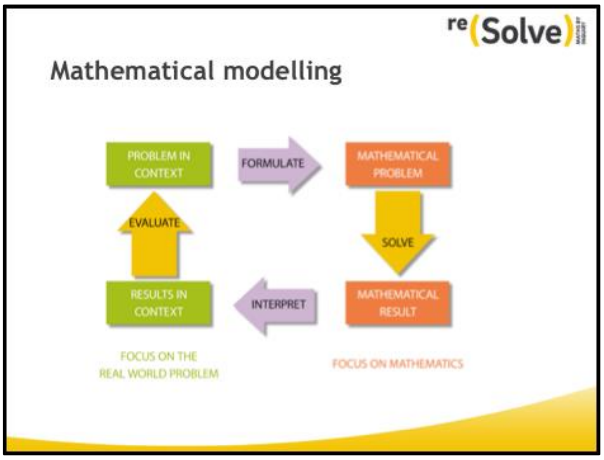
Show the slide Mathematical modelling.

Ask students to consider the questions:

- How realistic/unrealistic is their model now?
- What parts of the modelling process have they been involved with?
- What other factors might be taken into consideration later?



Expected Student Responses

The model is now more realistic but is probably still based on unrealistic assumptions.
We have been involved with all parts of the modelling cycle and we have been around the modelling cycle twice.

Signal that in the next lesson there will be an opportunity to consider an even more sophisticated model and consequently they will experience all parts of the modelling cycle again.

Pricing for Profit 2

Name: _____

(A) How selling price might affect sales and profit

How might the number of sales vary with selling price?

How will total profit be affected by selling price?

What would happen if you doubled the selling price? Would you double the profit? Explain your reasoning.

Sketch what you think is a realistic graph of the number of sales against selling price.

Plot the independent variable (in this case the price of the products) on the horizontal axis.

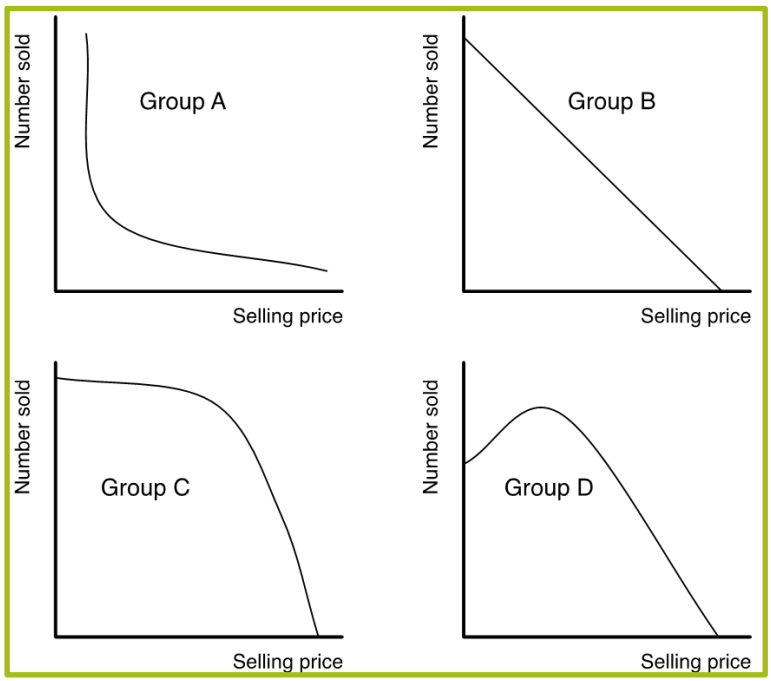
(B) Selecting relationships

Four groups of students produced these graphs to show how the number of sales varies with price.

Choose two graphs. For each of these:

- Tell the story and explain the thinking behind it.
- How realistic is the graph?
- Can you use algebra to describe it?

Graph 1



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Graph 2

Which graph is the most realistic representation of the situation? Explain.

For which graph is it easiest to write an algebraic relationship?

(C) Making a table and graph of number of sales and of profit

Assume that the number of sales decreases, as the price increases as shown in Group B's graph.
Make a table and graph to show how the **number of sales** varies with price. *Use a spreadsheet if available*.

Draw a table and graph to show how the **profit** varies with price. *Use a spreadsheet if available*.