Quick Start Lesson: The Modelling Process

Australian Curriculum: Mathematics (Year 9)

ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems. (Year 9)

Lesson abstract

Students are introduced to modelling using the context of bushwalking – considering how much time you should allow for bushwalking. Students develop their own 'rule' and then compare to Naismith's rule. The mathematical modelling cycle is then introduced and discussed. Students then make a simple model for growth of a family over generations, linking their work to the modelling cycle.

Mathematical purpose (for students)

Mathematical modelling provides insight into real world situations and helps us make decisions.

Mathematical purpose (for teachers)

This lesson is a 'short-cut' alternative to reSolve *Unit 1 Introduction to Modelling* for classes starting at a later modelling unit. The lesson introduces students to the modelling cycle and the key modelling activities of: making sense of real-life situations, identifying important factors, making assumptions, understanding the structure of the situation, developing a simple mathematical model leading to a mathematical solution, interpreting this solution in the real-life context, evaluating the model's effectiveness and considering how it might be improved.

Lesson length: 50 minutes approximately

Vocabulary Encountered

- Model, modelling
- Assumptions
- Formulating
- Interpreting
- Evaluating

Lesson Materials

- Slideshow *ST7_Modelling_QuickStartLesson.pptx*
- Student Sheet - 110 Years On (one per student)

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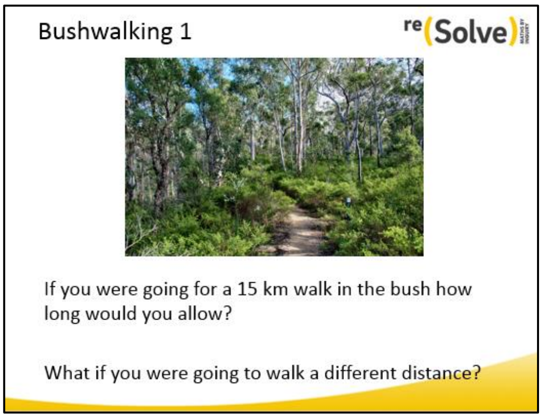
Lesson purpose and structure

- Bushwalking (Whole-class discussion together with some small-group work – 20 minutes)
- About modelling (Whole-class discussion - 10 minutes)
- Making your own model:110 Years On (small-group work – 15+ minutes)

This lesson is intended as an introduction to mathematical modelling for classes who need to skip reSolve *Mathematical Modelling Unit 1 (Introduction to Mathematical Modelling)* and start their substantive work with any other *Mathematical Modelling* units. Those units (Units 2 to 5) can be done in any order. The lesson introduces the idea of mathematical modelling, shows the modelling cycle used in other units, and introduces key terminology and central concepts (such as the need to make assumptions).

Modelling time for a bushwalk (about 20 minutes)

Use the slide Bushwalking 1 to introduce the context.



In this first section of the lesson introduce the context: bushwalking. It is likely that everyone will have done some bushwalking at some point. You may need to clarify for some students that bushwalking is the term used in Australia to include *hiking*, *tramping*, *hill walking*, *rambling* or *trekking*…. It covers everything from walking along a track in a park through to multi-day expeditions in dangerous areas.

Start by asking students to think about how long they would allow for a walk that was 15 km long.

The discussion should

- highlight the importance of estimating time (e.g. for convenience of transport, safety)
- share students' experiences of planning walks,
- identify some factors to consider (e.g. are there young children in the party, carrying a pack or not),
- identify the need to make some assumptions about the terrain, walking speed etc.

Suggest the usefulness of having a general rule to use for bushwalking and allow students some time to develop a rule in small groups.

Gather different rules from students: encourage different ideas. For each, write the rule down in the student's own words. Positively encourage students to verbalise the rules rather than give them only as formulae in terms of symbols. It may be that they are applying what is effectively the same rule but thinking about it differently. For example, they may say something like "allow one hour for every 5 km walked" or "for every 5 km it takes an hour" or "One kilometre takes about 10 minutes to walk."

When you have collected several different rules write down each one in symbols, in the form, "*T* =" where *T* represents time (in hours) for a bushwalk of distance *D* kilometres. Do this so that you can show that although rules may be expressed differently in words and make different assumptions, when written as a formula in mathematics they may have the same underlying mathematical structure, or perhaps a different structure.

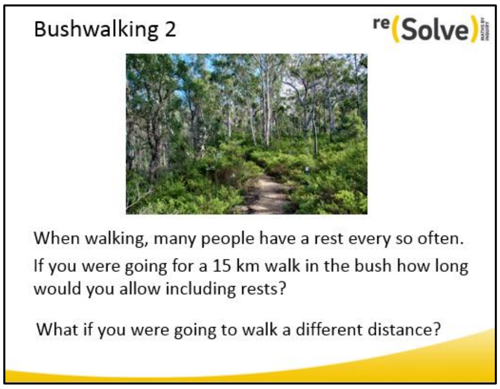
At this point it is worth pointing to the detail of each formula and highlighting that we need to divide the distance of the walk by the average number of kilometres per hour that we walk to find out the time taken. Although we have different 'rules', the basic mathematical structure is probably the same in each case.

Highlight that the general formula might be considered to be $T = \frac{D}{s}$ where T is the time of the walk in hours, *D* is the distance of the walk in kilometres and *s* the number of kilometres per hour that we walk (the walking speed). Point to the consistency of the units across the different values we are using. [hours = distance/ (distance / hour)]

Students will probably comment that these formulae only give approximations or a general idea about how long the walk will take. Students may suggest that you may want to allow some time to have a rest, lunch, to look at a view, etc. Making a better model is the next task.

Point out that starting with a simple model, then working step by step to make it more complex, is a useful model-building strategy.

Show the slide Bushwalking 2, which considers one of the factors so far ignored, that of taking a rest.



Students now improve their rule by adding a way of estimating how much time might be spent in resting on the walk. For example, they may suggest adding 10 minutes for every 5 km walked.

Ask them to consider how much time their new rule might add for a 15 km walk? What about different distances?

Enabling prompt

- What about twice the distance (30 km)? What about 10 km? How are you working it out?

In whole class discussion, return to the students' rules shared earlier and see how students developed them. For example, you may have rules such as "allow one hour and 10 minutes for every 5 km walked".

Again write down each one in symbols, in the form, "T =" .

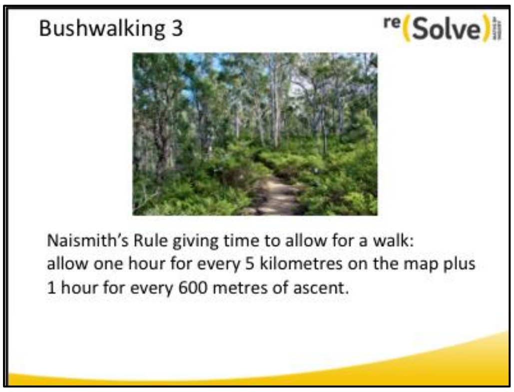
Highlight in each how the new detail has been taken into account.

In this case the general formula can be like any of the expressions below (show at least two):

- $T = \frac{D}{s} * \text{(1 hour+10 minutes)}$ (giving an answer in hours and minutes) or
- $T = \frac{D}{s} * \frac{70}{60}$, or
- $T = \frac{D}{s} * 70$, giving the answer in minutes rather than hours, or perhaps even
- $T = \frac{D}{s} + \frac{D}{6s}$ separating the walking and the resting times and using some algebra.

(Point out that writing the rule in the first form avoids the problem of 10 minutes being 0.167 of an hour.)

It is likely that students will have developed less sophisticated ways of expressing their thinking. Do not worry about this: keep the focus on the modelling by emphasising the connection between the real situation (bushwalking) and the maths. Point out, through questioning, how each term relates to the context (for example the significance of the 10 minutes in the brackets) and how the structure of these rules is in fact the same.



Show the slide Bushwalking 3. This introduces Naismith's Rule that is used around the world by walkers to estimate how long their walk might take them. As you can see it recognises that walking uphill takes longer than walking along flat ground and allows 1 hour extra for each 600 metres of ascent.

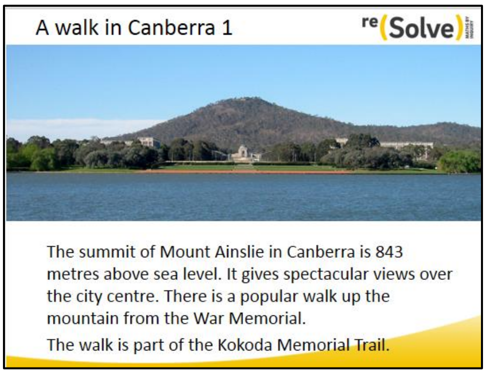
According to Wikipedia, Naismith was Scottish mountaineer who developed his rule in 1892. More complex personalised versions are used by serious walkers.

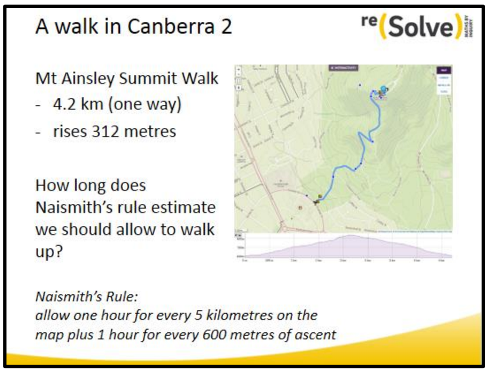
You may want to check student understanding of this by asking how long this 'rule of thumb' suggests we should allow for a 15 kilometre walk that includes 300 metres of ascent (3hours + ½ an hour).

Point out how this differs from the rules students developed (it does not only involve average speed).

Applying Naismith's rule

Show two slides A walk in Canberra 1 and then A walk in Canberra 2.





Students are asked to find the time that Naismith's Rule suggests should be allowed for the trip. They will need to take care with the calculations:

- 4.2 km is 0.84 of 5 km so time needed is 0.84 hours.
- 312 m is 0.52 of 600 m so time needed is 0.52 hours.

Total time needed is 1.36 hours (or 1 hour 22 minutes rounded to nearest minute, so about 1 hour 30 mins).

Alternatively, students could work using Naismith's Rule expressed in minutes, that is, "for every 5 kilometres on the map allow 60 minutes and allow 60 minutes for every 600 metres of ascent".

It will be worth pointing out that this still neglects other factors that may affect the length of time a walk takes; for example, taking a rest (unless it is included in the average speed), stopping to admire the view and so on.

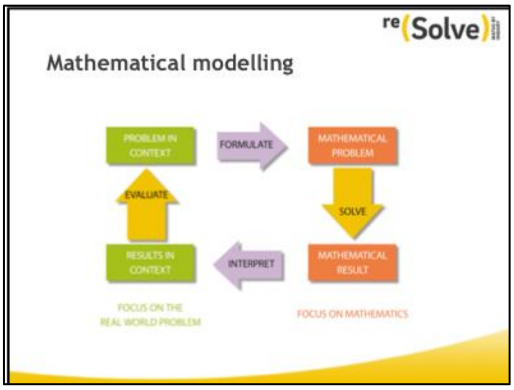
Note that the data supplied may not be accurate. This data is from a walkers' website, but other sites differ (e.g. length 4.5 km, rise 249m). Students might consider how important it is to search for accurate data. Probably it is not sensible to spend more than a few minutes in such a search because it would not make much difference to your plans.

Modelling (about 10 minutes)

The next phase of the lesson focuses on important features of mathematical modelling. This whole-class discussion is designed to provide some insight into how the above activity involves mathematical models and modelling. The model in the diagram is used in all the reSolve *Mathematical Modelling* units.

As the Teachers' Guide explained:

> *"A mathematical model is an abstract mathematical representation of how factors in a situation in the real world are structurally connected. It provides insight into the situation and can be used to answer certain questions that arise."*



Use the slide Mathematical modelling to prompt a whole-class discussion of how the modelling process relates to the work they have been doing. The two green boxes belong to the real world; the two orange boxes belong in the mathematical world. The purple arrows are transitions between the real world and mathematics. The yellow arrows take place within one world.

Work through the cycle starting at the 'Problem in context'.

Start by explaining that mathematical modelling is when we apply mathematics to make sense of, and answer questions that arise from, real-world situations (such as bushwalking).

In creating a rule for bushwalking there were many factors that we might have taken into account. Ask students to suggest some.

Expected responses

- How far the walk is.
- The (different) speed(s) one person might walk with, or that different people walk with.
- How much time people spend resting, perhaps related to fitness.
- If it takes more time to go up or down hill.
- If different people walk at different speeds.
- Different terrains (e.g. rough track, deep mud, many beautiful views, cliff edges).

Explain that, at first, we might only work with some of these factors – the most important, we hope. It is always a good idea to keep things simple at the beginning.

Look back at what students did to develop their first model: highlight which factors they took into account, most likely distance and speed. Point out how they ***formulated*** a mathematical way of working with these: developing a formula that had mathematical structure related to reality – the way to find time is to divide distance by speed. Also highlight that the students chose not to take into account other factors – particularly when they would make the mathematics more complex.

In other words, we ***made assumptions***. We assumed that the person walks with constant (average) speed during the day. Ask students, “what other assumptions did we make?”

Expected responses

- Everyone walks at the same speed.
- All people walk at a constant speed (5 kilometres per hour).
- People spend no time resting.
- It takes no more time to go up or down hill.
- People walk at the same speed through all different terrains.

We arrive at a mathematical formula – a model for the situation that will allow us to answer the question we were asked. We have a mathematical question (find T from D and s) which we ***solve*** to arrive at a mathematical answer (e.g. $T = 2.87$ hours).

We ***interpret*** this in terms of the situation. For example, to do the walk will take us 3 hours.

We ***evaluate*** this, asking the question “does this seem a reasonable outcome?” Ask your students to draw on their experience to ask if this seems reasonable. Clearly this may be reasonable for some people but not for others. This is when we need to consider whether any mistakes have been made, or whether any of our assumptions are unreasonable (e.g. assumed walking speed), or whether critical factors have been omitted.

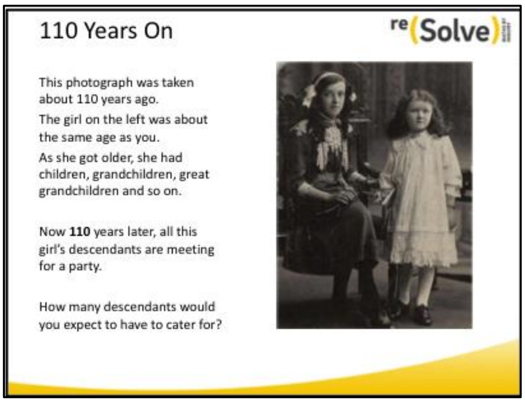
This may reveal a need to develop a better model. Before starting out, we need to consider if we will be able to cope with the new mathematics required, and if the improvement is likely to make a practical difference. Review how Naismith’s rule provides a more sophisticated model than their initial attempts. (It considers how walking uphill takes more time than walking along level ground.)

If we decide to improve the model, the cycle begins again.

Explain that the modelling cycle gives an overview of the types of activities involved when developing a mathematical model. The cycle doesn’t have to be strictly adhered to, but it does give insight into the sequencing of different activities.

110 years on

Show the slide 110 Years On. Use the remaining time for students to work on the task. Students can work in pairs or small groups.



Distribute Student sheet: 110 Years On.

This prompts students to:

i. Think carefully about the context by considering all the different factors they might take into account.
ii. Identify which factors they will include and which they will not worry about for now: effectively ignoring them or keeping them constant.
iii. ***Formulate*** their mathematical model - a relationship between the factors (variables).
iv. ***Solve*** their mathematical question by using mathematical techniques.
v. ***Interpret*** their answer.
vi. ***Evaluate*** the effectiveness of their model and consider how they might improve it.

Possible factors and assumptions

- how many children are born to the girl (assume 2)
- after how many years the girl has children (assume 10 years – when she is about 25)
- how many children are born to descendants in each generation (choose a constant value such as 2),
- whether or not each descendant has children (assume all do),
- the age gap between generations (assume this is constant – possibly 20, 25 or 30 years),
- any deaths of descendants (assume none, although this is clearly unrealistic given the time scale)
- major events like wars and famine which disrupt families (assume none),
- whether all descendants attend the party (assume all do)
- whether descendants attend with partners (assume not).

The mathematical structure of the problem involves growth (which is basically exponential in nature) over several generations. This is why the value of 110 was chosen – with the girl first having children after about 10 years and a generation gap of 20 years there are 5 generations, with 25 years there are 4 generations, with 30 years there are 3 generations.

With the simple assumptions suggested above (2 children per descendant) the expected outcomes are:

3 generations: (2 + 4 + 8 + 16 = 30),

4 generations (2 + 4 + 8 + 16 + 32 = 62),

5 generations (2 + 4 + 8 + 16 + 32 + 64 = 126)

As students work through the questions on the Student sheet: 110 Years On ask them to identify where they are in the modelling cycle diagram.

If students are making assumptions that would make the mathematics difficult allow this, but monitor the situation so that students don't spend too much time following an unproductive line of reasoning.

If time is short, focus especially on the first three parts of the worksheet. The important thing is to allow students to make a start by making sense of the situation and thinking through how they will make this into a mathematical situation and question. This provides engagement in, and insight into, this very important part of formulating a mathematical model.

Acknowledgement

The task '110 years on' originally appeared in a different form as part of the Bowland Maths Assessment tasks. It can be found at http://www.bowlandmaths.org.uk/assessment/110_years_on.html along with notes about using the task.

110 Years On

Name:

This photograph was taken about 110 years ago.

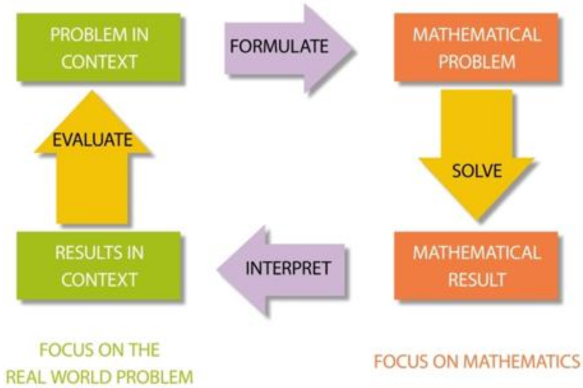
The girl on the left was about 15.

As she got older, she had children, grandchildren, great grandchildren and so on.

Now, 110 years later, all this girl's descendants are meeting for a party.

You need to decide how many descendants you will cater for.

As you work through the questions below, think about where you are in the modelling cycle.



1 What are the different factors you could take into account?

2 Which of these factors will you take into account, and which they will you not worry about for now? Why? (Will you ignore them or keep them constant?)

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3 What assumptions are you going to make?

4 Identify your mathematical question.

5 Solve this mathematical question

6. What does your answer tell you?

7 Evaluate the effectiveness of your model.
How might you improve it?