Introduction to Modelling
Lesson 1: Modelling a Traffic Jam

Australian Curriculum: Mathematics (Year 9)

ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. (Year 9)

- identifying direct proportion in real-life contexts

Lesson abstract

Students approach modelling by considering how many vehicles there might be in a 200-metre section of a blocked highway. They first identify factors that might be taken into account. They then consider how to calculate how many vehicles, making some assumptions and either researching or estimating the values. They write a description of how to tackle the problem using their approach and how to develop a more sophisticated model. Groups work out a first solution and think about the effects of the assumptions they made.

Mathematical purpose (for students)

Mathematical modelling starts with simple models that are improved.

Mathematical purpose (for teachers)

This lesson introduces students to the process of setting up and using a model of a real situation: a traffic jam. This involves ideas of proportionality, represented in words, graphs or algebra as appropriate. Students encounter important ideas in model formulation: simplifying a situation, identifying key variables, making assumptions, identifying relationships, developing a solution and reflecting on it. They first develop a solution to a simple problem – how many vehicles in a traffic jam – with brief consideration of the likely validity and accuracy of their result. They consider, briefly, how their solution might be affected if they were to change their assumptions. The mathematics includes proportional reasoning, estimation in common metric units, and optionally writing formulas.

Lesson Length 45 minutes approximately

Vocabulary Encountered
- Models
- Modelling
- Assumptions

Lesson Materials
- Slide show: ST7_Intro_To_Modelling_1a.pptx
- Student Sheet – Modelling a Traffic Jam (1 per group)
- Internet video of traffic jam (e.g. http://www.youtube.com/watch?v=cfoARdgGLOg)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

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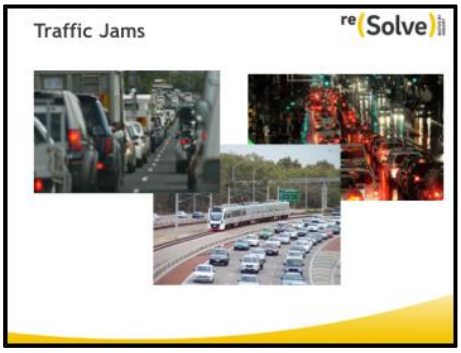
Lesson structure

- Setting up a first model (Whole-class discussion - 15 minutes)
- Modelling the traffic jam (Collaborative small-group work - 20 minutes)
- Reflecting on our models (Group, then individual work - 10 minutes)

Setting Up a First Model

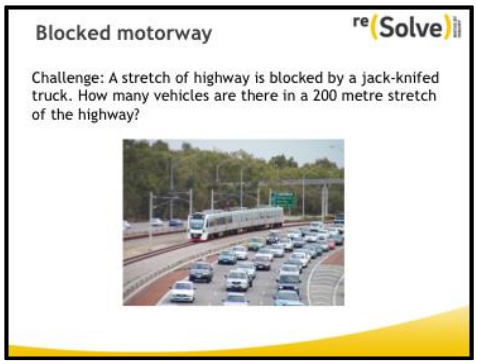
Show the slide Traffic Jams.



Suggest that if there is a traffic jam the police or TV and radio reporters may wish to get a quick idea of how many cars or people are involved.

If resources allow, start by showing this video sequence, or choose another of the many traffic jam videos on youtube.
http://www.youtube.com/watch?v=cfoARdgGLOg

Show slide Blocked highway.



Introduce the problem. The challenge is for students to find how many vehicles there might be in a 200-metre section of a blocked highway.

In this first part of the lesson the intention is that students focus on which *factors* they need to include to make a model from which an estimate can be derived, and think about sensible values for these *variables*.

Ask students to spend a few minutes to identify the factors they will need to take into account to work out roughly how many vehicles there are in the traffic jam. Ask them to distinguish which of the factors are essential to include and which they might include in a more sophisticated approach later.

Ask them to identify what they already know and what they will have to estimate or research (maybe informally).

Expected responses

Assumptions such as: Truck length about15 metres, Car length about 5 metres, Gap about 2 metres, all trucks in one lane

Factors that students might consider include.

- Typical lengths of a range of vehicles (cars, vans, trucks)
- The proportions of the different types of vehicles likely to be in the traffic jam
- Typical lengths of gaps between vehicles
- The number of lanes on the highway

Students should start to think about how exactly these pieces of information might be used to work out the number of vehicles. They will actually do this, in the next phase of the lesson but in this first phase it is important that they focus on the issue of identifying the factors that will affect their approach and how they might put the information together to work out a solution to the problem.

Ask each group to write a brief description to tell someone else the process they should use to model how many vehicles there might be in a traffic jam, and to estimate how accurate their answer is.

Discussing getting started

In whole-class discussion collect students' thinking to this point and consider the effect of varying different factors on the solution (e.g. a 200m traffic jam on a two lane highway involves twice as many vehicles as on a one lane highway).

In the discussion, encourage students to think about developing the simplest possible model to start with and then increasing its sophistication later. Include some discussion of how accurate any very early preliminary estimates are, listing uncertainties.

Modelling the Traffic Jam

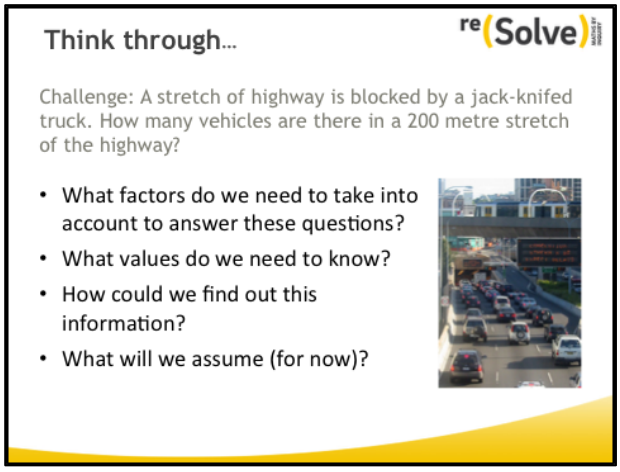
In this section, students estimate the number of vehicles in the traffic jam, going on to investigate the effect of varying the values that they have assumed.

Ask the pairs/groups now to work on the problem – that is, to come up with a first estimate of the number of cars in the 200 hundred metre section of the traffic jam. The images on Student Sheet – Modelling a Traffic Jam provide assistance with estimating various quantities.

Give students time for *productive struggle* with the problem. Circulate around the class and note which groups are considering the various factors so that you can choose who will most effectively contribute to the discussion in the next part of the lesson.

Groups should think about the various relationships that underlie their calculations. For example, they may conclude that the length of the traffic jam is inversely proportional to the number of lanes of traffic (without the technical language), or proportional to the length of a car plus the gap between cars (i.e. they assume that there are no trucks).

After they have had time to really get to grips with the problem you may want to use the enabling prompts and/or show the slide Think through…



Enabling Prompts

- What factors do we need to take into account to answer these questions?
- What values do we need to know?
- How could we find out this information?
- What will we assume (for now)?

If needed

- Would it help you get started to assume that there are only cars in the traffic jam?

Expected responses

- Assume 1 lane of trucks, 2 lanes of cars.
- Number of vehicles: Car lane: 200/(5+2) = approx 28, Truck lane: (200)/(15+2) = approx. 12
- Estimated total: 12 + 2x28 approx 70 vehicles

Any sensible assumptions and numbers are, of course, equally acceptable.

Extending Prompts

- Consider what would happen if you varied the gap between vehicles. What would be the effect? Can you express what happens mathematically?
- In the light of the uncertainties of the input assumptions, calculate roughly how accurate you estimate the results to be.

Expected responses

- Estimated accuracy: not better than 20%

Reflecting on our Models

The aim in this section is to bring the students' thinking together by presenting and reflecting together on their different assumptions and models, and the effect these have on their estimate.

Show the slide Present your solution.

Work with the whole class so that groups share their thinking on the problem. Ask a spokesperson from at least three groups to present their solutions. Use your earlier observations to make sure you draw on a wide range of thinking from across the class leading to very different solutions.

Tell the class that as the other groups present you want them to make a note of

- *what your group did that is the same*
- *what your group did that is different*
- *what you think the effect of these differences was on the different solutions?*

After several groups have presented

Show the slide What if....?.

Discuss the effect of making different assumptions on outcomes. It is important to emphasise the effect of varying a factor such as the gap between vehicles. For example, increasing the gap decreases the number of vehicles, decreasing the gap increases the number of vehicles. Numerical experiments can show how sensitive the result is to the assumptions. Results of the experiments could be graphed.

Individual Reflection

Show the slide Your report

Ask students to individually write a brief report to tell someone else the process they should use to answer the question of modelling how many vehicles in a traffic jam.

Students should think back over what they have done and make sure their report includes the aspects shown in the slide

In the next lesson we'll look more deeply into the modelling process.

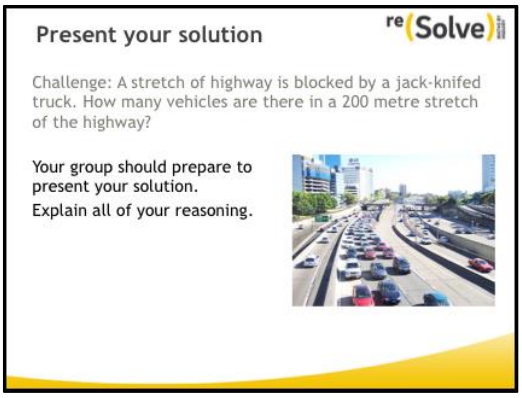
Extending the challenge (if time allows)

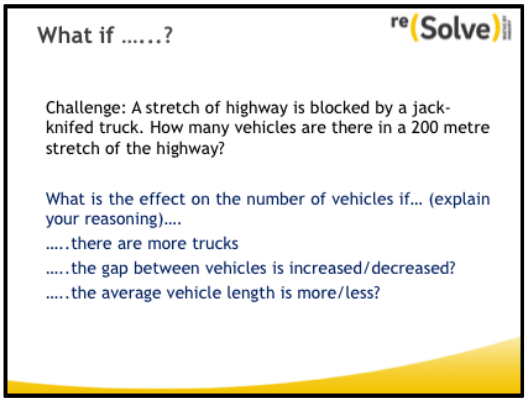
If your class has made good progress, extend the challenge with either of the slides Extending the problem (A) or (B).
(A) is easier. (B) looks forward to Lesson 2.

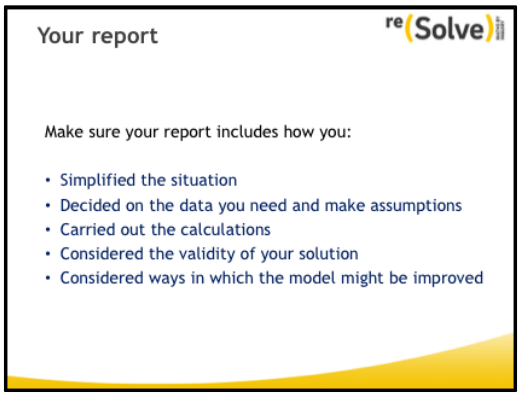
(A) What is the effect of increasing the size of the gap between vehicles on the number of vehicles in a 200 metre line?

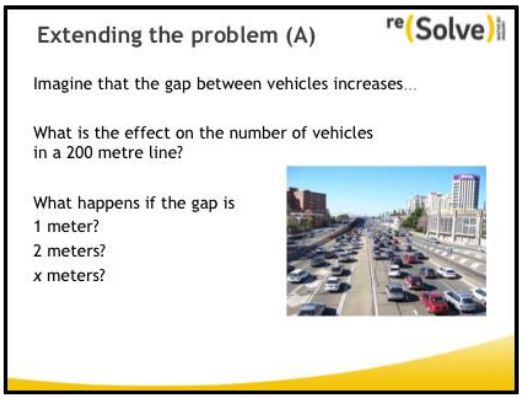
What happens if the gap is 1 metre? 2 metres? *x* metres?

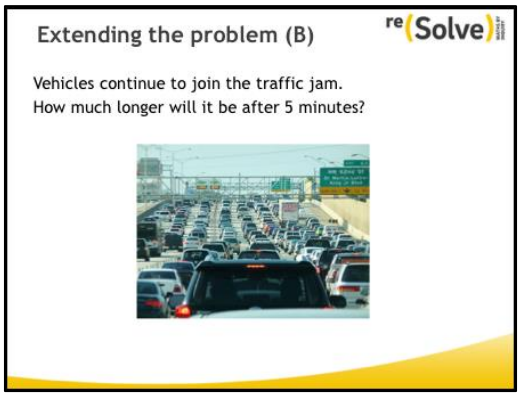
(B) Vehicles continue to join the traffic jam. How much longer will it be after 5 minutes? 10 minutes, 15 minutes and so on?











Modelling a Traffic Jam

Name: _____

A stretch of highway is blocked by a jack-knifed truck.

How many vehicles are there in a 200 metre stretch of the highway?

Use these pictures and your other knowledge to help you answer the question.

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