# Introduction to Modelling
## Lesson 5: Improved Traffic Jam Model

### Australian Curriculum: Mathematics (Years 8 and 9)

**ACMNA208**: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. (Year 9)

- identifying direct proportion in real-life contexts

**ACMNA 192:** Simplify algebraic expressions involving the four operations. (Year 8)

### Lesson abstract

Students return to the traffic jam situation from Lesson 1, seeking to transfer to traffic flow the understandings gained in the theme park context. They look at what happens as the traffic jam clears, developing a more sophisticated model that considers inflow and outflow. They map their work onto the modelling diagram, reflecting on the factors involved and how they influence stages in the modelling process.

### Mathematical purpose (for students)

An understanding of the modelling process can be developed by reflecting on your experiences of modelling traffic flow and theme park queues.

### Mathematical purpose (for teachers)

The aim is to consolidate understanding of the modelling process by relating modelling activity to the modelling diagram and reflecting on the challenges and opportunities that modelling presents. By the end of this lesson, students will have made simple models for the traffic and theme park situations, and will have improved both of them. This means they have a reasonable amount of experience to reflect on.

The mathematics of flow rates is again central, involving proportional reasoning and rate concepts. Estimation with common metric units is involved, and graphing and algebra may be used. There are opportunities for some students to develop relatively sophisticated models, including multiple factors, perhaps expressed algebraically.

Lesson Length 45 minutes approximately

Vocabulary Encountered

- Models
- Modelling
- Assumptions

Lesson Materials

- Slide show: ST7_Intro_To_Modelling_5a.pptx
- Student Sheet – Mathematical Modelling Cycle (reuse from lesson 3)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

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## Lesson structure

- Generating variables for a new model (Teacher led - 10 minutes)
- Constructing a model (Collaborative small-group work - 15 minutes)
- Making the model more sophisticated (Collaborative small-group work - 10 minutes)
- Linking your modelling to the diagram (Collaborative small-group work -5 minutes)
- Reflecting on modelling (Collaborative small-group work -5 minutes

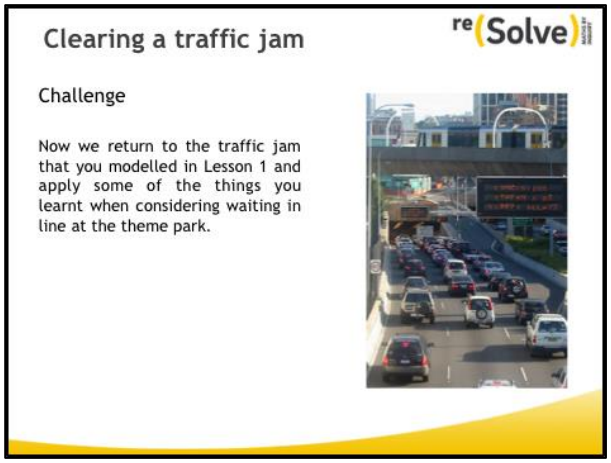
# Generating variables for a new model

This first section returns to the traffic jam situation of Lesson 1 and sets out the challenges of extending the model to cover flows in and out of the traffic jam as it clears.

Show the slide Clearing a traffic jam.

Explain that the aim is to draw on the insights gained when modelling queuing for the theme park ride, to develop a more sophisticated model for the traffic jam.

Remind students that in Lesson 3 they went round the modelling cycle a second time to improve the model for theme park queues and that's what they are now going to do for traffic jams.

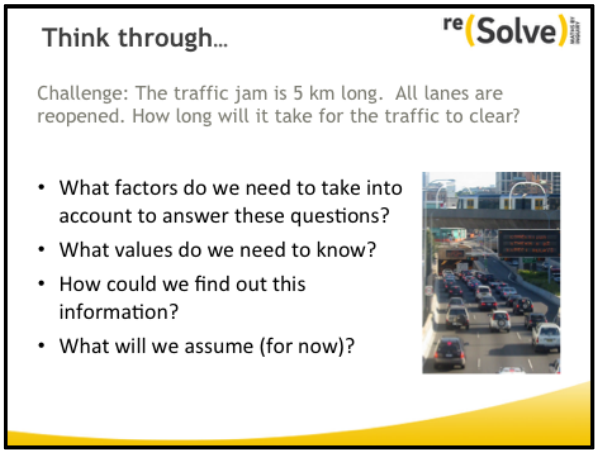



Show the slide Think through… which sets out the new challenge.

Students begin to work on finding out how long the 5km traffic jam will take to clear after all lanes are reopened.

**Enabling prompts**

- What factors are involved in this new situation?
- What information will you need to answer this question?
- What can you find out?
- What will you assume (for now)?




As usual, students should be prepared to share their reasoning with the whole class.

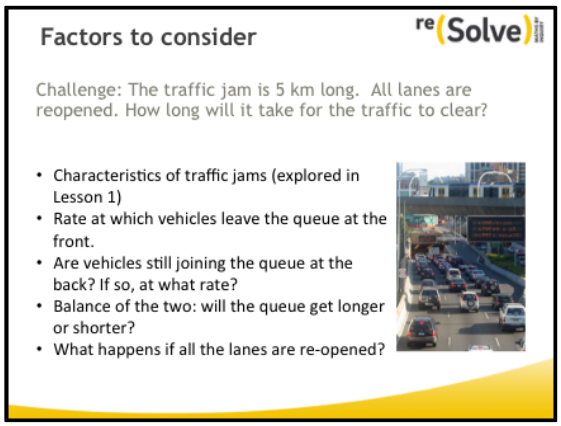
## Identifying the variables

Encourage students to discuss the situation in their groups, identifying the factors that are important

Show the slide Factors to consider.

**Enabling prompts**

- This slide lists the key things for you to consider.
- Think through what happens when a stationary queue starts to move.




## Sharing thinking

Collect ideas from the group about what they will need to consider for their model and how they will find the information

- by carefully considered estimates or
- by research (e.g. on the web, ask expert, …)

# Constructing a Model

Now the class has shared ideas, each group can move on to

- creating their own more sophisticated models
- drawing conclusions about the time taken to clear the traffic jam

Students may find this particularly difficult. To be able to support them it is important that you work through developing the model yourself before the lesson. Use the 'expected responses' below to inform your thinking. Some thoughts about how you might support students are given in the 'expected responses'.

### Expected responses

(Students should work with assumed numerical values before trying to do this algebraically!)

Rate at which vehicles leave the queue at the front depends on:

- driver reaction times: $t_r$ seconds ~ 2 seconds *(*In a first attempt student might assume that there is no reaction time and all cars start moving together)*
- the *n*th car won't move until $nt_r$ seconds after the front car moves
- When will the car 5 km from the obstruction begin to move? The value *of n* comes from the model in Lesson 1 – the number of cars in one lane of 5km length) *(*students can treat this as a single lane as they may assume that all lanes will leave at the same rate with no drivers switching lanes)*

Drivers have to drive to the where the obstruction was, at average speed of (say) 30 km/h.

- > 5/30 hours = 10 minutes (for the drivers at the back of the queue). So in the simplest model without reaction times it would take 10 minutes for the back car to drive to the where the obstruction was.
- The reaction time can now be introduced. There are 1750/3 vehicles per lane. So it will be (1750/3)*2 seconds before the back drivers can start to move, approx. 20 minutes.
- So time to clear approx. 30 minutes with three lanes open.

### Extending Prompts

- Adjust your model to include cars still arriving at the back of the traffic jam.

## Sharing thinking

Gather feedback from groups about their predictions about the time taken to clear the jam.

Explore differences in approach and results.

The key concepts are

- wait time which depends on the number of vehicles ahead and driver reaction times
- drive time to go from where you are stopped to the place where the obstruction was.

For the nth vehicle in a lane it is roughly

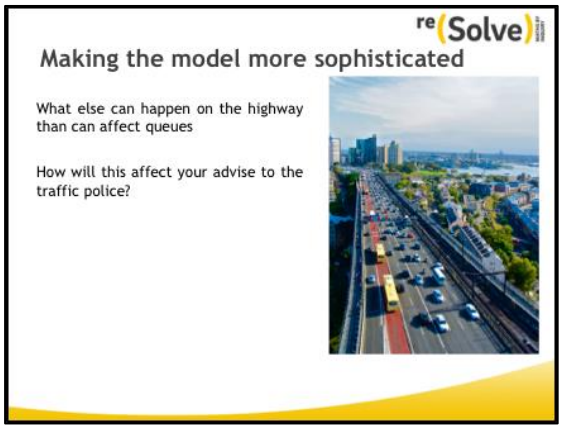
Time to pass obstruction = $nt_r + nL$

where $L$ is the average (vehicle length + separation). Essentially this is the average distance from front bumper to front bumper

# Making the Model More Sophisticated (Optional)

Use this phase of the lesson if you have time. The aim is to give students an opportunity to reflect on their models and to consider how it might be developed so that it is more sophisticated and better reflects the reality of traffic jams.

Show the slide Making the model more sophisticated.




Points raised may include:

- Different times of day – rush hour problems.
- Minor accidents in queue (e.g. lane changing) could cause further problems.
- Coping with long-term queues: people's need for food/drink and protection from heat/cold.
- Use of deviations from the highway.
- Having a place where people can make a U-turn and leave the queue.

## Sharing thinking

Ask groups to consider how good they think their model of traffic flow is.

*You might want to ask students to reflect on the idea that they have been modelling two situations in which flow is important (flow of traffic and flow of people)*

Where else may we meet the idea of flow?

(Some possible answers: water flow in pipes or rivers, gas flow in pipes, flow of electricity, cash flow, data flow on the internet, flow of products in a factory, flow of blood in the human body)

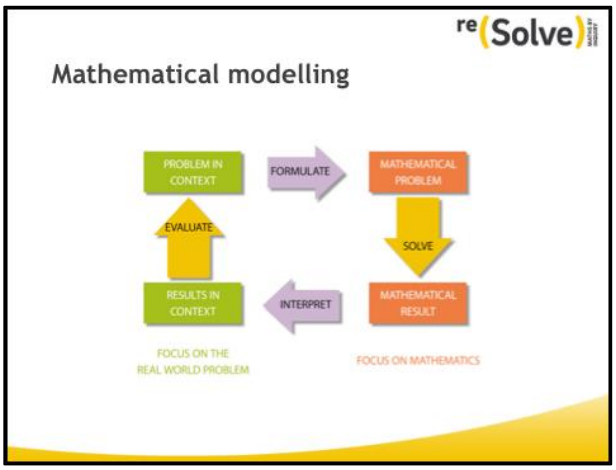
When is flow important, or potentially dangerous?

(Some possible answers: water flow in rivers is good for irrigation and for a healthy river, but too much flow can lead to flooding. The flow of data on the internet is often thought of as 'traffic': internet providers need to plan how to cope with peak times when lots of people are using the internet.)

# Linking your Model to the Modelling Diagram

In lesson 3, students linked their modelling of theme park queues to the modelling diagram. Here the aim is for them to do so for their traffic jam activity, now including both their Lesson 1 model and the more sophisticated model of this lesson. Students are to think more deeply about the modelling process as they have experienced it.

Show the slide Mathematical modelling.




Students can look again at the handout from Lesson 3.

Explain that in Lesson 3 we talked about the modelling process and linked first models of theme park queueing to the modelling diagram. Ask them now to relate their model of clearing the traffic jam to the modelling process - the four state boxes in the diagram and the process arrows that connect them.

### Enabling Prompts

- Can you identify where you were **formulating** the model by making assumptions?
- What exactly is your **model**
- Where exactly were you **solving**?
- What is your mathematical result?
- What is the implication of this in the reality of the traffic jam?
- Where were you **interpreting**?
- Where were you considering how good your model is? (i.e. where were you evaluating

## Sharing thinking

Call upon one group spokesperson to report on their analysis and use this to stimulate discussion of their experience in linking to the diagram. Choose this group carefully as you circulate so that you can have a stimulating discussion with the whole class.

Ask the class how well they managed to link their work to the modelling diagram and which parts they found straightforward and which were challenging.

# Reflecting on Modelling

In this final section, students reflect on their modelling experience as a whole and draw out key practical ideas about how to tackle modelling.

Show the slide Modelling real world problems

Ask students to discuss in pairs for a short while and then gather responses.

Important ideas to emphasise are:

- Keep the model very simple to start with.
- Consider using estimated values (or values found through research) for everything before allowing factors to vary.
- If you can, develop an algebraic model (perhaps using a spreadsheet).
- Think about trying to develop a graph.
- Always interpret mathematical answers in terms of the real situation.
- Evaluate how good your model is.