re(Solve) MATHS BY INQUIRY

MATHEMATICAL MODELLING

How Risky is Life?

Lesson 5: How Risky for Me?

Australian Curriculum: Mathematics (Year 9 and 10)

ACMNA208: Solve problems involving direct proportion (Year 9)
ACMSP226: Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'. (Year 9)
ACMSP228: Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly and from secondary sources (Year 9)
ACMSP253: Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (Year 10)

Lesson abstract

This lesson explores variation and prediction. Students use a computer simulation to explore the random variation that may be expected in the data, getting a feel for whether reported changes (e.g. in number of drownings) are significant, or likely to have happened by chance. The unit closes by reflecting on the extent to which overall probabilities apply to individuals. Finally, students summarise their mathematical understanding of risk in a poster.

Mathematical purpose (for students)

Data varies from year to year because of random fluctuations and because of real changes. Simulation can help decide what is real and what is random.

Mathematical purpose (for teachers)

The aim is that students get a feeling for random variation and come to see that available data can provide *best estimates* to inform future decisions and an informed scepticism is sensible towards dramatic statements on risk. First students distinguish random fluctuations in data from real changes from year to year. Is one year's data relevant for future years? Is population data relevant for me? A computer simulation supports an exploration of the degree of random variation that may be expected in the data. This helps make judgements about the size of variation that might indicate a real change.
Finally, students make a poster that communicates their mathematical understanding about making decisions in everyday life. The aim is to surface all the understanding about ideas of risk and probability that students have engaged with when working on the unit and communicate them in concise, clear language and diagrams.

Lesson structure

- Why does data fluctuate from year to year? (Whole-class discussion - 15 minutes)
- Exploring random fluctuations (Paired work and whole-class discussion - 20 minutes)
- How far does population data apply to me? (Whole-class discussion – 10 minutes)
- Make a poster (30 mins)
- Critique posters (15 mins)

Lesson Length *90 minutes approximately, allowing time to make posters*

Vocabulary Encountered

- Random variation
- Simulation
- Significance

Lesson Materials

- Slide show *ST7_Risk_5a_How_Risky_For_Me.pptx*
- Student Sheet 1- Real or Random? (one per group)
- Student Sheet 2 - Assessing Posters (one per group)
- Access to http://resolve.edu.au/st-apps/ or downloaded zip file http://www.resolve.edu.au/sites/default/files/simulating-random-variations.zip (student access preferable)
- Data sheets handed out in previous lessons
- Poster paper, pens, (graph) paper, scissors, glue etc.

Why Does Data Fluctuate from Year to Year?

This section helps students understand that data will vary from year to year for two quite different reasons.

- There are random 'chance' fluctuations in data that we can do nothing about. This lesson will help us to estimate the amount of random change we should expect from year to year and get a feel for how much variation might be expected.
- There are underlying changes in the real situation. For example, many people may decide to stop smoking lowering the rates of a wide variety of diseases including cancer, or the government might lower speed limits lowering car accidents. Basic sanitation has dramatically increased life span over the last century.

A topical area where this is often questioned is that of climate change. Are changes in climate that we see from year to year just due to random variation or is there some underlying change?

In this first section of the lesson, focus students on the issue of random variation. Set students thinking by indicating:

- In 2015, the population of Australia was about 25 million.
- There were 265 murders

Ask students to discuss this question in pairs, the share responses.

- How many murders would you expect there to be this year? Write down your estimate and a number of murders that would surprise you. Give reasons.

Expected responses

Estimate	Reason
250	It will be about the same. I would be surprised by 1000, because that would definitely mean things were getting worse.
300	The trend is probably rising. I would be surprised by 500, because I do not think things are changing very fast. I would also be surprised by 200 because I do not think things are getting better.
You can't tell	It is just random chance. No one knows when murders will be committed.

Ask students to vote with a show of hands on the number of murders they would expect (in brackets is an idea of how likely each range is).

- How many people think that next year’s number of murders will lie:

  between 100 and 500? (You could bet your life on it?)
  between 150 and 350? (Almost certain?)
  between 200 and 300? (Very likely?)
  between 230 and 300? (Likely?)
  between 262 and 268? (Unlikely?)

Prompt further discussion by asking:

- What about the year after? (Same answer?)

- Why do you think the numbers fluctuate from year to year?

Make sure points the following emerge from the discussion, and especially note that there may be changes that affect the situation as well as random fluctuations.

Changes in the real underlying causes

- The number of murders may increase if guns or other weapons become more commonplace.
- If other crimes decrease, like organized crime and drugs, then murders may reduce as well.
- In using data from the past for prediction, you need to be vigilant – think about how the situation may have changed and how big an effect this may have had.

Random fluctuations

- Even when nothing has changed, there will be some random variation. On average, about the same number of people will want to murder someone, but it will vary.

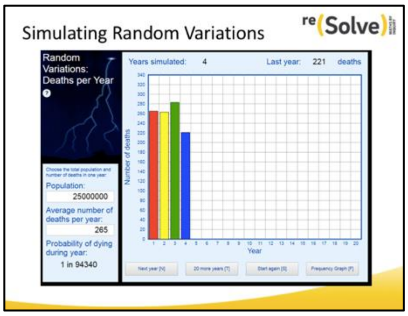
Summarise to bring out the distinction between changes in the real situation and random fluctuations. It is important to understand this distinction. Make the point that if the news says "Murder on the increase" we want to know whether this is likely to be just random fluctuation (so it might change back next year) or whether there is an underlying cause.

The size of fluctuations that we might expected is addressed in the rest of the lesson.

Exploring Random Fluctuations in Data

This section aims to give students some understanding of random fluctuations, and a heuristic feeling for their likely magnitude. It uses the app “Simulating Random Variation” that can be accessed directly from http://resolve.edu.au/st-apps/ on computers or tablets or phones (although phone screen sizes are really too small).

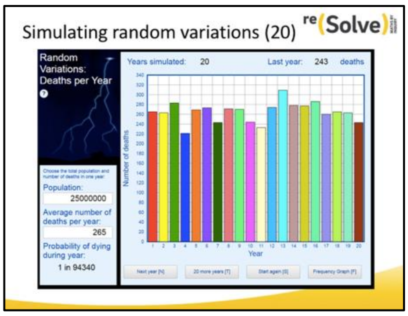
If online use is not desirable, the app can be downloaded as a zip file from http://www.resolve.edu.au/sites/default/files/simulating-random-variations.zip. Two clicks going through the downloaded folders to “start.html” should start the app. However, on some computers to get the proper display shown in the slideshow, you will need to download the zip file to your computer, extract files (do not just open the zip), then start.



Introduce the software app "Simulating random variations", using the slides ***Simulating random variations*** and ***Simulating random variations (20)***, or demonstrate directly with the app.

In the "Population" box, enter the total population, 25 000 000 and in the "Deaths" box enter, say, 265 (for the number of murders). Point out that, automatically, the computer calculates the proportion of people that were murdered (1 in 94 340). Check that students understand where the value of 94 340 comes from (25,000,000÷265 = 94339.6).

(Of course, the app does not only work for murders!)

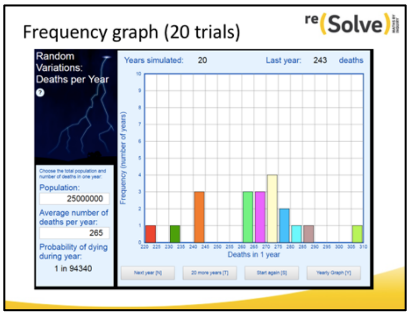


Clarify how the data (number of murders in the year) has been obtained by the computer simulation. Students can imagine that the computer has 25 million pieces of paper in a hat and 265 of them say M ('murdered'). For each person in Australia, the computer randomly selects a piece of paper from the hat, writes down what it gets (M or not) and then puts the paper back in the hat. When it has finished, it counts the number of M’s. The assumption is that everyone has the same chance of being murdered. This program shows what happens with just random variation from trial to trial, and no underlying trend for change. This simulation is a simple mathematical model of the number of deaths in a population.

Ask students to explore this – preferably using their own device. Ask them to click on the "Next year" button several times and explain that, as they do so, the computer is running a simulation to see what the number of murders might be. Emphasise that the computer is *not* making predictions here; it is just simulating a random process where the population is 25 million and the probability of something happening is about 1 in 100,000.

Students should find that in most of the trials, the number of murders is between 240 and 290, but there are some outside this range.

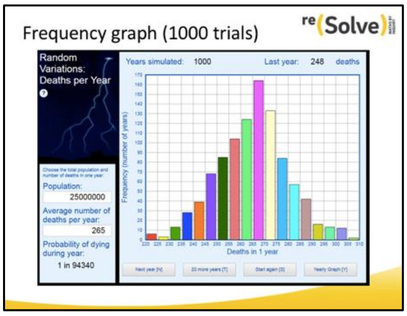
Going deeper with the frequency graph

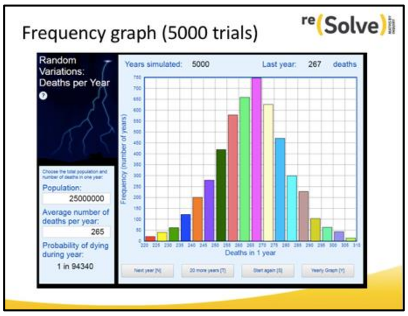


Use the app or the slide ***Frequency graph (20 trials)*** to discuss the frequency graph of the number of times each number of deaths was obtained.

This slide shows the number of murders when the simulation was run 20 times. This can be thought of as running the simulation for 20 different years (with constant population and death rate), or alternatively, running the simulation for one year 20 times. Either way we can see the effect of randomness. Ask students to interpret what has happened. (ANS: Most of the answers seem to be around 240 to 290, with a few outside this range.)

Repeatedly click on the "20 more trials" button to see how the frequency graph changes after many more random trials. Use the app or the slides ***Frequency graph (1000 trials)*** and ***Frequency graph (5000 trials)*** to emphasise how the distribution becomes more 'bell-shaped' the more trials that are run. (In fact, it is moving towards the normal distribution.)





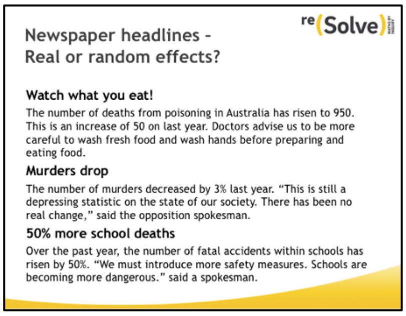
Real or random changes? Test with the app.

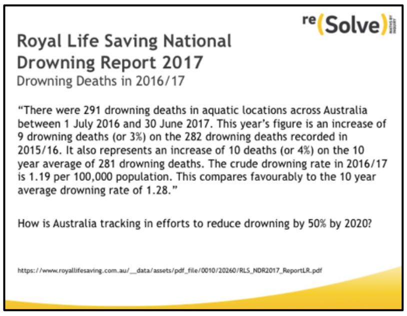
In almost every scientific study or experiment, the results must be checked mathematically to make sure that a change is due to a real cause, not just a random fluctuation. When scientists say a result is *significant*, they don't necessarily mean *big* or *important* - just that they are very sure that they have measured something real – not a random fluctuation.

Hand out Student Sheet 1- Real or Random? and ensure that students have access to the Australian deaths data from Lesson 3.

Show slide ***Newspaper headlines – Real Or random effects?*** and later slide ***Royal Life Saving National Drowning Report 2017.***

Ask the students, working in pairs, **to use the app** to decide whether the newspaper is trying to create a story out of nothing more than random fluctuations in the data. They can also use data handed out in previous lessons.





Expected Responses

Watch what you eat! *The number of deaths from poisoning in Australia has risen to 950. This is an increase of 50 on last year. Doctors advise us to be more careful to wash fresh food and wash hands before preparing and eating food.* Enter 25 million and 900 deaths into the simulation app. Note how frequently the simulation gives 950 or more deaths. It appears to be in about 10% of the trials, so there is a reasonable chance that this high number has just occurred by chance. The advice to wash hands is important anyway. The important thing is to reduce deaths, and this has not occurred.
Murders drop *The number of murders decreased by 3% last year. "This is still a depressing statistic on the state of our society. There has been no real change," said the opposition spokesman.* Running the simulation with 269 murders per year for Australia, showed that the number of murders dropped below 260 (a 3% drop) in about 10% of trials. The spokesman is right to say there is no evidence of a real change.
50% more school deaths *Over the past year, the number of fatal accidents within schools has risen by 50%. "We must introduce more safety measures. Schools are becoming more dangerous." said a spokesman.* Being able to say whether this data indicates a trend, rather than random fluctuations, depends on the annual number of fatal accidents in schools. So this needs to be estimated first. The chart "Base risk of death by age" showed that school age children have a low risk of death – it appears to be about 3/10000 averaging over school ages. There were 73199687 Australians in the 1 – 24 age group in 2015, so estimate that there are 4 million school children. Hence there would be about 1200 deaths of school age in Australia. We also know (Causes of death table) that only about one third are due to accidents, so approximately 400 deaths each year. School goes for about a fifth of children's waking time. So this gives an estimate of 80 deaths of children at school each year. In running the simulation a thousand times, the number of deaths did not reach 120 (a 50% rise), and only once went over 110. So, for a 50% rise, it is very likely that there was a non-random change. Note that if we were to make different estimates, we might come to a different conclusion, e.g. for 8 deaths per year, the simulation went to 12 or more (50% increase) quite frequently. Note also that a non-random change might have in fact been due to a 'random event' such as a plague, or a tsunami hitting a school. It is not necessarily an ongoing change.
Royal Life Saving National Drowning Report 2017 - Drowning Deaths In 2016/17 *There were 291 drowning deaths in aquatic locations across Australia between 1 July 2016 and 30 June 2017. This year's figure is an increase of 9 drowning deaths (or 3%) on the 282 drowning deaths recorded in 2015/16. It also represents an increase of 10 deaths (or 4%) on the 10-year average of 281 drowning deaths. The drowning rate in 2016/17 is 1.19 per 100,000 population. This compares favourably to the 10-year average drowning rate of 1.28.* *How is Australia tracking in our efforts to reduce drowning by 50% by 2020?* The change of 3% in 291 deaths by drowning is probably not a real change: changes this large happen frequently in our model because of random fluctuations. However, although the number of drownings has increased, the rate per 100 000 has decreased. This indicates that the population has grown. The improvement in the rate per 100 000 population may be more important that the change of 9 deaths. There may have been a drop in drownings. Next year's data may show a trend in the same direction.

Summarise

If necessary, remind students that these are simulations, using random numbers. The computer can't really work out how many people will die next year. But, if you are looking at real numbers, and see a change, it is very important to be able to know if it could just be a random variation. In any serious research, the results must be carefully checked to make sure that they can't be explained by chance.

Spend a few minutes in discussion with the whole class to summarise the work in this part of the lesson.

Does Population Data Apply to Me?

The numbers we have been looking at are from the whole population. We now consider how far these probabilities apply to an individual, and indeed whether it even makes sense to apply them to an individual.

Discuss the following questions:

- Do these estimates of risk (of all causes of death) apply to you and to me?
- How typical is each of us of the Australian population?
- Of course, everyone is an individual with their own characteristics – male or female, in good health or not, risk-taking or cautious. But how far are these differences likely to affect your risk level?
- Being more specific, the data suggests that the base risk of dying in the next year for a 15-year old boy is about 1 in 5,000. Is this a reliable estimate of the risk for students in our class?

Points that should emerge include:

For unexpected death:

- While a cautious approach to life can reduce risks of accidental death, excessive caution (e.g. never going out on your own) has other undesirable effects on health and opportunity to enjoy life.
- Being cautious is unlikely to change these risks by a huge amount (e.g. for more than half the people killed in road accidents, it was not their fault).
- Sometimes it is easy to modify behaviour, and sometimes it is hard and has undesirable costs.

For illness:

- Medicine is learning more all the time about the factors that affect chances of disease. The factors can be lifestyle (e.g. smoking), environmental (e.g. asbestos exposure) or genetic (e.g. some diseases run in families). Some of these are very strong risks (e.g. smoking), but very few are straight “cause and effect”.
- Is there any more detailed data that would allow a better estimate? If so, would it change the estimate enough to make caution worth the effort? (Consider decisions for an individual, and for the community as a whole.)
- Some important data is readily available – male/female and age, and the effects of smoking are the prime examples.

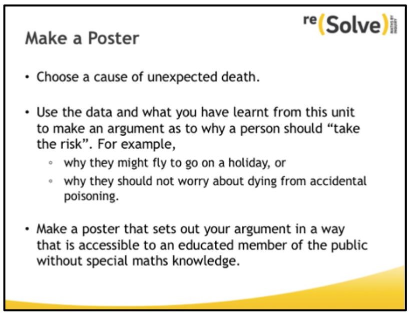
Generally, the population data provides a good rough estimate that enables you to:

- gain a realistic assessment of the various risks
- bear the numbers in mind when taking your life decisions
- bear the numbers in mind when taking decisions for a community.

Unless we can find and analyse data for sub-populations, **the base risk provides the best estimate available.**

Making a Poster and Assessing Others

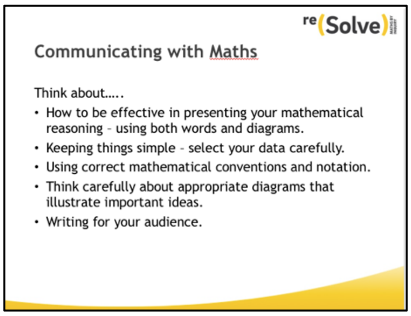
Finally, pairs of students make a poster (in class or at home) to illustrate what they have learned. Show slide ***Make a Poster***. The poster should encourage members of the public to “Take the Risk”. The challenge is to take a possible cause of unexpected death such as those they have considered during their work on the unit and to develop a poster that clearly communicates why they shouldn’t worry if they “Take the Risk”.

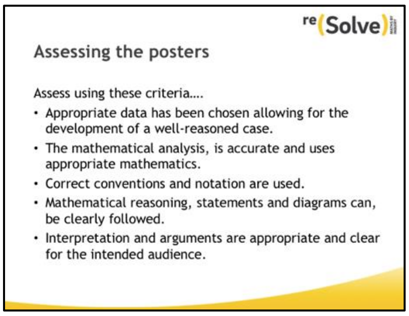


They should start thinking about how the key ideas that they have learned about when working on the unit might put someone’s mind at rest if they, say, travel by air, or travel on the road.

Students should choose their cause of death carefully. It is good if different groups select different causes.

Show the slide ***Communicating with Maths***, which highlights important aspects that students should consider when developing their poster. Show and discuss the slide ***Assessing the posters***. The poster has to explain their ideas to an educated member of the public who does not have much mathematical insight in this area. Students should make the mathematical ideas as clear as possible. Diagrams may be useful.





Assessing posters

When the posters are complete, arrange for students to critique each others’ posters. One way of doing this is for small-groups to exchange posters and for each group to make written comments about the others’ poster using the criteria on Student Sheet 2 - Assessing Posters.

Allow time for final review.

Real or Random?

Name: ______________

Are the claims made in these newspaper reports likely to show real trends or simply random variation?

Investigate by using the app "Simulating Random Variation" to conduct simulations to find out whether random variation is likely to cause such a large effect.

Watch what you eat! The number of deaths from poisoning in Australia has risen to 950. This is an increase of 50 on last year. Doctors advise us to be more careful to wash fresh food and wash hands before preparing and eating food.
Murders drop The number of murders decreased by 3% last year. "This is still a depressing statistic on the state of our society. There has been no real change," said the opposition spokesman.
50% more school deaths Over the past year, the number of fatal accidents within schools has risen by 50%. "We must introduce more safety measures. Schools are becoming more dangerous." said a spokesman. (You will need to also look at the Australian data from Lesson 3)
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Simulating Random Variation

App for computers, tablets, phones: http://resolve.edu.au/st-apps/

Downloaded zip file for offline computer use comes from http://www.resolve.edu.au/sites/default/files/simulating-random-variations.zip

[Download the zip file to your computer, extract files (do not just open the zip), then start.]

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

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Assessing Posters

Name: ____________

Comments on success and on how the poster can be improved

Name of Group/Poster being assessed:

Chosen and used appropriate data allowing for the development of a well-reasoned case?	
Presented mathematical analysis that is appropriate and accurate?	
Used correct mathematical terms and notation?	
Presented mathematical reasoning, statements and diagrams that are easy to follow?	
Presented interpretation and arguments for a definite conclusion that are appropriate and clear for the intended audience?	

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