Year 5 Exemplar
What Else Belongs?

Australian Curriculum: Mathematics (Year 5)

ACMNA098: Identify and describe factors and multiples of whole numbers and use them to solve problems.

- Exploring factors and multiples using number sequences.
- Using simple divisibility tests.

Abstract

In this task, students find common properties of three given numbers. They notice and describe properties of number; such as size, order, composition, place value, multiples, factors, even or odd. The students justify why some numbers have similar or dissimilar properties. There is a strong emphasis on Analysing – seeing what is the same and what is different. The task can very readily be adapted to other content and year levels.

Mathematical purpose (for students)

We will find many properties that numbers share.

Mathematical purpose (for teachers)

Teachers support and challenge students to:

- Notice and describe common properties, including multiplicative properties (Analysing)
- Write a statement (conjecture) about the common property (Generalising)
- Explain the common property and find further cases (Generalising)
- Test their conjecture by verifying that each number belongs (Justifying)

Time needed 60 minutes approximately

Vocabulary Encountered

- convince me
- justify
- explain why
- testing/verifying ideas
- because

Materials

- Student Sheet 1 – What Else Belongs? (optional,1 per student)
- Reasoning Prompt Cards or Poster (see Teachers' Guide *ST5_Reasoning_TeachersGuide.docx*)
- Assessment Sheet (1 per student)

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

What Else Belongs: The Lesson

Reasoning Task

I wonder: could these numbers 36, 12, 18 ***belong*** *together? Why?*
What other numbers do you think could belong with these numbers?
How do you know that all these numbers fit with your reason?
Use words, numbers, or drawings to explain.

Introducing the Reasoning Task

To orient students' thinking, present them with a number, such as 10, and ask them to discuss with a partner and share everything they know about that number. Then present them with the main task and clarify any questions without giving too much away.

During the working time, draw students' attention to the fact that there are many common properties shared by these three numbers. For example, they have even ones digits (6, 2, 8) and odd tens digits (1 and 3). So, this task has no one right answer. Challenge students to find mathematically interesting answers.

Also, be clear about the emphasis that you want placed on justifying. For example, do you want students to demonstrate why 36 is an even number, or just take this as commonly shared knowledge. This choice depends on lesson goals.

Reasoning Prompts

For more prompts related directly to this reasoning task, see this table.

- What is the same about …? (Analysing)
- Sort or organise the following according to … (Analysing)
- What follows from this? What do you think will happen next if we do this? (Analysing)
- What is the pattern here? (Generalising)
- Are there other examples that fit the rule? (Generalising)
- Convince me. (Justifying)

What is the same and different about …?	Sort or organise the following according to …	What follows from this? What do you think will happen next if we do this?	What is the pattern here?	Are there other examples that fit the rule?	Convince me…

Enabling Prompts

- How might numbers such as 6 and 8 belong together? Then the student re-engages with the main task.
- List everything you know about the number (12) and then one other number (18 or 36). Then students pick out some things that these two numbers have in common; why they can be on the same team.

Extending Prompts

- What numbers would *not* belong if this is the reason?
- Write down as many reasons as you can find that 36, 12 and 18 belong together.

Summary Phase

Invite students to share their solutions to develop a whole class mathematical discussion. The Formative Assessment Table shows the likely variation in responses. You might:

- Summarise the class contributions like the teacher shown below, in a table with 5 columns (Names of students, Suggested new number, Suggested reason for belonging, Other numbers that belong, Numbers that do not belong).
- Encourage students to explain each other's thinking.
- Ask
    - What is one thing you know now about finding common properties that you did not know before?
    - What have you learned about explaining your reasoning to others?

Further Activities

Follow-Up Tasks

- This task is very flexible and sets of numbers can be created to go with almost any number topic at any year level. For example, use {0.25, 1.5, 3.75} or {3/6, 2.5, 18/4}.
- Create your own set of "What else belongs?" with a minimum of three numbers for a friend to work on. Make sure you write down your reason on a secret sheet of paper. Remember your friend might find a different reason for the numbers to belong together, so you will have to see if that is also a correct reason!

Alternate Tasks

- *Above and Below the Line* has similar underpinnings to *What Else Belongs?*. The task is appropriate for middle and upper primary students. https://topdrawer.aamt.edu.au/Reasoning/Activities/Above-and-below-the-line

Formative Assessment

The following table shows some responses that students commonly give to this problem. These responses demonstrate the variety of levels for each reasoning action. Studying these sample responses can prepare the teacher for identifying their students' reasoning during the lesson. Suitable prompts are suggested to support or extend such students' reasoning.

Many of the possible responses in the table are linked to full work samples from students. Each work sample has been annotated by the teacher using the Rubric. A copy of the teachers' assessment sheet shows what the teacher recorded about reasoning during and after the lesson, and the recommendations the teacher made about how to further that student's reasoning.

ANALYSING Possible Student Response	Level	Suggested Prompts
They are all numbers.	Not evident	What is the same and different about these numbers…?
In the ones place they are all even and in the tens, they are odd."	Beginning	How do you know these are odd numbers? How do you know this is an even number? Can you show me how you get that? (expect drawings, gestures)
They are all even. **(See** Annotated Work Sample 1**)**	Developing	How do you know these are all even? What is the same? What is different?
Counts by 1 s, 2 s, 3 s and 6 s. "12 × 1 = 12, 4 × 3 = 12, 12 × 5 =60; 2 × 14 = 60 (sic); 3 × 6 = 18; 6 × 6 = 36; 3 × 12 = 36". "6 goes into that [pointing at 36] 6 times; 6 goes into that [pointing at 12] 2 times; 6 goes into that [pointing at 18] 3 times"	Developing	I'm wondering what **reasons** there might be for these numbers **belonging** together in this group. Why do you think these numbers **belong** together? What is your **reason**?
GENERALISING **Possible Student Response**	**Level**	**Suggested Prompts**
Count on by 2 from each number to find other numbers that fit with their rule **(See** Annotated Work Sample 1**)**	Developing	How does 12 fit with your reason? Convince me.
The student offers 216 as another number that was in the counting pattern and explains: *"Yeah, and I just got to 108 and doubled it."* *"That is - skip counting by 6s"*	Consolidating	Convince me … How can we be sure…? How do you know…? What is the pattern here?
They are all multiples of 2, 3 or 6." **(See** Annotated Work Sample 3**)**	Consolidating	
My other group of numbers can be double, triple or six times these numbers and they will belong."	Consolidating	
JUSTIFYING **Possible Student Response**	**Level**	**Suggested Prompts**
They're all over 10, under 40 and they each have 2 digits	Beginning	How else could you sort these numbers? What does each number have in common?
Identifies each number from the group in the '2s' and '3s' counting patterns. **(See** Annotated Work Sample 2**)**	Consolidating	Is there another reason that these numbers could fit together in a group? What do you notice about the numbers in the two groups that you have made?
"**Any** number that is a multiple of 2, 3 and 6 will belong." Provides further examples.	Extending	Offer extending prompt or follow up task.

Annotated Work Sample 1

OUR REASON

These numbers belong together because ....

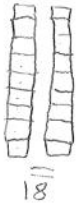
These numbers are all even

Other numbers that belong with this group are ...

32 14 20
34 16 22
36 18 24
38 20 26
40 28
38

How do you know that all these numbers belong and fit with your reason? Use words, numbers or drawings to explain.

~~18~~ 16+2=18



even to each

18

Deakin University Mathematics Reasoning Research Group

ANALYSING: **Notices** a common numerical or spatial property.

GENERALISING: **Communicates** a rule about a *property* using words, diagrams or number sentences.

Count on by 2 from each number to find other numbers that fit with their rule

GENERALISING: **Extends** the pattern using an example to **explain how** the rule works.

JUSTIFYING: **Does not justify** (only one case verified).

Even is defined as numbers that can be divided by 2

GENERALISING: Explains what the rule means using one example.

Uses a number sentence, diagram and words to explain the meaning of the rule.

Even is defined as adding 2 to another number

ANALYSING: Developing

GENERALISING:

Developing/ Consolidating

JUSTIFYING: Not evident

Teacher Prompt

How do 12 and 30 fit with your reason? Convince me.

Note: This student was working with the set {12, 18, 30}.

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

re(Solve) MATHS BY INQUIRY

Work Sample 1 Rubric

Student Name: WORK SAMPLE 2 Reasoning Task: WEB ? Date:

Observation of student's reasoning:

* Used number sentences to explain the rule
* explored odd/even numbers

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.** only one case verified
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument.** • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Develop justification using prompts e.g Convince me... "why..."

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Annotated Work Sample 2

GENERALISING: Communicates a rule about a ***property*** using words.

OUR REASON

These numbers belong together because .... they are all even numbers and from the threes counting patten

Other numbers that belong with this group are ...

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30

How do you know that all these numbers belong and fit with your reason? Use words, numbers or drawings to explain.

GENERALISING: Extends the number pattern using an example to explain how the rule works.

Explore two rules – even numbers and counting patterns.

ANALYSING: **Notices** more than one common property by systematically generating further cases and/or listing and considering a range of known facts.

Even numbers and 3s counting pattern.

JUSTIFYING: Verifies truth of statements by using a common property, rule or known facts that confirms each case. May also use materials and informal methods.

Identifies each number from the group in the '2s' and '3s' counting patterns.

ANALYSING: Consolidating

GENERALISING: Developing/ Consolidating

JUSTIFYING: Consolidating

Teacher Prompt

Is there another reason that these numbers could fit together in a group?

What do you notice about the numbers in the two groups that you have made?

Note: This student was working with the set {12, 18, 30}.

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

re(Solve) MATHS BY INQUIRY

Work Sample 2 Rubric

Student Name: **Reasoning Task:** **Date:**

Observation of student's reasoning:

* Explored two rules: even numbers & counting patterns

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Use prompts to further develop generalisation & justifications e.g "Is there another reason..."

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Annotated Work Sample 3

ANALYSING: **Notices** more than one common property.

Notices an additive relationship between numbers

GENERALISING: **Communicates** a rule about a **property** using words.

Using words to explain counting patterns

ANALYSING: Consolidating

GENERALISING: Developing/Consolidating

JUSTIFYING: Consolidating

Teacher Prompt

Explain why numbers in the twos and threes counting pattern are also in the sixes counting pattern.

OUR REASON

These numbers belong together because ....

This number counts by 1's, 2's, 3's and 6's. These numbers belong with 12 and 18 both equal to 30 and 30 + 30 = 60.

Other numbers that belong with this group are ...

30's 6's 3's 7's 2's 60's the 60's

there is the 5 at the end of each of them numbers because

How do you know that all these numbers belong and fit with your reason? Use words, numbers or drawings to explain.

6+6+6+6+6=30

3+3+3+3+3+3+3+3+3+3=30

1+1+1+1+1+1+1+1+1+1

GENERALISING: **Explains** the rule for finding one term in the pattern using a number sentence

Using number sentences to explain the common property using one case for each property.

Note: This student was working with the set {12, 18, 30}.

Whole class discussion:

Six goes into 30 five times, six goes into that [12] two times, 6 goes into that [18] three times.

JUSTIFYING: **Verifies** truth of statements by using a common property, rule or known facts that confirms each case. May also use materials and informal methods.

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

re(Solve) MATHS BY INQUIRY

Work Sample 3 Rubric

Observation of student's reasoning:

* Noticed additive relationship
* Used words to explain patterns
" Number sentences to explain common properties

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Extending prompt * Alternate / follow up tasks
* Develop watertight argument.

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

What Else Belongs?

Name: _____

36	12	18

MY REASON

- These numbers belong together because…

- Other numbers that belong with this group are…

- I know that all these numbers belong together because…
  (use words, numbers, or drawings to explain)

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

Student Name: **Reasoning Task:** **Date:**

Observation of student's reasoning:

	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify**.
BEGINNING	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • A**rgument** is not coherent or does not include all steps.
DEVELOPING	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
CONSOLIDATING	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
EXTENDING	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt