Year 6 Exemplar
Area and Perimeter

Australian Curriculum: Mathematics (Year 6)

ACMMG137: Solve problems involving the comparison of lengths and areas using appropriate units.

Abstract

In this task, students respond to a conjecture related to a common misconception: that a rectangle with a larger perimeter will always have a larger area. Students will learn that it is sufficient to offer one counter example to refute a conjecture or a general statement that makes a claim about all cases.

Mathematical purpose (for students)

You only need one counter example to disprove a conjecture about all cases.

Mathematical purpose (for teachers)

Teachers support and challenge students to test a conjecture and to justify by:

- Using results of trials to refute the conjecture (Justifying)
- Using understanding of area and perimeter of rectangles to select good examples (Analysing)

Time Needed 60 minutes approximately

Vocabulary Encountered

- compare and contrast
- convince me
- justify/explain why
- test/verify ideas
- because
- proof
- conjecture

Materials

- Square tiles or squared paper (optional)
- Student Sheet 1 – Area and Perimeter (optional: 1 per student)
- Reasoning Prompt Cards or Poster (see Teachers' Guide *ST5_Reasoning_TeachersGuide.docx*)
- Assessment Sheet (1 per student)

We value your feedback after these tasks via https://www.surveymonkey.com/r/RJC6FPC

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Area and Perimeter: The Lesson

Reasoning Task

> *Nathan said: "When you increase the perimeter of a rectangle, the area always increases."*
>
> *Is this statement true for all cases?*
>
> *Explain why Nathan's statement is correct, or incorrect.*

Revise the terms *area* and *perimeter* as necessary without giving away too much. Pose the task with minimal initial instruction.

Some students will be assisted by using squared paper to draw rectangles and count area and perimeter. Small square tiles could also be used to make and modify rectangles.

For this task the logical argument comes through a counter-example – here, a rectangle with a larger perimeter than another but a smaller area. This case would be a counter-example to Nathan's conjecture. Because Nathan has made a claim about all rectangles, students only need to find one example that does not fit, to prove conclusively that Nathan is wrong. Some students might see that the first five cases they test all support Nathan's conjecture, but this is not a proof that Nathan's conjecture is true: students need to be prompted to look further.

Reasoning Prompts

For more prompts in the context of this task, see this table.

- What stays the same and what changes (when you make rectangles with perimeter of 20 units)? (Analysing)
- Is it just sometimes true, or is it always true? (Analysing)
- When is it true? (Analysing)
- Explain why does this work? (Justifying)
- How do you know? (Justifying)

What stays the same and what changes?	Is it just sometimes true, or is it always true?	When is it true?	Explain – why does this work?	How do you know?

Enabling Prompts

- Choose one rectangle to start with, then think how we can change the dimensions of a rectangle to increase the perimeter.
- Can you provide examples to show that Nathan might be correct?
- Have you searched for examples that show Nathan might be incorrect?

Extending Prompts

- Can you find a rule that say how to change the dimensions of a rectangle to make the perimeter increase but the area decrease?
- What rectangle with perimeter 20 cm has the smallest area?

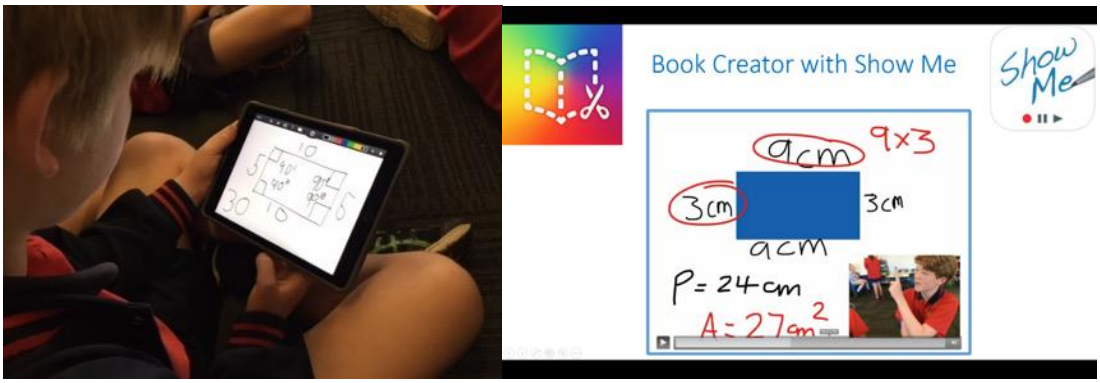
Summary Phase

Invite students to share their solutions in order of complexity to develop a whole class mathematical discussion. Highlight the fact that one counter-example will prove that Nathan is not correct, and explain why this is. Note also that it is interesting to find out when Nathan's statement is correct – in other words to improve Nathan's conjecture. The Formative Assessment Table shows the likely variation in responses. You might:

- Encourage students to explain each other's thinking.
- Ask:
    - "What is one thing you know now about testing conjectures that you did not know before?"
    - "What have you learned about explaining your reasoning to others?"

Teacher Notes

- Using digital technology apps such as *Geoboard* and *Show Me*, students investigate and communicate their reasoning. Students then present their findings using *Book Creator* that can be added to a digital portfolio such as *SeeSaw*.



Further Activities

Having listened to a range of students' strategies in the initial task, students can apply those strategies to follow-up tasks as shown below.

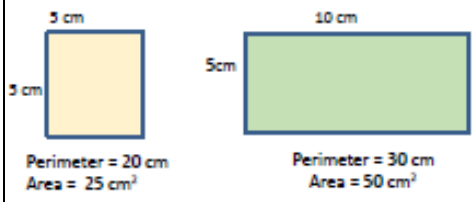
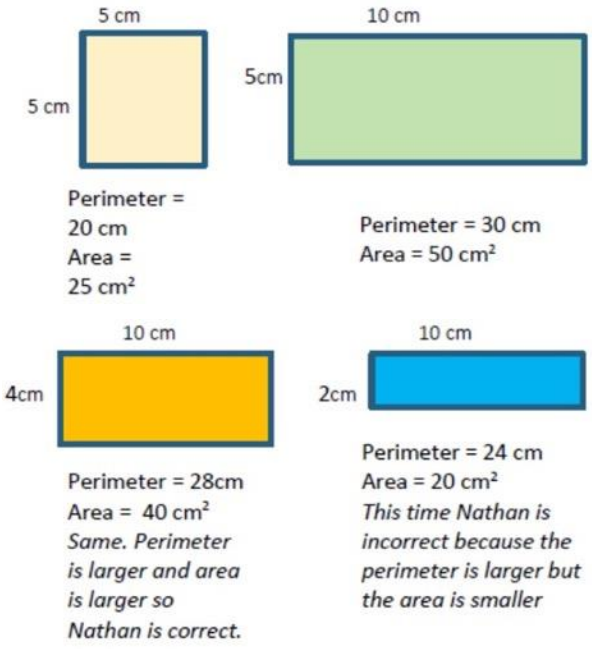
1. Would Nathan's statement be true for triangles? (ANS: No – a long thin triangle can have a big perimeter and small area).
2. Is there any type of shape for which Nathan's statement is true? (ANS: squares, equilateral triangles, other regular polygons, rectangles with one side of 2.83 cm, circles, etc.)

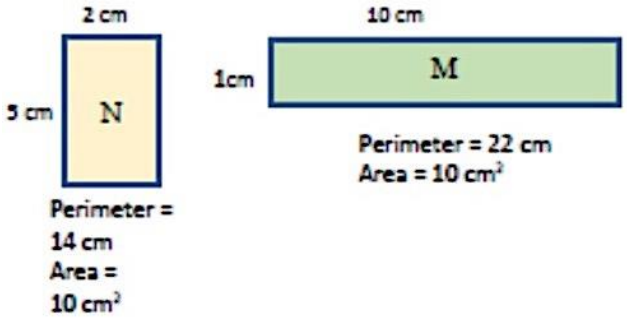
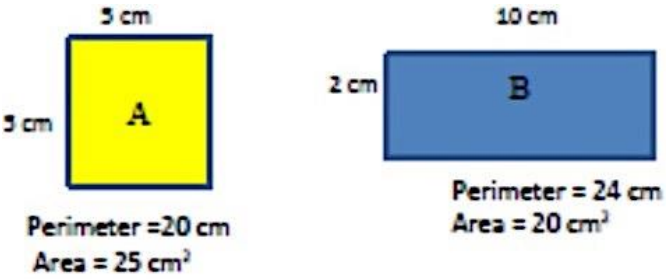
Formative Assessment

The following table shows some responses that students commonly give to this problem. These responses demonstrate the variety of levels for each reasoning action. Studying these sample responses can prepare the teacher for identifying their students' reasoning during the lesson. Suitable prompts are suggested to support or extend such students' reasoning.

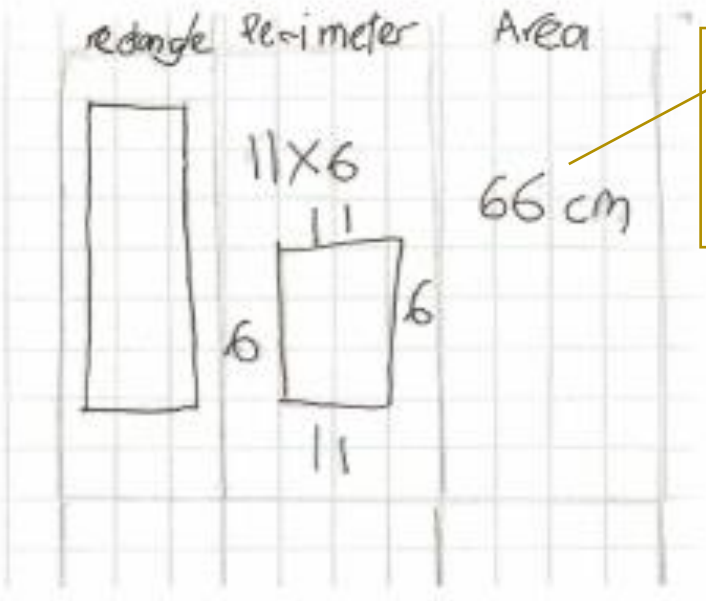
Many of the possible responses in the table are linked to full work samples from students. Each work sample has been annotated by the teacher using the Rubric. A copy of the teachers' assessment sheet shows what the teacher recorded about reasoning during and after the lesson, and the recommendations the teacher made about how to further that student's reasoning.

ANALYSING **Possible Student Response**	**Level**	**Suggested Prompts**
"Nathan is wrong." **(See Annotated Work Sample 1)**	Not Evident	What is perimeter? How can we calculate the perimeter of a rectangle? What is area? How can we calculate area of a rectangle? Can you work out the area and perimeter of more rectangles? What stays the same and what changes?

 The student notices similarities across examples and concludes that as the perimeter increases so does the area. **(See Annotated Work Sample 2)**	Beginning	What other rectangles could you create to test this conjecture? (Note: if the perimeter is only allowed to increase by making sides longer – not shorter – then Nathan's conjecture is true.) Counter-examples will only come by increasing one side and decreasing the other less.)
The student creates examples and sorts and classifies examples according to the perimeter total and the resulting area. **(See Annotated Work Sample 3)**	Developing	What stays the same and what changes?
The student notices the effect of increasing/decreasing the length and width on the rectangle on the final area of the rectangle. **(See Annotated Work Sample 4)**	Consolidating	Explain why does this result work?
JUSTIFYING **Possible Student Response**	**Level**	**Suggested Prompts**
Nathan is correct. Increasing the perimeter means the area increases – but student does not provide examples	Not evident	Offer enabling prompt. Can you show with examples? Is that always true?
The perimeter is larger so the rectangle is bigger and the area is larger - so Nathan is correct.	Beginning	What other rectangles could you create to test this conjecture?
The area is not always bigger than the perimeter. **(See Annotated Work Sample 3)**	Developing	What stays the same and what changes? Is it just sometimes true, or is it always true? When is it true?
	Developing	(Student compared the other three rectangles to the one with perimeter 20cm.) You have found examples which fit Nathan's conjecture and examples which do not fit. Does this make Nathan's statement true or false?

Nathan is incorrect. I can find an example to show that when the perimeter of a rectangle increase, the area stays the same. Rectangle M has a larger perimeter (22cm) than rectangle N (14cm) but both rectangles have the same area (10 cm^2).  (See Annotated Work Sample 4)	Consolidating	You have found a counter example to show that Nathan is incorrect. Extension task: What if we consider other shapes such as triangles?
Nathan is incorrect because I can find an example to show that when the perimeter of rectangles increases, the area decreases. For instance, Perimeter of rectangle B is larger than of rectangle A but the area of rectangle B is smaller than that of rectangle A  	Consolidating	You have found a counter example to show that Nathan is incorrect. Extension task: What if we consider other shapes such as triangles?
Nathan is incorrect. Nathan's statement may hold true in some cases, like in the example that Nathan showed, it is not always true for every case. I can find an example where the perimeter increases but the area stays the same or decreases. This is a counter example. As long as I have ONE counter example, it is enough to refute Nathan's conjecture. I have proved that when you increase the perimeter the area does not always increase. (See Annotated Work Sample 5)	Extending	What do you notice about the dimensions of the rectangles that show the conjecture is correct and incorrect?

Annotated Work Sample 1



Analysing: Does not notice numerical or spatial structure of examples or cases.

Revision of concept of perimeter and area needed.

Justifying: Does not justify.

ANALYSING: Not evident

JUSTIFYING: Not evident

TEACHER PROMPTS:

What is perimeter? How can we calculate the perimeter of a rectangle?

What is area? How can we calculate area of a rectangle?

Can you work out the area and perimeter of more rectangles?

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Work Sample 1 Rubric

Student Name: WORK SAMPLE 1 Reasoning Task: AREA + PERIMETER Date:

Observation of student's reasoning:

- Not connecting perimeter to area
- Couldn't explain why Nathan wrong

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Revise concept - use geoboards / enabling prompts to support explanation

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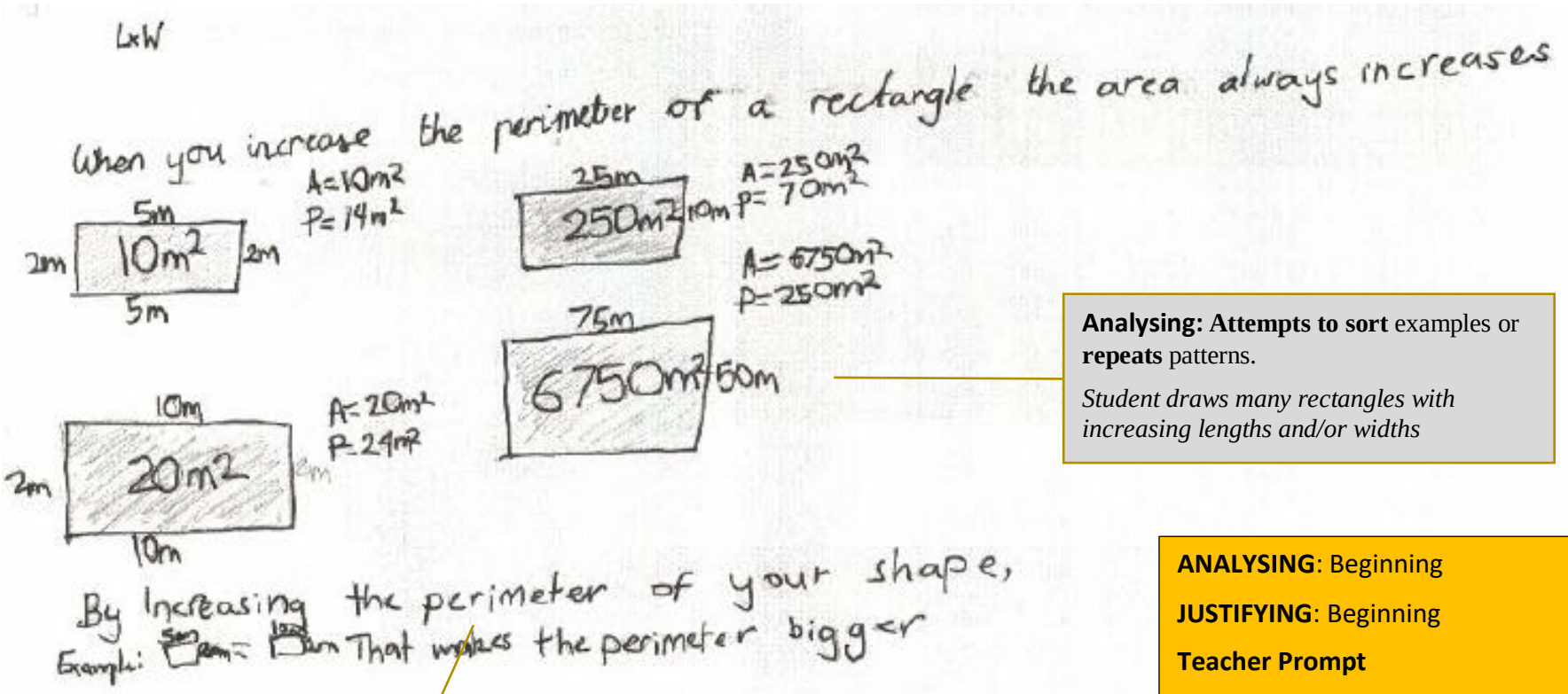
Annotated Work Sample 2



Analysing: Attempts to sort examples or **repeats** patterns.

Student draws many rectangles with increasing lengths and/or widths

Justifying: Describes what they did and **recognises** what is correct or incorrect.

Student makes a seemingly correct statement but the argument is incoherent

ANALYSING: Beginning

JUSTIFYING: Beginning

Teacher Prompt

What other rectangles could you create to test this conjecture?

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Work Sample 2 Rubric

Student Name: Work Sample 2 Reasoning Task: Area + Perimeter Date:

Observation of students' reasoning:

Examples show that Nathan is correct

Argument incoherent

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Encourage find non-examples & develop justification - look at prompts.

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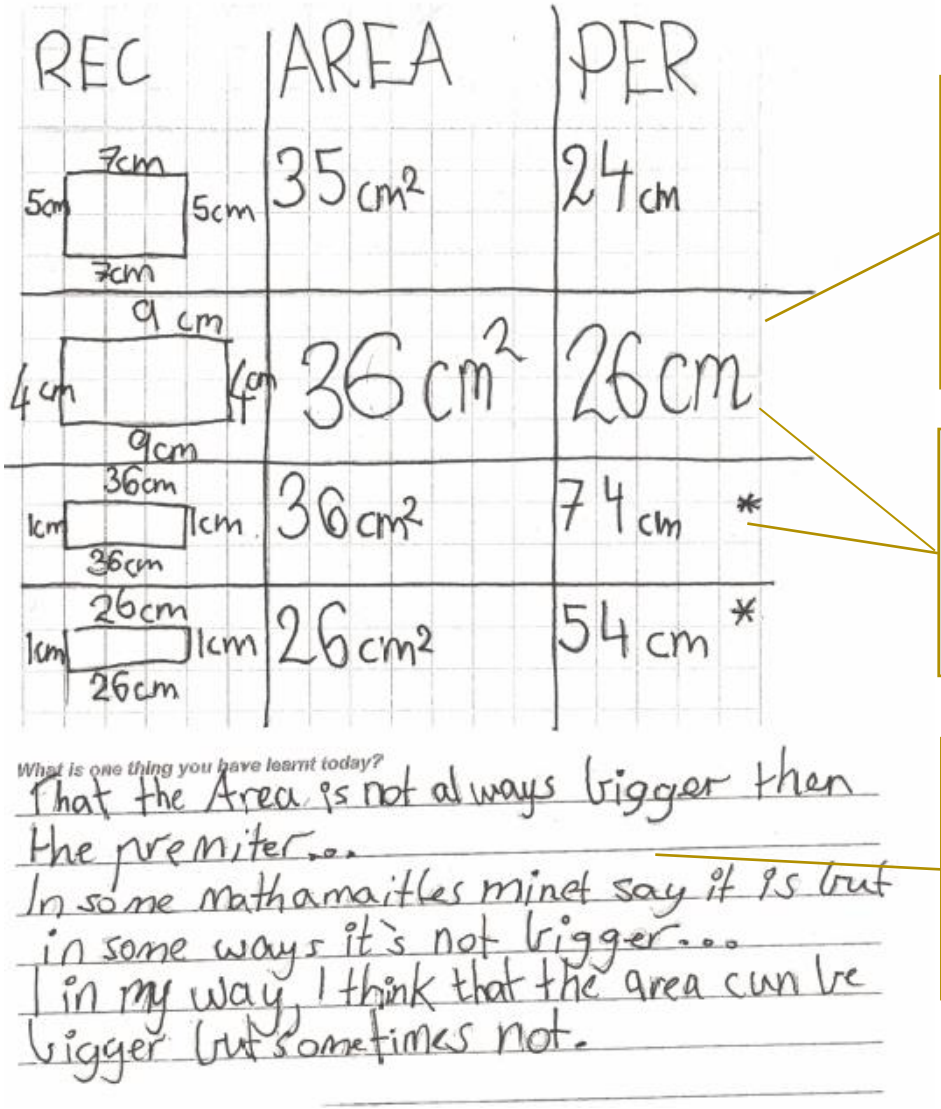
Annotated Work Sample 3



REC	AREA	PER
7cm, 5cm, 5cm, 7cm	35 cm^2	24 cm
9 cm, 4cm, 4cm, 9cm	36 cm^2	26 cm
36cm, 1cm, 1cm, 36cm	36 cm^2	74 cm *
26cm, 1cm, 1cm, 26cm	26 cm^2	54 cm *

What is one thing you have learnt today?

That the Area is not always bigger then the premiter...
In some mathamaitles minet say it is but in some ways it's not bigger...
In my way, I think that the area can be bigger but sometimes not.

Analysing: Notices a common property, or **sorts and orders** cases, or **repeats and extends** patterns. **Describes** the property or pattern.

Student knows the properties of area and perimeter and used a table to record the cases.

Justifying: Recognises what is correct or incorrect.

Student notices counter examples - that when perimeter increases the area does not always increase.

Justifying: Starting statements in a logical argument are correct.

Student attempts to form an argument involving a counter example.

ANALYSING: Developing

JUSTIFYING: Developing

Teacher Prompts

What stays the same and what changes?

Explain why does this result work?

Is it just sometimes true, or is it always true? When is it true? How do you know?

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Work Sample 3 Rubric

Student Name: Student Worksample 3 Reasoning Task: Area + Perimeter Date:

Observation of student's reasoning:

The student randomly tested ideas & found a counter example (e.g $A = 26cm^2$, $P = 54cm$) but the argument is not coherent.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Encourage student to find more counter examples & systematically sort cases to develop the argument.

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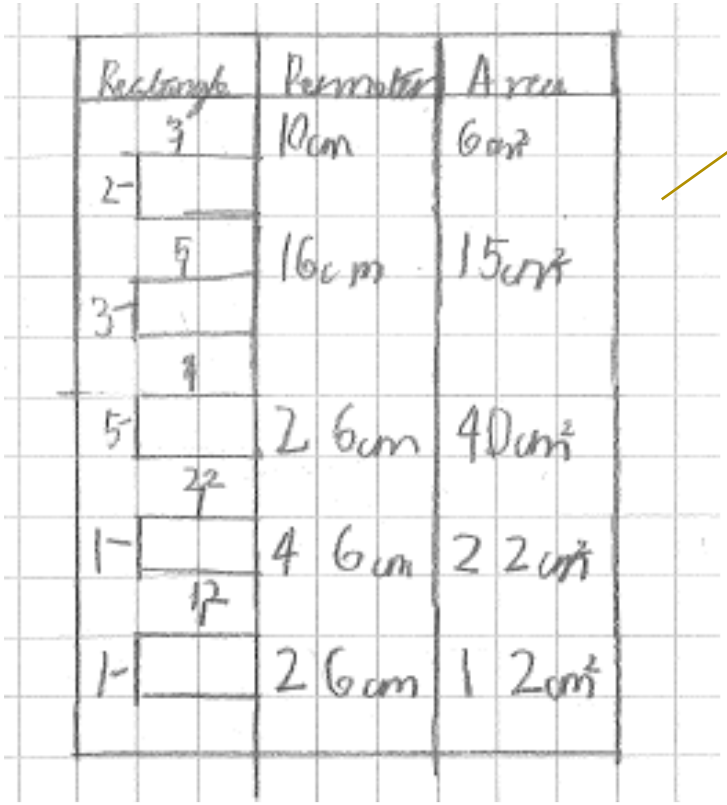
Annotated Work Sample 4



Rectangle	Perimeter	Area
3 / 2	10cm	6cm²
5 / 3	16cm	15cm²
1 / 5	26cm	40cm²
22 / 1	46cm	22cm²
12 / 1	26cm	12cm²

Analysing: Systematically searches for examples, extends pattern or analyses structure to form a conjecture.

ANALYSING: Consolidating

JUSTIFYING: Consolidating

Teacher Prompts

Is it just sometimes true, or is it always true?

Convince me.

Justifying: Verifies truth of statements by confirming all cases or **refutes** a claim by using a counter example.
Uses a correct **logical argument.**

Student needs to look at more examples, and think about what might make the result true, to give a correct conjecture and water tight argument. Decreases of the sides do not always result in lower area, as is claimed here.

What is one thing you have learnt today?

I learnt that if you get a rectangle and increase both sides the area will increase, but if you decrease one side and increase the other the area will not increase.

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Work Sample 4 Rubric

Student Name: Work Sample 4 Reasoning Task: Area + Perimeter Date:

Observation of student's reasoning:

Systematically looking for counter example
Used "if...then" to justify

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Need more counter examples for water tight argument "Convince me!"

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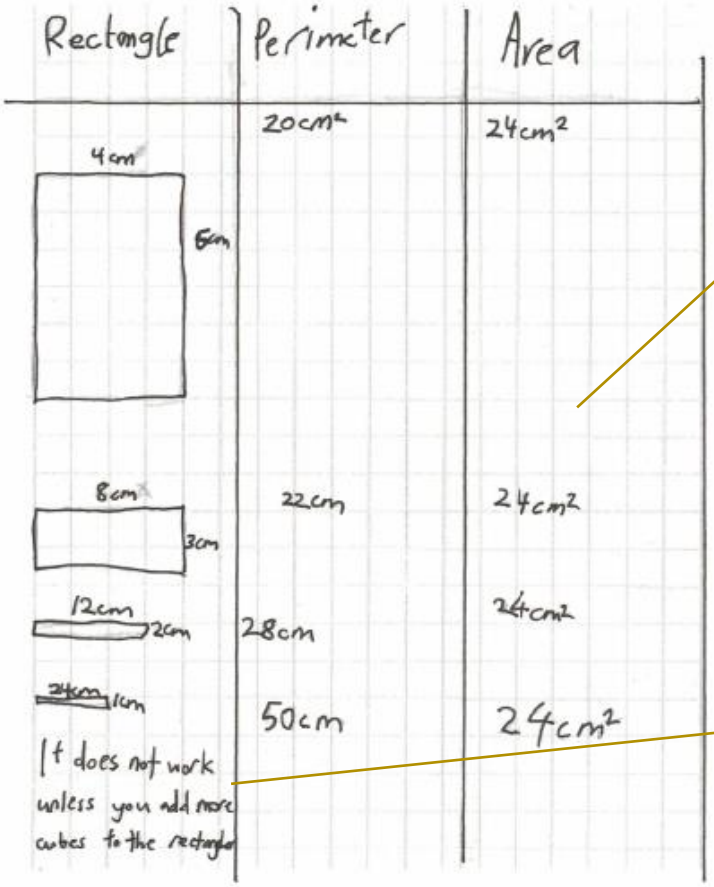
Annotated Work Sample 5



Analysing: **Notices and explores** relationships between properties.
Student explores lengths and widths that gives the same area but the perimeter increases.

ANALYSING: Extending

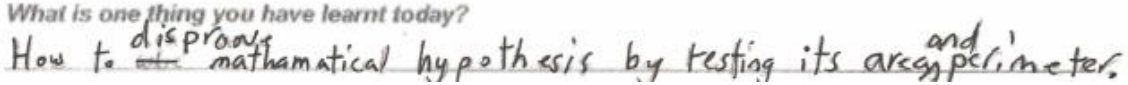
JUSTIFYING: Extending

Teacher Prompt

Does Nathan's conjecture work for triangles?

Justifying: Uses a **watertight logical argument**.

The student developed a watertight logical argument by showing a counter-example.



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Work Sample 5 Rubric

Student Name: Work Sample 5 Reasoning Task: Area + Perimeter Date:

Observation of student's reasoning:

Used 24 tiles ($A = 24 cm^2$) to model different examples.

Developed watertight argument by testing/verifying cases.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Explore other shapes e.g triangles

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Area and Perimeter

Name:_____

Nathan said: “When you increase the perimeter of a rectangle, the area always increases.”

Is this statement true for all cases?

Explain why Nathan’s statement is correct, or incorrect.

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Student Name: **Reasoning Task:** **Date:**

Observation of student's reasoning:

	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify**.
BEGINNING	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • A**rgument** is not coherent or does not include all steps.
DEVELOPING	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
CONSOLIDATING	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
EXTENDING	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

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