Cornering

Lesson 1: Sharing the Road

Australian Curriculum: Mathematics (Year 8-10A)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area. (Year 8)

Lesson abstract

Students discuss the meaning of the "Do not overtake turning vehicle" sign found on some trucks and buses, and recall instances when a truck or bus has had difficulty getting around a corner or roundabout. They use a simple paper scale model to visualise what happens when long vehicles turn corners, and discuss their findings, and the limitations of the model. A model with steerable wheels is suggested.

Mathematical purpose (for students)

To explore some unexpected results when long vehicles like trucks and buses turn corners.

Mathematical purpose (for teachers)

Students develop their understanding of this complex context and problem. The motion of turning vehicles is difficult to visualise and prone to misconceptions. The paper scale model provides an accessible entry point to the modelling process, and helps students see that the length of a vehicle affects how much width it takes up as it turns. Students are prompted to think more deeply about how vehicles move, e.g. which wheels steer, and how drivers decide how far to turn the steering wheel.

If desired, the first part of Lesson 2 (introducing and planning the bicycle experiment) can be taught as part of this lesson. A bicycle or scooter or toy car will be required for demonstration. Test your vehicle beforehand to make sure that it works as required for the experiment.

Lesson Length 35 minutes approximately

Vocabulary Encountered

- Modelling
- Scale model

Lesson Materials

- Slide show *ST7_Cornering_1a.pptx*
- Video: https://www.youtube.com/watch?v=QJuo-R7eo-Y
- Student Sheet 1 – Trucks and Corners (1 per group)
- Scissors (one pair per group)
- Rulers (one per group)

We value your feedback after these lessons via https://www.surveymonkey.com/r/J8GPD7Z

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

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Lesson structure:

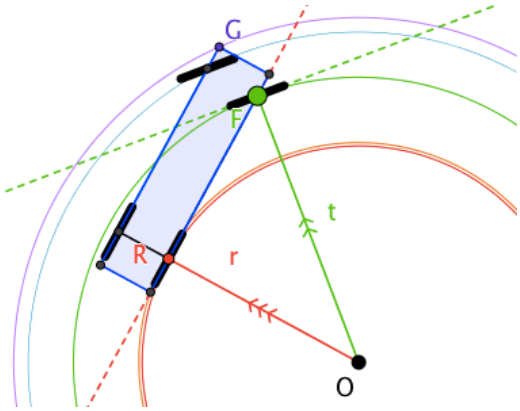
Sharing the Road (Class discussion – 10 mins)
A First Model (Collaborative small group work - 10 minutes)
The Modelling Cycle (Class discussion - 15 minutes)

Background for Teachers

Studying the behaviour of vehicles is complicated. There are lots of different situations that could be investigated, and many factors involved in making a model of each of these situations. We have chosen to investigate a reasonably simple situation: ***rigid vehicles making tight turns at low speed***. Here we can focus on the geometry of the path of the vehicle, rather than on the physics of motion (acceleration, centre of gravity etc).

Rear Wheels 'Cut-in'

This unit is built around one geometric result – the rear wheels of a turning rigid vehicle do not follow the same path as the front wheels, but follow a concentric arc of smaller radius. We call this 'cutting in' and it has implications for road use and road safety. It is illustrated in the diagram opposite (from Lesson 4) where $r < t$.



The front outside corner of a vehicle can also cause problems as it turns, but this is more intuitive and as such is more likely to be expected by other road users.

Both these results affect 'how much road' a vehicle uses as it turns.

Products: Safety Advice and Road Design Advice

Students can make two products in this unit: a presentation (or poster) for other road users on how to be safe around long vehicles, and a recommendation for council on roundabout dimensions.

The required products are loosely specified to allow students some freedom in how they investigate and what they produce. Students can be encouraged to focus on just one aspect, or to be more comprehensive in their approach, with the depth of investigation suited to each group. You may decide to 'play the role' of customer (e.g. the council) and have a meeting with each group to discuss what they are planning to produce, and whether it meets your requirements (this would fit best in Lesson 4).

Road safety guidelines can be found on various government sites e.g. https://www.towardszero.vic.gov.au/safe-vehicles/safe-trucks

The actual guidelines for designing roundabouts (which cover many other factors such as flow rates and lane changing) can be found online here https://austroads.com.au/publications/road-design/agrd04b (register for a free account to download). Each state will have its own guidelines as well, e.g. http://www.rms.nsw.gov.au/business-industry/partners-suppliers/documents/technical-manuals/busrouteassessments-i.pdf which has some useful information and ideas about long buses.

Assumptions

The models in Lessons 2-5 are still simplified with a variety of assumptions which may arise in class discussion:

- Wheels move only in the direction they are facing (i.e. they do not slip sideways at all). At each instant they are rolling in the direction they are pointing. This means that a wheel is drawn tangent to its path.
- Turning a corner is like moving in a circular arc (i.e. it is as if the steering wheel is moved instantly to the correct angle and stays there throughout the turn).
- (It is not necessary to discuss this assumption unless students query it.) In these models, both front wheels turn by the same amount, and the 'centre of rotation' of the vehicle is found using just two wheels (the right front and right rear; the centre of rotation is the intersection of the normal from the wheels). You can see this in the model pictured above. In reality, the front wheels of a car are adjusted to move slightly differently to each other, enabling normals from *all four wheels* to intersect at the centre of rotation.

Articulated Vehicles

This unit is about rigid, not articulated, vehicles. Students may find footage of semi-trailers and other articulated vehicles. These still illustrate the issues that long vehicles have turning corners, although the behaviour of the sections (e.g. prime mover and trailer) is more complex that what is analysed in this unit. Students may realise that one reason articulated vehicles exist is to cope with the difficulty in getting long rigid vehicles around corners: if a 'bendy bus' was not bendy it would not get its 18m length around a small intersection!

Path Radius and Turning Circle

In this unit, we use the term *path radius* to describe the radius of the circular arc being traced by any point on a vehicle. Different points on the vehicle (e.g. the two headlights) can have different path radii when a vehicle corners. The headlight on the 'inside' of a turning car will trace out a circular arc with a smaller radius than the other one.

The *turning circle* of a vehicle is the smallest possible circle the vehicle can make, i.e. with the steering wheel turned as far as it will go. The name is not used consistently– sometimes the number given is the radius, sometimes the diameter. Sometimes it refers to the path of the outer front wheel, and sometimes to the path of the outermost part of the body of the car. Sometimes it is wall-to-wall, and sometimes curb-to-curb (allowing the front corner to go over the curb). Often it is just an indication of whether the turning circle is large or small (or tight).

Road planners use the term *turning radius* to mean the path radius. We avoid using *turning radius* so that there is no confusion with the more commonly used *turning circle*.

Other Notes

- Drivers of long vehicles, knowing that their inner rear wheels will 'cut in' as they turn, will sometimes compensate for this by starting their turn from further out, even from outside their lane. This could be thought of as a secondary result of the geometry of turning, as it results from deliberate driver action. This idea may come up in discussion.
- To avoid scaring students unnecessarily, reassure them that we are only talking about low speeds. High speed roads (and even moderate speed roads) are designed to not have tight bends, so that this issue of rear wheels 'cutting in' is not relevant. When the sign says 'do not overtake turning vehicle' it doesn't mean you should be worried about a vehicle going around a slight bend! Encourage students to observe trucks on the road to reassure themselves that slight bends do not result in the back wheels 'cutting in' noticeably.
- Sensitivity is required should students bring up fatal accidents that have occurred around turning vehicles.

Introduce the problem

Show slides 2-3 of *ST7_Cornering_1a_Sharing_The_Road.pptx*.

- *Have you ever seen one of these signs? Where?* (Ans: on a truck or bus)
- *What do you think it means?* (Ans: don't go past this truck while it is turning left or right)

Slides 4-5: *Why do you think this sign is needed?*

- *Have you ever observed a truck or a bus turning a corner? Why should we be careful near turning trucks or buses?*

Expected Student Responses:

- School bus runs over kerb or edge of roundabout.
- Trucks are wide and take up a lot of the lane.
- Back/side/front of bus or truck swings out further than you expect.
- Pedestrians must be careful too, especially when waiting to cross at a corner.

As part of the discussion, show students this video of car getting (only mildly) squished against the kerb as a truck turns a corner: https://www.youtube.com/watch?v=QJuo-R7eo-Y

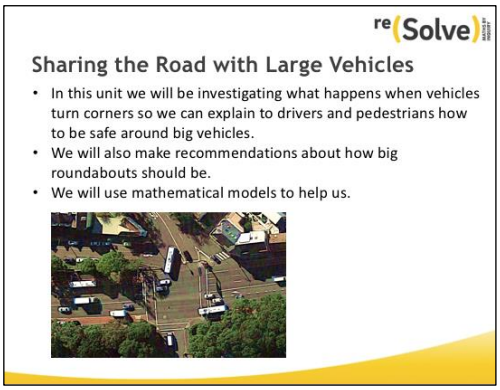
Aim of the Unit and Final Products

Show slide 6 of *ST7_Cornering_1a_Sharing_The_Road.pptx*.

In this unit we will be investigating what happens when vehicles turn corners, so we can explain to drivers and pedestrians how to be safe around big vehicles.

We will also be making some recommendations about how roads should be designed to make turning easy for large vehicles.

The underlying mathematical task is to describe the space that is needed by a turning vehicle. We use mathematical models to help us.



A First Model

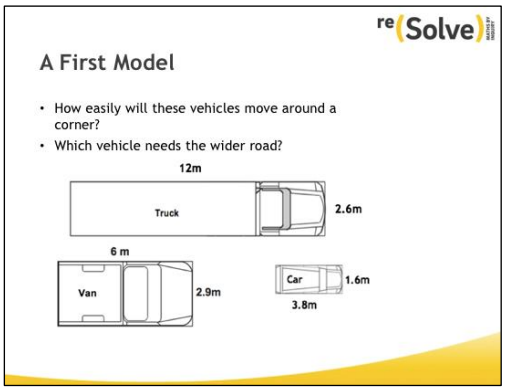
Why do trucks and buses go outside their lane?

Hand out Student Sheet 1 – Trucks and Corners and show slide 7.

Advise students that since this situation is complicated, we'll start with a simple physical model instead of a fully mathematical model.

Students cut out the truck, the van and the car, and move them around the corner, noting how much the vehicles move outside their lane. They estimate how much wider the lane would need to be for the vehicles to remain wholly within the lane.

As students are working, ask them to also think about the limitations of this simple model.



Bring students together to discuss their findings. Some discussion about how cars turn can be included (only front wheels turn; if the wheels are able to turn more, then the vehicle can go around a tighter corner, etc).

Expected Student Response

- The truck went outside its lane. We couldn't get it around the corner without the back right corner sticking out, because the truck was very long.
- The van could almost get around the corner without going outside the lane.
- The car stayed in its lane easily.
- For the truck to get around the corner the lane would need to be about 2m wider. We estimated this using the width of the car which is 1.6m. (Students will have different estimates and justifications).

Discussion

How could we make this model better so that we can describe the space required for turning more accurately?

- It is hard to be sure about the path of the vehicle – paper can turn many ways.
- The truck model doesn't show the back wheel position. Wheel position is important, because the wheels influence how the truck turns.
- Clarify with the class that on most road vehicles only the front wheels turn (i.e. only the front wheels are steerable), and the back wheels stay in place, parallel to the body of the car.

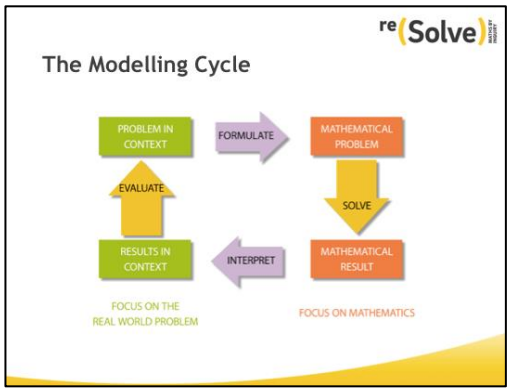
If it is not suggested, introduce the idea of a model that has wheels! Students may suggest:

- A model or toy car (point out that it will need steerable front wheels to be a good model)
- A scooter or bicycle (even though they only have 2 wheels, we can still gain insights from watching them turn a corner)

Tell the class that next lesson we will be using a bicycle or scooter (or toy car) to model what happens when a vehicle turns a corner.

The Modelling Cycle

Show slide 8 of *ST7_Cornering_1a_Sharing_The_Road.pptx*.



Students should be aware that they have been using a simple model that begins to make sense of the real-world situation. Ask students to consider what modelling they have been doing so far, and to explain clearly what the real-world situation is.

Expected Student Response

- We have been using a simple paper scale model of a vehicle, and manually turning corners to see what space they need as they turn.
- Perhaps we have turned the paper in ways that a vehicle might not be able to turn. Our model needs wheels!
- Already we have seen that the width of the lane needed for turning does not only depend on the width of the vehicle, but also depends on its length.

Ask students to consider (in pairs) whether they think that there should be restrictions on the dimensions of vehicles allowed to use certain roads. Ask them to justify their answer, and share with other pairs.

Expected Student Responses

- Yes: Large vehicles can be dangerous to cyclists and pedestrians at road junctions.
- Yes: They sometimes cause traffic jams because they are slow to turn corners, particularly in busy cities.
- Yes: It is not practical to make all roads wide enough for big trucks to go around the bends.
- No: Shops need goods to be delivered by trucks. The roads must be made wide enough.
- No: People need buses to get to work, school, etc. The roads must be made wide enough.

Depending on your local council, there may be restrictions on trucks and/or buses in some areas. You might like to have students research this. (The reasons may not be related to turning, but to other factors such as noise.)

Looking ahead

Show slide 9, and explain that before the next lesson, students need to:

- Observe how buses and trucks get around tight corners.
- Ask friends and family for stories about buses and trucks turning.
- Ask a (car) driver how they steer around a corner. How do they know how far to turn the steering wheel?

Note: This last question is to help with the planning of the bicycle experiment (or scooter experiment). Students will be experimenting in one of two ways, either:

- Turning the handlebars to a chosen angle, and taping them so they do not move, then walking the bike along and marking the path of the back and front wheels. With the handlebars fixed, the front wheel will move in a circle, with the back wheel making a smaller concentric circle. Students will need to find the centre of these circles and measure the radii.
    - This option is like a driver turning the wheel to a chosen angle, and then just moving forward.
    - It does not assume that the paths will be circular.
- Making a large circle on the ground using chalk and a string, and then walking the front wheel of the bicycle around it, marking the path of the back wheel as they go. After a short time, the back wheel will follow a smaller concentric circle. Students will need to measure the radii of the circles.
    - This option is like a driver following the curve of the corner.

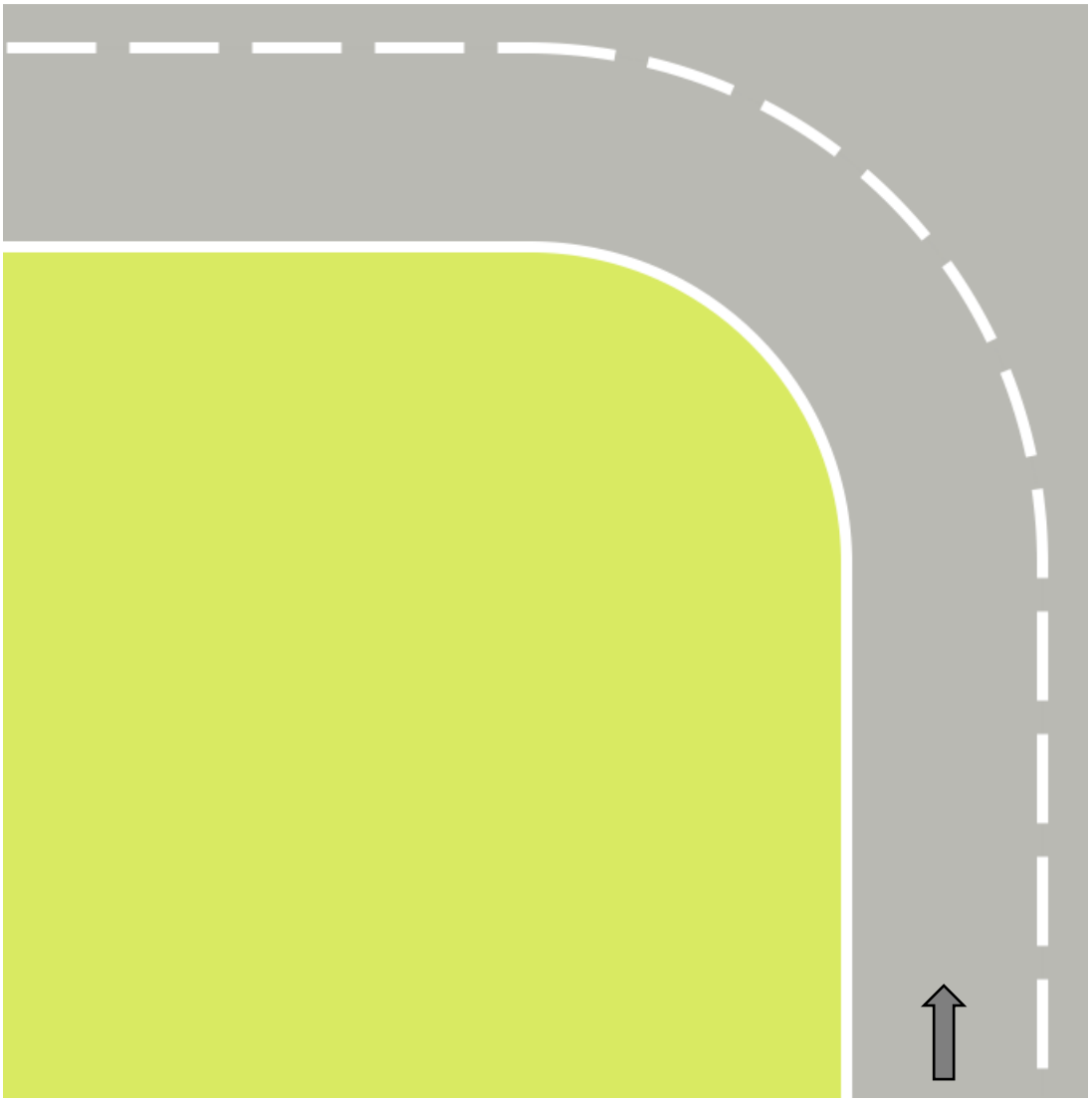
ORGANISE SUFFICIENT STUDENTS TO BRING BIKES, SCOOTERS OR TOYS TO THE NEXT LESSON

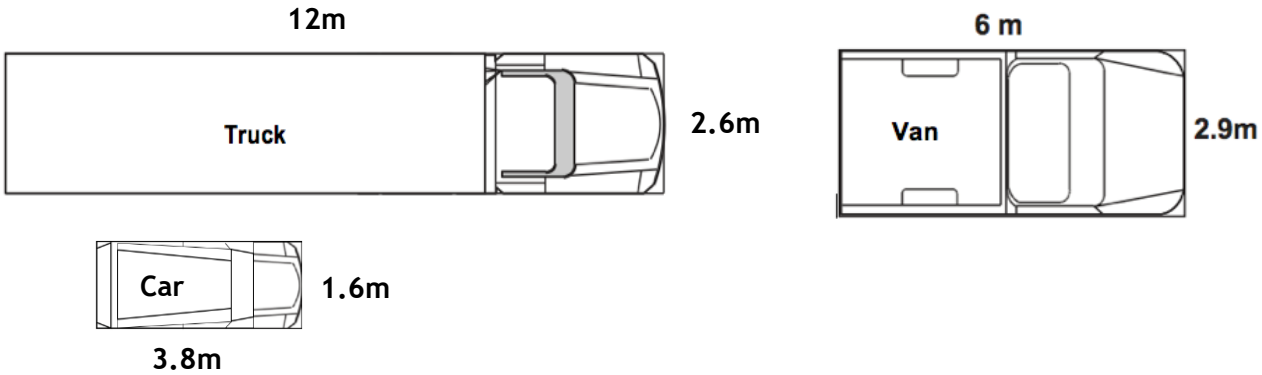
Trucks and Corners

Name: _____

Below are scale drawings of a truck, a van, and a car. Cut them out and move them around the 'corner'.

- Do they go outside their lane and/or over the kerb?
- Which vehicle goes the furthest outside the lane?
- Approximately how much wider would the lane need to be to avoid this?





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