Year 6 Exemplar
Painted Cube

Australian Curriculum: Mathematics (Year 6)

ACMNA133: Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence.

- identifying and generalising number patterns.
- investigating additive and multiplicative patterns such as the number of tiles in a geometric pattern, or the number of dots or other shapes in successive repeats of a strip or border pattern looking for patterns in the way the numbers increase/decrease.

Abstract

The Painted Cube task is rich and complex, providing students with opportunities to explore a variety of patterns that can be described spatially, numerically and symbolically. There are good opportunities for using visualisation. Students learn to work systematically by keeping a clear record of results which will encourage them to develop and test conjectures and to ask themselves questions about further cases.

Mathematical purpose (for students)

To analyse a situation to find a generalisation, and then explain why it is true.

Mathematical purpose (for teachers)

Teachers support and challenge students to analyse a spatial structure and:

- Notice features and use patterns to find further results. (Analysing)
- Form and test conjectures about these patterns. (Justifying)
- Express a conjecture verbally or in a written statement using words or symbols. (Generalising)
- Express a generalisation using words or algebraic symbols (Generalising)

Time Needed 120 minutes approximately

Vocabulary Encountered

- explain why
- testing/verifying ideas
- because
- if… then…

Materials

- 27 small cubes per pair and one 3x3 cube for class
- 4 small cubes, coloured on 3 faces, 2 faces, 1 face and no faces
- Student Sheet 1 – The Painted Cube (1 per student, printed on two separate sheets, not back to back)
- Extra plain or isometric paper as required
- Reasoning Prompt Cards or Poster (see Teachers' Guide *ST5_Reasoning_TeachersGuide.docx*)
- Video *ST5_Reasoning_6a_Paint.avi* (optional)
- Assessment Sheet (1 per student)

We value your feedback via https://www.surveymonkey.com/r/RJC6FPC

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Painted Cube: The Lesson

Introducing the Reasoning Task

- Show the large cube. Invite students to brainstorm the features of the large cube (number of edges, corners/vertices, faces, volume perhaps).
- Show how it is made of, or can be cut into, 3 x 3 x 3 small cubes.
    - Ask how many mini cubes altogether (27).
    - Make sure that students understand that the small cubes are not only on the 'outside' of the large one, but also inside it.
    - (Alternative, cut a cube of cheese into 27 small cubes.)
- Drop the large cube into a pretend paint can. Pull the "painted" cube out, and pretend to cut it into the small cubes.
    - An animation of this can be found at https://nrich.maths.org/2322
- In pairs, students discuss whether all the small cubes are painted, and where the paint will go on each of the small cubes.
- In class discussion, draw out that some of the cubes will be painted on some faces, but none will be painted on all 6 faces. Some small cubes might be painted on 0, 1, 2, or 3 faces, but not on 4, 5, or 6 faces. The point here is to raise the possibilities, not to get exact answers, in preparation for the student sheet which only asks for 0, 1, 2, 3 painted faces. Don't give too much away.
- Hand out Student Sheet 1 - The Painted Cube, and set students to work on the Reasoning Task.
- To help students visualise the small cubes, show the mini cubes coloured on 3, 2, and 1 faces.

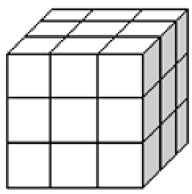
Reasoning Task

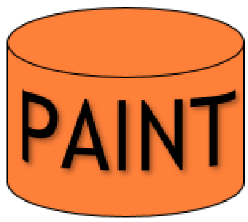
Imagine a cube made up of 27 smaller cubes (3 x 3 x 3).

Imagine that you fully immerse the cube in paint.

Now separate it into 27 small cubes. You will notice that some of the small cubes have been painted on some faces.

Which small cubes have been painted on 3 faces, on 2 faces, on 1 face, and not painted at all? How many cubes of each type are there?





First fill in the grid below for a 3 x 3 x 3 cube.

Then complete the grid for different size cubes, 2 x 2 x 2, 4 x 4 x 4, 10 x 10 x 10, etc.

Reasoning Prompts

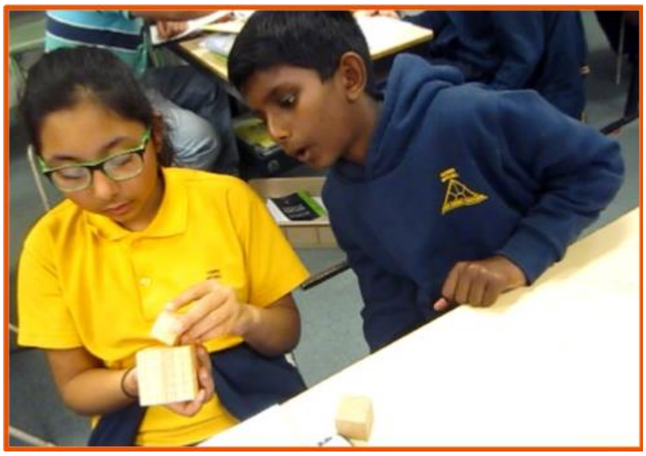
For more prompts in the context of this task, see this table.

- What stays the same and what changes? (Analysing)
- If we change this what will happen? (Analysing)
- What is the pattern here? (Generalising)
- Is that … (pattern) always going to work? (Generalising)
- What happens in general? (Generalising)
- What is the rule? (Generalising)
- How do you know…? (Justifying)

What stays the same and what changes?	Alter an aspect of something to see an effect. If we change this what will happen?	What is the pattern here?	Is that… (pattern) always going to work?	What happens in general?	What is the rule?	How do you know?

Enabling Prompts

- Offer cubes for students to manipulate.
  - Where are the cubes with no faces painted?
  - Where are the cubes with 1, 2, 3, faces painted?
- Look for multiples of 12 and 6 within the results and try to picture where these numbers come from.



Extending Prompts

- Complete the grid for a size "n" cube.
- How do the rules relate to the features of cubes?
- A large cube has 488 painted small cubes. What size is the large cube? (ANS: 10 x 10 x 10 cube with an 8 x 8 x 8 unpainted cube inside it. 1000 – 512 = 488)

Summary Phase

Invite students to share their solutions in order of complexity to develop a whole class mathematical discussion. The Formative Assessment table shows the likely variation in responses

It may be useful to show the short video *ST5_Reasoning_6a_Paint.avi*. The students shown in the video (and image above) use blocks of different sizes to show that the visible faces of the large cube form a ‘shell’ around a smaller, unpainted central cube. They clearly explain their reasoning about the numbers of faces to put in the table.

You might:

- Encourage students to explain each other’s thinking.
- Ask:
  - What is one thing you know now about making and testing conjectures that you did not know before?
  - What helped you to write the rules?
  - What have you learned about explaining your reasoning to others?

Further Activities

More information on the **Painted Cube** can be found at: http://mathematicscentre.com/taskcentre/160paint.htm and https://nrich.maths.org/2322

The Partly Painted Cube:

A cube is made from some smaller cubes. Some of the faces of the large cube were painted, and then taken apart. 45 cubes had no paint on them at all. What size was the big cube and how many faces were painted?

The full problem, illustrations and solution can be found at https://nrich.maths.org/6903

Formative Assessment

The following table shows some responses that students commonly give to this problem. These responses demonstrate the variety of levels for each reasoning action. Studying these sample responses can prepare the teacher for identifying their students’ reasoning during the lesson. Suitable prompts are suggested to support or extend such students’ reasoning.

Many of the possible responses in the table are linked to full work samples from students. Each work sample has been annotated by the teacher using the Rubric. A copy of the teachers’ assessment sheet shows what the teacher recorded about reasoning during and after the lesson, and the recommendations the teacher made about how to further that student’s reasoning.

ANALYSING **Possible Student Response**	**Level**	**Suggested Prompts**
"They will always be painted because they are dipped in paint".	Not Evident	Suggest student uses cubes actively. If you dip this cube in paint, what do you notice if you pull out the individual cubes? Do you notice there are cubes with different number of faces painted? Can you identify where they are located?
"The number of cubes painted on 3 faces is always 8". **(See** Annotated Work Sample 1**)**	Beginning	What stays the same and what changes? (Analysing)
"The number of cubes painted on 3 faces is always 8. The number of unpainted cubes in a 4x4x4 cube is the same as the total number of cubes in a 2x2x2 cube". **Notices** a common property related to the total number of cubes, the number of cubes painted on faces. However, there are some mistakes in the analysis. **(See** Annotated Work Sample 2**)**	Developing	Where are the cubes with no faces painted? Where are the cubes with 1, 2, 3 faces painted?
The student **systematically lists** numbers of cubes. However, need to observe student constructing this table to see when they used a rule that they noticed rather than counting using concrete materials. **(See** Annotated Work Sample 3**)**	Consolidating	What is the pattern here? What happens in general?
The student notices that the number of cubes painted on three faces is always 8 and **explores the relationships** between the number of cubes painted on 0/1/2 faces. **(See** Annotated Work Sample 4**)**	Extending	How could you demonstrate/show/prove that it is true?
GENERALISING **Possible Student Response**	**Level**	**Suggested Prompts**
Does not communicate a general rule or pattern. **(See** Annotated Work Sample 1**)**	Not evident	Let's have a look at 2x2x2 and 3x3x3 cubes. Can you please point to where the cubes with 1, 2, 3 faces painted are? What do you notice?
"I notice the number of unpainted cubes is the same as the total number of cubes from 2 cubes ago" Note: This is an important step to form conjectures about calculating how many cubes will have 2 faces painted. **Draw attention** to the number of unpainted cubes and **attempts to communicate** a rule for a pattern. **(See** Annotated Work Sample 2**)**	Beginning	What stays the same and different about…? How are they related to what you see?
"The number of cubes painted on 2 faces is going up by 12". **Communicates a rule and explain** with an example what the rule means for aspects of the cube. **(See** Annotated Work Sample 3**)**	Developing	What is the pattern? What do you notice? How are they related to what you see?

<table>
<tr>
<td>

CUBE SIZE	Total number of small cubes	Number of cubes painted on 3 sides	Number of cubes painted on 2 sides	Number of cubes painted on 1 side	Number of cubes not painted at all
n x n x n	n^3	8	length-2 x12		$[\text{length}-2]^3$

Communicates a rule using mathematical terms, symbols or diagrams.
(See Annotated Work Sample 4**)**</td>
<td>Consolidating</td>
<td>Are there other examples that fit the rule?

How would the table of results look for an n by n by n cube?</td>
</tr>
<tr>
<td>

CUBE SIZE	Total number of small cubes	Number of cubes painted on 3 sides	Number of cubes painted on 2 sides	Number of cubes painted on 1 side	Number of cubes not painted at all
n x n x n	n^3	8	$12(n-2)$	$6(n-2)^2$	$(n-2)^3$

Develops an algebraic expression/formula and can explain and apply it to other examples</td>
<td>Extending</td>
<td>Convince me your expression is true for all cases.
How do the rules relate to the features of the cubes we noted earlier?</td>
</tr>
<tr>
<th colspan="3">JUSTIFYING</th>
</tr>
<tr>
<th>Possible Student Response</th>
<th>Level</th>
<th>Suggested Prompts</th>
</tr>
<tr>
<td>Does not justify
(See Annotated Work Sample 1)</td>
<td>Not evident</td>
<td>How can you convince me that the number of cubes painted on three faces are always 8?</td>
</tr>
<tr>
<td>The cubes in the middle won’t be painted.</td>
<td>Beginning</td>
<td>Can you convince me that the cubes in the middle won’t be painted? Is that true for any size cube?</td>
</tr>
<tr>
<td>Starting statements in a logical argument are correct and accepted by the classroom. E.g. “It repeats by 8 because there are 8 corners.”
(See Annotated Work Sample 3)</td>
<td>Developing</td>
<td>When is it true?</td>
</tr>
<tr>
<td>The student verifies truth of statements by using a common property, rule or known facts to confirm the total cubes painted on 0/1/2 faces.</td>
<td>Consolidating</td>
<td>Can you generalise these patterns?</td>
</tr>
<tr>
<td>The student verifies that the rule is true for all cases.
E.g.
Unpainted cubes: (n-2)^3 So for example if you have a 4x4x4 and a 2x2x2 cube, the 2x2x2 cube will fit inside the 4x4x4 cube leaving 1 block on each side. So the number of cubes in the 2x2x2 is the number of cubes not painted. If I test this on any other type of cube, it will be true.
1-face painted is 6(n-2)^2 or (n-2)^2 x 6. In each face of a cube, you need to take one away from each corner because these are painted on three faces. There will be n-2 cubes along each edge. Because it is a square then you have to square it and there are 6 faces in a cube so you need to multiply by 6.
2-faces painted is (n-2) x 12 - or 12(n-2). We look at each edge but you don’t include the corners. So, if there are n-2 cubes on each edge, then you have to times it by the 12 edges.
3-faces painted is always 8 because only the corners will ever have 3 faces painted.</td>
<td>Extending</td>
<td>Convince me your expression is true for all cases.

How do the rules relate to the features of the cubes we noted earlier?

Try an extension problem.</td>
</tr>
</table>

Annotated Work Sample 1

ANALYSING: **Notices** the number of cubes painted on three faces is always 8.

Imagine a cube made up of 27 smaller cubes (3 x 3 x 3).
Imagine that you dip the cube in paint.

If you now separate it into 27 small cubes, you will notice that some of the small cubes are painted. Which small cubes have been painted on all 3 sides, on 2 sides, on 1 side, and not painted at all - and how many are there?

Fill in the grid below for a 3 x 3 x 3 cube.
Consider and complete the grid for different size cubes, 2 x 2 x 2, 4 x 4 x 4, 10 x 10 x 10, etc.

CUBE SIZE	Total number of small cubes	Number of cubes painted on 3 sides	Number of cubes painted on 2 sides	Number of cubes painted on 1 side	Number of cubes not painted at all
2 x 2 x 2					
3 x 3 x 3	27	8	12	6	6
4 x 4 x 4		8			
5 x 5 x 5		8			
6 x 6 x 6		8			
7 x 7 x 7		8			
8 x 8 x 8		8			
9 x 9 x 9		8			
10 x 10 x 10					
...					
15 x 15 x 15					
20 x 20 x 20					

Create a rule for predicting the answers for larger cubes without counting all the small cubes?
Show you working on the next sheet.

CUBE SIZE	Total number of small cubes	Number of cubes painted on 3 sides	Number of cubes painted on 2 sides	Number of cubes painted on 1 side	Number of cubes not painted at all
n x n x n					

Student Name:

For columns A,B,C,D, and E, describe the patterns that you see. What changes, and what stays the same?

A

B

GENERALISING: **Does not communicate** a common property or rule for pattern.

C

D

JUSTIFYING: **Does not justify.**

E

ANALYSING: Beginning

GENERALISING: Not evident

JUSTIFYING: Not evident

Teacher Prompt

How can you convince me that the number of cubes painted on three faces is always 8?

Use 4x4x4 cube and show me how you find the cubes painted on one face only, two faces and no faces at all in this cube? What do you notice?

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Work Sample 1 Rubric

Student Name: WORK SAMPLE 1 **Reasoning Task:** PAINTED CUBE **Date:**

Observation of student's reasoning:

Noticed the number of cubes painted on 3 sides is always 8 but experienced difficulties in other areas.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

+ Go back to enabling prompts & reasoning prompts

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Annotated Work Sample 2

ANALYSING: **Notices** similarities across examples.
The student notices that the number of cubes painted on three faces is always 8. The student also notices the total number of cubes and the number of cubes painted on two faces and one face and not painted at all. There are some mistakes in the analysis.

Imagine a cube made up of 27 smaller cubes (3 x 3 x 3).
Imagine that you dip the cube in paint.

If you now separate it into 27 small cubes, you will notice that some of the small cubes are painted. Which small cubes have been painted on all 3 sides, on 2 sides, on 1 side, and not painted at all – and how many are there?

Fill in the grid below for a 3 x 3 x 3 cube.
Consider and complete the grid for different size cubes, 2 x 2 x 2, 4 x 4 x 4, 10 x 10 x 10, etc.

CUBE SIZE	Total number of small cubes	Number of cubes painted on 3 sides	Number of cubes painted on 2 sides	Number of cubes painted on 1 side	Number of cubes <u>not</u> painted at all
2 x 2 x 2	8	8	0	0	0
3 x 3 x 3	27	8	12	6	1
4 x 4 x 4	64	8	28	24	4
5 x 5 x 5	125	8	54	54	27
6 x 6 x 6		8			
7 x 7 x 7		8			
8 x 8 x 8	384	8			
9 x 9 x 9		8			
10 x 10 x 10		8			
...					
15 x 15 x 15		8			
20 x 20 x 20	80000	8			

Create a rule for predicting the answers for larger cubes without counting all the small cubes?
Show you working on the next sheet.

CUBE SIZE	Total number of small cubes	Number of cubes painted on 3 sides	Number of cubes painted on 2 sides	Number of cubes painted on 1 side	Number of cubes <u>not</u> painted at all
n x n x n					

GENERALISING: **Draws attention to** or **attempts to communicate** a common property or repeated component of a pattern using oral language.

Student Name:
For columns A,B,C,D, and E, describe the patterns that you see. What changes, and what stays the same?

A The number of non Painted squares is always the cube 2 below's total cubes.

B It is allways 8 for column B
Cubes always have 8 corners

C

JUSTIFYING: **Recognises** what is correct or incorrect using materials, objects or words.

D

E

ANALYSING: Developing

GENERALISING: Beginning

JUSTIFYING: Developing

Teacher Prompt:

Where are the cubes with no faces painted?

What stays the same and what changes? (Analysing)

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Work Sample 2 Rubric

Student Name: WORK SAMPLE 2 **Reasoning Task:** PAINTED CUBE **Date:**

Observation of student's reasoning:

- Noticed number of cubes on three sides is always 8
- Beginning to form a conjecture about being '2 less'

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Need to explore patterns using materials & prompts.

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Annotated Work Sample 3

ANALYSING: **Noticing more than one** common property by systematically generating further cases and/or listing and considering a range of known facts or properties.
The student systematically lists numbers of cubes. However, need to observe student constructing this table to find out when they used a rule that they noticed rather than only counting cubes.

Imagine a cube made up of 27 smaller cubes (3 x 3 x 3).
Imagine that you dip the cube in paint.

If you now separate it into 27 small cubes, you will notice that some of the small cubes are painted. Which small cubes have been painted on all 3 sides, on 2 sides, on 1 side, and not painted at all - and how many are there?

Fill in the grid below for a 3 x 3 x 3 cube.
Consider and complete the grid for different size cubes, 2 x 2 x 2, 4 x 4 x 4, 10 x 10 x 10, etc

CUBE SIZE	Total number of small cubes	Number of cubes painted on 3 sides	Number of cubes painted on 2 sides	Number of cubes painted on 1 side	Number of cubes <u>not</u> painted at all
2 x 2 x 2					
3 x 3 x 3	27	8	12	6	1
4 x 4 x 4	64	8	24	24	4
5 x 5 x 5	125	8	36	54	27
6 x 6 x 6	216	8	48	96	64
7 x 7 x 7	303	8	60	175	125
8 x 8 x 8					
9 x 9 x 9					
10 x 10 x 10					
...					
15 x 15 x 15					
20 x 20 x 20					

Student Name:

For columns A,B,C,D, and E, describe the patterns that you see. What changes, and what stays the same?

A

JUSTIFYING: Starting statements in a **logical argument** are correct and accepted by the classroom.

B It's repeting 8 because there are only 8 corner

C It's going up by 12 every bigger shape.

GENERALISING: **Communicates the rule** about a pattern using words.

D If you got for example a 3x3x3 cube and don't count the single outline of o face that leaves 1 so times it by 6 and you have your anser.

E

GENERALISING: **Explains** what the rule means using one example

ANALYSING: Consolidating

GENERALISING Developing

JUSTIFYING: Developing

Teacher Prompt

Can you write a rule to find the number of cubes painted on two faces for any size cube? Can you describe the pattern for the number of cubes with one painted face?

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Work Sample 3 Rubric

Student Name: WORK SAMPLE 3 **Reasoning Task:** PAINTED CUBE **Date:**

Observation of student's reasoning:

- Systematically lists facts
- Explains meaning of rule e.g 3×3×3 cube
- Notices increase of 12.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Explore ways to devise a rule using informal to more formal methods.

→ Prompts

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Annotated Work Sample 4

Imagine a cube made up of 27 smaller cubes (3 x 3 x 3).
Imagine that you dip the cube in paint.

If you now separate it into 27 small cubes, you will notice that some of the small cubes are painted. Which small cubes have been painted on all 3 sides, on 2 sides, on 1 side, and not painted at all – and how many are there?

Fill in the grid below for a 3 x 3 x 3 cube.
Consider and complete the grid for different size cubes, 2 x 2 x 2, 4 x 4 x 4, 10 x 10 x 10, etc

CUBE SIZE	A Total number of small cubes	B Number of cubes painted on 3 sides	C Number of cubes painted on 2 sides	D Number of cubes painted on 1 side	E Number of cubes not painted at all
2 x 2 x 2	8 8	8	0	0	0
3 x 3 x 3	27	8	12	6	1
4 x 4 x 4	64	8	24	24	8
5 x 5 x 5	125	8	36	54	27
6 x 6 x 6	216	8	48		64
7 x 7 x 7	343	8	60		125
8 x 8 x 8	512	8	72		216
9 x 9 x 9	729	8	84		343
10 x 10 x 10	1000	8	ab		512
...					
15 x 15 x 15	3375	8	156		169
20 x 20 x 20	8000	8	216		324

Create a rule for predicting the answers for larger cubes without counting all the small cubes? Show you working on the next sheet.

CUBE SIZE	Total number of small cubes	Number of cubes painted on 3 sides	Number of cubes painted on 2 sides	Number of cubes painted on 1 side	Number of cubes not painted at all
n x n x n	n^3	8	length-2 x12		$[\text{length}-2]^3$

ANALYSING: **Notices more than one common** property by systematically generating further cases and/or listing and considering a range of known facts or properties.

The student notices that the number of cubes painted on three faces is always 8 and identifies the number of cubes painted on 0/1/2 faces.

ANALYSING: **Repeats and extends patterns** using both the numerical and spatial structure.

ANALYSING: Consolidating

GENERALISING: Consolidating

JUSTIFYING: Consolidating

Teacher Prompt

Do you notice any patterns in the number of cubes painted on 1 face?

How can you convince me that the pattern always true?

GENERALISING: **communicates a rule** using mathematical terms, symbols or diagrams.

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re(Solve) MATHS BY INQUIRY

Work Sample 4 Rubric

Student Name: WORK SAMPLE 4 **Reasoning Task:** PAINTED CUBE **Date:**

Observation of student's reasoning:

↑ Analysing - noticed various patterns in table. Beginning to develop a 'rule' to test conjecture "length - 2 × 12"

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. * • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. * • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Formalise rule & apply to test further cases & to develop a 'watertight' argument.

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The Painted Cube

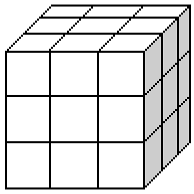
Name: ______________

Imagine a cube made up of 27 smaller cubes (3 x 3 x 3).

Imagine that you fully immerse the cube in paint.

Now separate it into 27 small cubes. You will notice that some of the small cubes have been painted on some faces.

Which small cubes have been painted on 3 faces, on 2 faces, on 1 face, and not painted at all? How many cubes of each type are there?



First fill in the grid below for a 3 x 3 x 3 cube.

Then complete the grid for different size cubes, 2 x 2 x 2, 4 x 4 x 4, 10 x 10 x 10, etc.

CUBE SIZE	Total number of small cubes **(A)**	Number of cubes painted on 3 faces **(B)**	Number of cubes painted on 2 faces **(C)**	Number of cubes painted on 1 face **(D)**	Number of cubes <u>not</u> painted at all **(E)**
2 x 2 x 2					
3 x 3 x 3					
4 x 4 x 4					
5 x 5 x 5					
6 x 6 x 6					
7 x 7 x 7					
8 x 8 x 8					
9 x 9 x 9					
10 x 10 x 10					
…					
15 x 15 x 15					
20 x 20 x 20					

Create a rule for predicting the answers for larger cubes without counting all the small cubes. Show your working on the next sheet.

CUBE SIZE	Total number of small cubes **(A)**	Number of cubes painted on 3 faces **(B)**	Number of cubes painted on 2 faces **(C)**	Number of cubes painted on 1 face **(D)**	Number of cubes <u>not</u> painted at all **(E)**
n x n x n					

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For columns A, B, C, D, and E, describe the patterns that you see.
What changes, and what stays the same?

A

B

C

D

E

Student Name: **Reasoning Task:** **Date:**

Observation of student's reasoning:

	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify**.
BEGINNING	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
DEVELOPING	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
CONSOLIDATING	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
EXTENDING	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

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