How Risky is Life?
Lesson 4: Phoney figures?

Australian Curriculum: Mathematics (Year 9 and 10)

ACMNA208: Solve problems involving direct proportion (Year 9)

ACMNA210: Express numbers in scientific notation (Year 9)

ACMSP226: Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'. (Year 9)

ACMSP247: Use the language of ‘if ....then, ‘given’, ‘of’, ‘knowing that’ to investigate conditional statements and identify common mistakes in interpreting such language (Year 10)

ACMSP253: Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (Year 10)

Lesson abstract

The lesson first relates the analysis in previous lessons of the unit to the modelling process and diagram. Students then move on to a series of exercises in critiquing interpretation and evaluation of data, done mainly in think-pair-share groups. The aim is to practise looking at data sensibly, seeing the story presented in a direct reading of data but then looking through it make more sophisticated interpretations.

Mathematical purpose (for students)

Beware of taking data at face value and learn to avoid common pitfalls.

Mathematical purpose (for teachers)

The lesson first reviews the modelling cycle and develops further understanding of data analysis as descriptive modelling. The second part of the lesson focusses on developing awareness about how data might be accidentally misinterpreted or deliberately misused in advertising and commerce, politics and elsewhere. Student practise skills for looking at data sensibly, including (a) searching for additional variables to explain surface ‘face-value’ data features, and (b) thinking about both absolute numbers and relative proportions.

Lesson Length 45 minutes approximately (+optional 10 minutes)

Vocabulary Encountered

- Data-driven modelling
- Critiquing inferences

Lesson Materials

- Slide show *ST7_Risk_4a_Phoney_Figures.pptx*
- Student Sheet 1 - Phoney Figures? (one per student or per group)

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

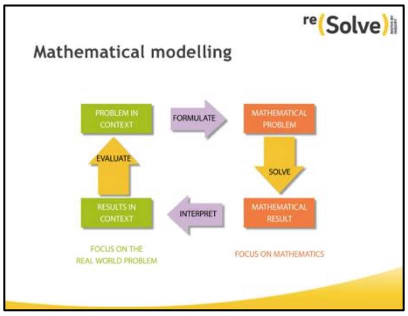
aamt

Lesson structure

- Reviewing the modelling process (Teacher introduction and group work - 15 minutes)
- Critiquing and improving inference from data (Collaborative think-pair-share - 3x10 minutes)

Reviewing the Modelling Process

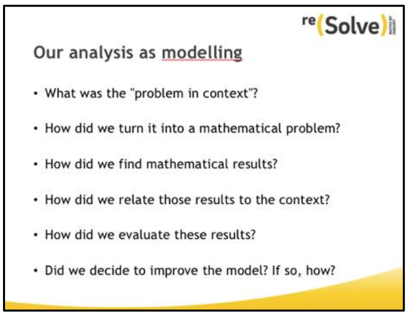
In this section we relate the work that students have done in the first three lessons to the processes of mathematical modelling as set out in the modelling diagram.



Introduction

Show the slide ***Mathematical modelling***

Explain that our thinking about risk has focused on data and what it can tell us. Now our task is to relate what we have done to the modelling processes set out in the standard modelling diagram.



Show the slide ***Our analysis as modelling***

Ask students:

What was the "problem in context"?

How did we turn it into a mathematical problem?

How did we find mathematical results?

How did we relate those results to the context?

How did we evaluate these results?

Did we decide to improve the model? If so, how?

Working in groups ask students to consider each of these questions and be prepared to explain their reasoning to the class.

Expected responses

- What was the "problem in context"?
  How risky is life? We first narrowed the question to probability of death within a year from unnatural causes. We noted the difference between the hazard (how serious the event is) and probability of occurrence.
- How did we turn it into a mathematical problem? (Formulate the model)
  We found data that was relevant to the problem and focused on the data that seemed to have meaning.
- How did we find mathematical results?
  We displayed the data in a variety of ways, first looking for those displays that clarified the size of the risk (notably the 'Swan diagram').
- How did we relate those results to the context?
  We often had to think about the exact definition of each category of data.
- How did we evaluate these results?
  We saw that death from unnatural causes is extremely unlikely in Australia, so a more detailed model was needed to answer our fundamental question "How risky is life?"
- Did we decide to improve the model? If so, how did we do it, and what else might be done.
  Yes. We broadened the question to include death from all causes, went around the modelling cycle again, and saw that illnesses were the major cause at all ages - but that even these were a small statistically small risk until about age 60. We could next have gone into more detail than that, looking at specific illnesses and how risks could be minimized, e.g. by lifestyle choices. It is also important to look at hazards other than death. Car accidents or near drowning, for example, can have a major impact on people’s lives and deserve to be counted when answering “how risky is life?”. We have not done that yet.

A Note on Data driven modelling

Modelling that is driven by data has some differences to analytical modelling (e.g. in the other units of this special topic). Key features of data-driven modelling are that:

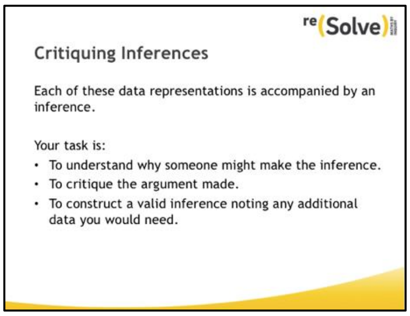
- it is descriptive, not immediately concerned with underlying mechanisms.
- formulation is about focusing/simplifying the questions of interest, then finding data that will help answer them - a two-way process, since there will be a lack of data for some questions.
- the variables are theoretically-chosen to match the questions, but relationships are established by the data then selected.
- solving is about finding ways to organise and present the data in a way that will make interpretation (and explanation) clear and credible.
- interpretation and evaluation are similar to those processes in other variants of modelling.
- as in other variants of modelling, evaluation will also emphasise considerations like:
    - How reliable is the data?
    - Is the population representative?
    - Are the samples adequate?
    - Are the results significant – are they real effects or just due to random variation? (We'll return to this in Lesson 5.

Critiquing and Improving Inferences from Data

In this section students will look at data from a variety of contexts and inferences drawn from it in a critical way, noting unfounded inferences and suggesting how they may be made valid. Organise this using think-pair-share including the whole-class in the shared discussion.

Students might work individually, or in small groups with Student Sheet 1 - Phoney Figures? or participate in a class discussion using the slide show. Whether students work in small groups first or discuss as a whole class, the responses need to be reviewed as demonstrated below.

Show slide ***Critiquing inferences***.

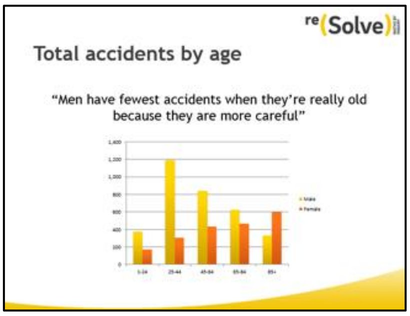


Explain that we will look at some data representations and the inferences that someone suggests the data shows.

The task is:

- To understand why someone might make that inference.
- To critique the argument made.
- To construct a valid inference noting any additional data you would need.

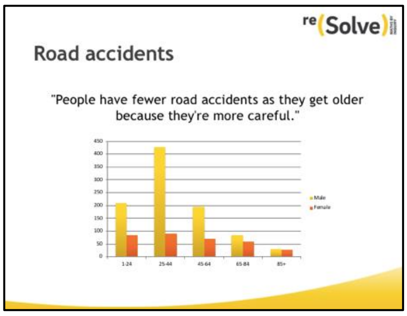
Show slide ***Total accidents by age***.



The factors that undermine the simple explanation that older men are more careful include:

- There are fewer older than younger men to have the accidents; we saw data on relative sizes of the populations with a big drop for men over 85. (Calculating as percentages of the populations is needed.)
- There may be other reasons for fewer accidents for each person. Older people tend to travel less. Retired people don't have risks at work.
- Of course, older people may also be more careful.
- The steady rise in women's accidents is interesting, perhaps suggesting greater frailty.

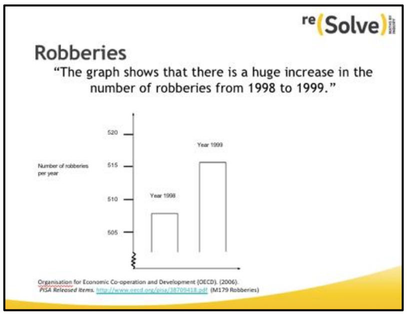
Show slide *Road accidents*.



The additional factors that undermine this simple interpretation include:

- Older people probably drive less – for example, work often involves people in their middle years (particularly men) driving substantial distances.
- The peak for men aged 25-44 may show less careful driving but more data is needed to be sure. (Other data, reflected in insurance premiums, supports this explanation.)
- Of course, older drivers may also be more careful - but they have slower reactions.

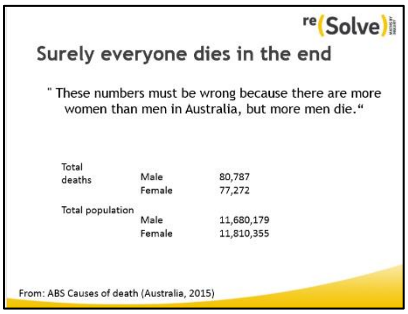
Show slide *Robberies*.

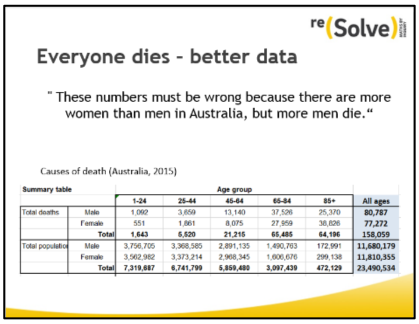


The key issue here is the suppression of the zero. The change in numbers, around 10 in 500, is relatively small though it doesn't look like it – a widely used distortion. This links to Lesson 5 where the scale of random variation is investigated.

Show slide **Surely everyone dies in the end**, and after some discussion Everyone dies – better data.

This is a subtle one. Ensure that students understand the data table. Factors that undermine the simple argument include the changing total population and the different times in which each group was born.





Summary table		Age group					
		1-24	25-44	45-64	65-84	85+	All ages
Total deaths	Male	1,092	3,659	13,140	37,526	25,370	**80,787**
	Female	551	1,861	8,075	27,959	38,826	**77,272**
	Total	**1,643**	**5,520**	**21,215**	**65,485**	**64,196**	**158,059**
Total populatio	Male	3,756,705	3,368,585	2,891,135	1,490,763	172,991	**11,680,179**
	Female	3,562,982	3,373,214	2,968,345	1,606,676	299,138	**11,810,355**
	Total	**7,319,687**	**6,741,799**	**5,859,480**	**3,097,439**	**472,129**	**23,490,534**

Phoney Figures?

Name: ____________

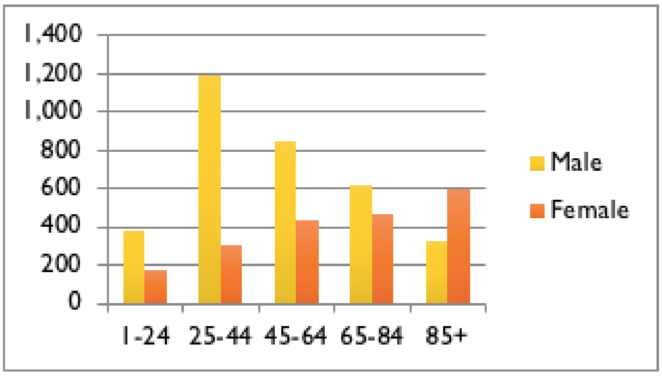
Each of these data representations is accompanied by an inference.

Your task is:

- To understand why someone might make the inference;
- To critique the argument made;
- To construct a valid inference noting any additional data you would need.

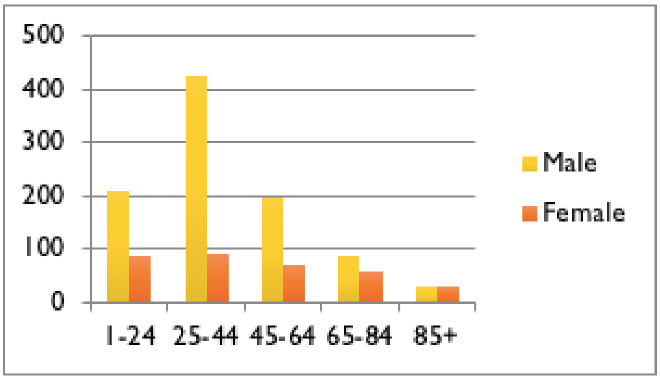
All accidents

"Men have fewest accidents when they're old because they have become more careful."



Road accidents

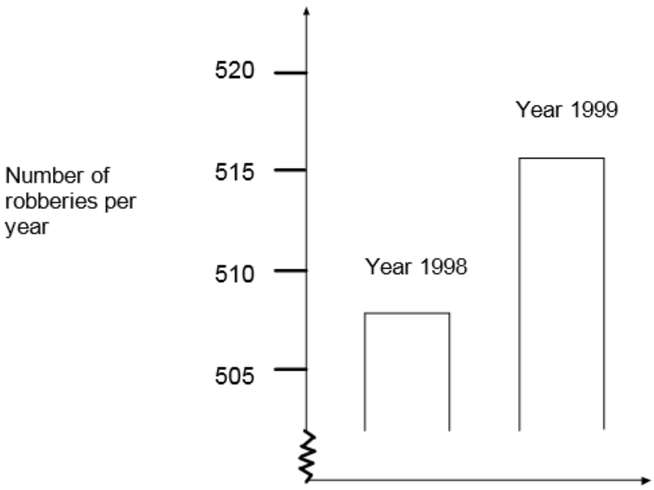
"People have fewer road accidents as they get older because they're more careful."



Robberies

A TV reporter showed this graph and said:

"The graph shows that there is a huge increase in the number of robberies from 1998 to 1999."



Organisation for Economic Co-operation and Development (OECD). (2006). *PISA Released Items*. http://www.oecd.org/pisa/38709418.pdf (M179 Robberies)

Surely everyone dies in the end

The data below has been calculated from 'Causes of death (Australia, 2015)'.

"The numbers must be wrong because there are more women than men in Australia but more men than women die."

Total deaths	Male	**80,787**
	Female	**77,272**
Total population	Male	**11,680,179**
	Female	**11,810,355**

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training