Year 3 Exemplar
Number Towers

Australian Curriculum: Mathematics (Year 3)

ACMNA054: Recognise and explain the connection between addition and subtraction.

- Demonstrating the connection between addition and subtraction using partitioning or by writing equivalent number sentences.

ACMNA055: Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation.

- Combining knowledge of addition and subtraction facts and partitioning to aid computation.

Abstract

Number Towers gives students an opportunity to develop and test conjectures and form generalisations by reasoning mathematically about numerical structures with addition. The task promotes careful analysis of a mathematical structure. Students need to experiment systematically, keep track of results, and choose cases carefully to test the rule. The task can be adapted for older students by using fractions or decimals.

Mathematical purpose (for students)

To develop and test conjectures and then explain and justify to others.

Mathematical purpose (for teachers)

Teachers support and challenge students to develop, test and explain conjectures by:

- Conducting trials using number facts and additive properties of numbers (Analysing)
- Comparing and contrasting examples to develop a conjecture (Analysing, Generalising)
- Explaining conjecture using an example (Generalising)
- Testing and explaining conjectures using examples to verify using trials (Justifying)
- Using understanding of additive relationships to justify conjectures (Justifying)

Time Needed 100 minutes approximately

Vocabulary Encountered

- compare and contrast
- convince me
- justify/explain why
- testing/verifying ideas
- because/if… then…

Materials

- Student Sheet 1 – Number Tower Template (1 per student)
- Student Sheet 2 – Largest Tower (1 per pair)
- Student Sheet 3 – Jane's Conjecture (1 per student)
- Reasoning Prompts Cards or Poster (see Teachers' Guide *ST5_Reasoning_TeachersGuide.docx*)
- Assessment Sheet (1 per student)

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Number Towers: The Lesson

Introducing Number Towers

A 'Number Tower' is made by placing numbers in the boxes of the tower template. Any five numbers are chosen for the bottom row of boxes. Then each of the boxes on the next row is calculated by adding the numbers in the two adjacent number boxes below it. All the boxes of the number tower below are filled in this way.

For example, the number 50 on the top of the Number Tower below is calculated from 28 + 22.

The filled-in template on the left is not a number tower. For example, 102 is not equal to 5 + 12.

Number Tower

		50		
	28		22	
17		11		11
9	8		3	8
3	6	2	1	7

Not a Number Tower

		258		
	120		75	
47		39		102
11	20		5	12
3	6	2	1	7

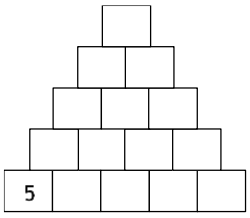
- Begin the lesson by showing students what a Number Tower is, giving examples and non-examples like those above.
- Ask students to make a Number Tower for themselves and get a partner to check it. Suggest that they choose small numbers on the bottom row. When everyone is clear what a Number Tower is, introduce the Reasoning Task.

Reasoning Task

Jane makes a Number Tower by starting with the numbers 1, 2, 3, 4 and 5 on the bottom row, then adding each pair of numbers to get the next row up.

Jane thinks that to make the largest total at the top, you need to put the largest number (5) as the first number on the bottom row.

Do you agree with Jane?
Why or why not?



- Hand out Student Sheet 1 – Number Tower Template and Student Sheet 2 – Largest Tower and Student Sheet 3 – Jane's Conjecture.
- Set students to work on the reasoning task in pairs using the numbers {1, 2, 3, 4, 5}.
- Suggest students try different arrangements and keep a good record of what happens. As they work, they should look for reasons to explain the findings.
- Alternatively, teachers might allow students explore ways to make the largest number first using Student Sheet 1 – Number Tower Template and Student Sheet 2 – Largest Tower and then engage in the main task (Student Sheet 3 – Jane's Conjecture).
- After 10-15 minutes, check-in with the class and invite them to share the largest number (see Student Sheet 2 – Largest Tower) they have created so far and notice any similarities and/or differences. They should share any ideas and observations they have made that might explain their findings.
- This task is useful for encouraging students to use the reasoning language of "If…then…"

Reasoning Prompts

For more prompts in the context of this task, see this table.

- What is the same and different about …? (Analysing)
- Alter an aspect of something to see an effect. If we change this what will happen? (Analysing)
- What is the pattern here? (Generalising)
- Is that … (pattern) always going to work? (Generalising)
- What happens in general? (Generalising)
- What is the rule? (Generalising)
- Can you go through that step by step? (Justifying)

What is the same and different about …?	Alter an aspect of something to see an effect. If we change this what will happen?	What is the pattern here?	Is that…(pattern) always going to work?	What happens in general?	What is the rule?	Can you go through that step by step?

Enabling Prompts

- What totals can you make by pairing the numbers 1-5 on the bottom row?
- Offer students a calculator.
- Reduce the number of levels of the tower.
- Where do you put the numbers to get the biggest total for a 2-layer tower, or for a 3-layer tower?

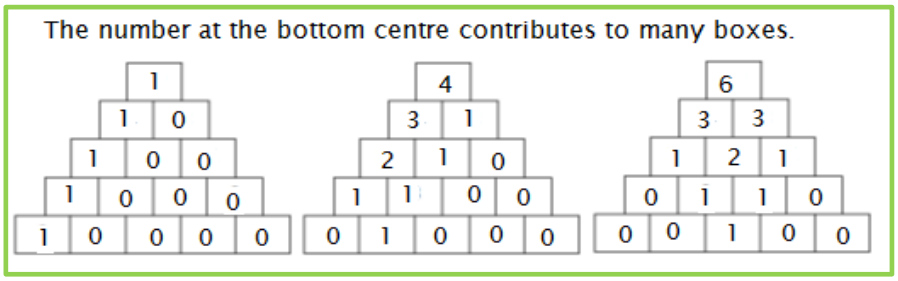
Extending Prompts

- Is this true for any set of numbers? Convince me.

Summary Phase

The value in the task comes from students sharing and discussing their conjectures and their reasons. Invite students to share their solutions in order of complexity to develop a whole class mathematical discussion. The Formative Assessment Table shows the likely variation in responses. You might:

- Encourage students to explain each other's thinking.
- Ask:
  - "What is one thing you know now about making and testing conjectures that you did not know before?".
  - "What have you learned about explaining your reasoning to others?"
- To assist in explanations of why the largest number needs to go in the middle, it is helpful to look at the Number Towers below that show the 'influence' of the placement of a number on the final total.



Further Activities

Follow Up Task Variations

Students can apply the strategies that they have heard and been shown to finding the best arrangement of any set of numbers.

- Decimals or fractions can be used if addition of these numbers is a current classroom goal. For example, {0.1, 0.2, 0.3, 0.4, 0.5}.
- Students can choose their own sets of numbers such as {4, 5, 6, 7, 8} or {2, 7, 10, 11, 24}. Keep the numbers reasonably small to start.
- A very revealing choice is the set {1, 2, 3, 4, 100}. This choice makes it easy to trace how the 100 makes its contribution to the total.

Formative Assessment

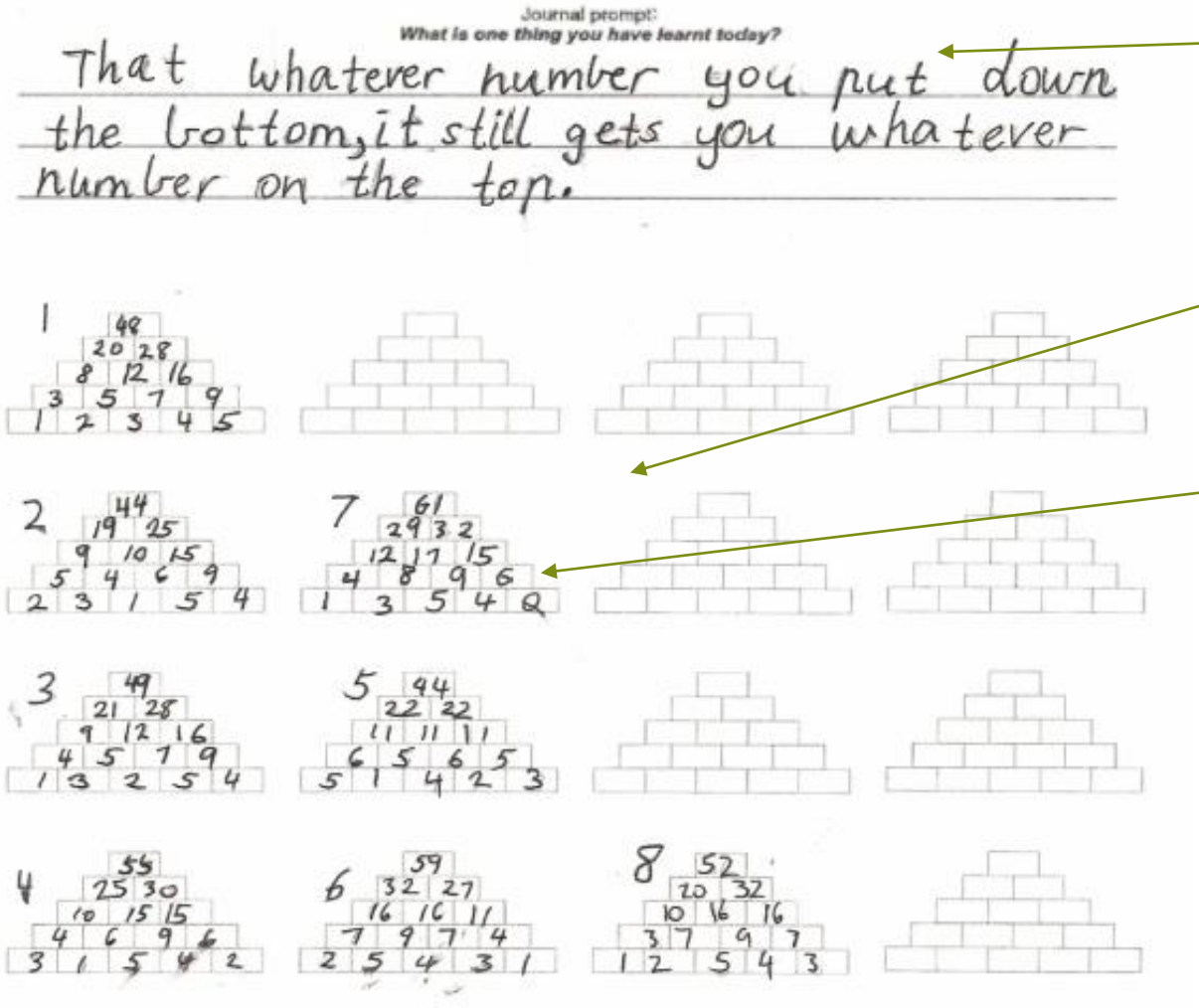
The following table shows some responses that students commonly give to this problem. These responses demonstrate the variety of levels for each reasoning action. Studying these sample responses can prepare the teacher for identifying their students' reasoning during the lesson. Suitable prompts are suggested to support or extend such students' reasoning.

Many of the possible responses in the table are linked to full work samples from students. Each work sample has been annotated by the teacher using the Rubric. A copy of the teachers' assessment sheet shows what the teacher recorded about reasoning during and after the lesson, and the recommendations the teacher made about how to further that student's reasoning.

ANALYSING Possible Student Response	Level	Suggested Prompts
Creates a tower but does not attend to the rules for constructing each layer of addition.	Not Evident	Offer enabling prompt (simplified tower).
The student creates a range of number towers and notices similarities across the examples.	Beginning	What is the same and different about …? What stays the same and what changes?
Attempts to sort the number towers based on a particular position of a number. **(See Annotated Work Sample 2)**	Beginning	Alter an aspect of something to see (such and such) effect. If we change this what will happen?
The student orders the towers from highest to lowest totals to show what stays the same and what is different. **(See Annotated Work Sample 3)**	Developing	What do you notice?
The student investigates how the position of numbers effects the distribution of subsequent totals and then the final total. **(See Annotated Work Sample 4)**	Consolidating	What happens in general? Are there other examples that fit the rule?
The student describes the "effect" that the position of the number on the bottom row has on subsequent totals. **(See Annotated Work Sample 6)**	Extending	Propose a more challenging task.
GENERALISING **Possible Student Response**	**Level**	**Suggested Prompts**
Whatever number you put down the bottom, it still gets you whatever number at the top. **(See Annotated Work Sample 1)**	Beginning	What is the same and different about …? What stays the same and what changes?
When you put a 4 in the bottom left corner, it will always equal 49. **(See Annotated Work Sample 2)**	Beginning	Is it just sometimes true, or is it always true?
If you put a 5 in the middle you will get 61 on top.	Beginning	Is it just sometimes true, or is it always true?
You don't always have to put the bigger numbers on the side to make a bigger number on top. When I tested the bigger number on the side and it didn't work then we tested the little numbers [on the side] and it worked.	Developing	Is that arrangement always going to work?
The student describes that by placing the larger numbers in the 5 in the middle results in a larger number on top. **(See Annotated Work Sample 3)**	Developing	Is that … (pattern) always going to work?

I disagree with Jane because 3, 4 and 5 need to go together to make a big number and if the 5 is on the corner, it can only go with one number and if it's in the middle they all connect. **(See Annotated Work Sample 4)**	Consolidating	How could we demonstrate / show / prove that it is true? Is that order of numbers the only one that works?
No – my reasons are: If we had 5 in the middle, the 5 would go into the next two equalling a higher number, but if we have it on the side it only goes into half the numbers so it isn't as high. **(See Annotated Work Sample 5)**	Consolidating	How could we demonstrate / show / prove that it is true? Is that order of numbers the only one that works?
You have to have the highest number in the middle because that's the one that affects the most numbers that will come afterward. (Clearly articulated conjecture). Tests and refines conjecture on further examples. **(See Annotated Work Sample 6)**	Extending	Is it just sometimes true, or is it always true?
JUSTIFYING **Possible Student Response**	**Level**	**Suggested Prompts**
No Jane is incorrect. You can make different numbers.	Not Evident	Why did you think it was not possible?
Jane is right.	Not Evident	How do you know? What combinations can you make?
I used addition.	Beginning	What is the same and different about (each tower)?
Jane is not right because I tried it her way and also with a lower number and I had a larger number when I put the smaller number on the side.	Beginning	How can we be sure …?
You can't have the big numbers on the side if you want big numbers…We tried trial and error, then we put the five in the middle and the 4 next to the five. Lastly, we put the small numbers at the side. **(See Annotated Work Sample 3)**	Developing	Why does this arrangement work?
You have to have the highest number in the middle because that's the one that effects the most numbers that will come afterwards. In the same way, the lowest numbers will have to be on the very ends because it will have the least effect on the numbers on top. **(See Annotated Work Sample 6)**	Consolidating	How could we demonstrate / show / prove that it is true? Is that … (pattern) always going to work?
We get the largest total when the largest number is in the middle of the bottom row (it contributes to the total six times), and the next two largest numbers on either side of it (they each contribute four times to the final total).	Extending	Is this true for any set of numbers?

Annotated Work Sample 1



JUSTIFYING: **Describes** what they did and why it may or may not be correct.

The student has explained the process they used to derive the number they used at the top of the tower. The next step is to explore what influences the total.

ANALYSING: Notices similarities across examples. Recalls and **repeats** patterns displayed visually or through use of materials. *In this case they have systematically changed the arrangement on the bottom row.*

GENERALISING: **Draws attention to** or **attempts to communicate** a common property or repeated components of a pattern by using drawing, and concrete materials. *They have created several examples.*

ANALYSING: Beginning

GENERALISING: Beginning

JUSTIFYING: Beginning

Teacher Prompt

What stays the same and what changes?

What is the same and different about …? (Analysing)

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Work Sample 1 Rubric

Student Name: WORK SAMPLE 1 Reasoning Task: NUMBER TOWERS Date:

Observation of student's reasoning:

Could explain what they did (process) to get number on top but not 'why' some numbers were larger.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify.**
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

* Explore 'why'
* What stays the same / what changes?
* What is same / different about...

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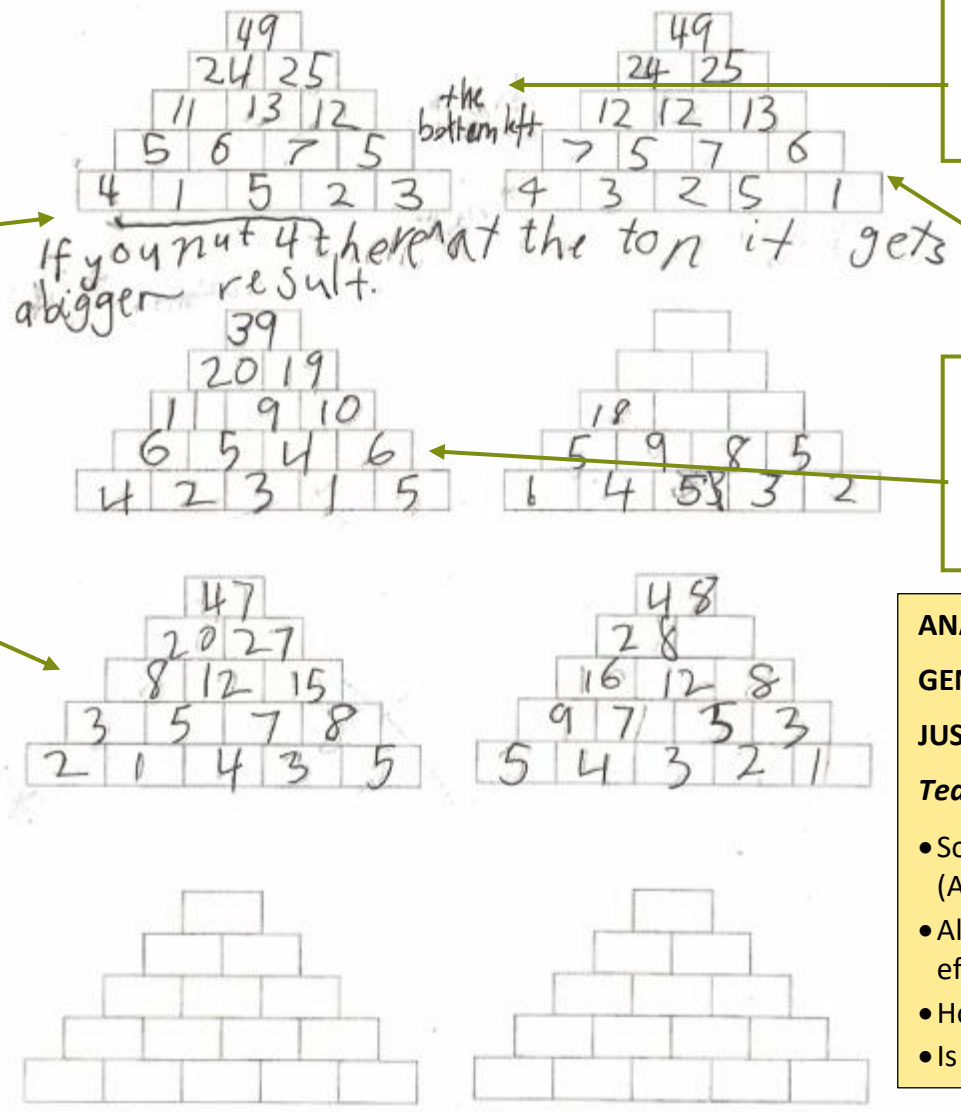
Annotated Work Sample 2



"If you put 4 there, at the bottom left, at the top it gets a bigger result"

GENERALISING: Draws attention to or **attempts to** communicate a common property or repeated components of a patter using materials. *(Testing the idea of the placement of 4)*

JUSTIFYING: Describes what they did and why it may or may not be correct (*because 4 on the left end doesn't always give a bigger*

JUSTIFYING: The **argument may not be coherent** or include all steps in the reasoning process. *This example refutes earlier claim of "when you put a 4 on the bottom left hand corner, it will get a bigger number on top" but not recognised by student.*

ANALYSING: Notices similarities across examples.

Attempts to **sort** cases based on a common property**.** *(The position of the 4)*

ANALYSING: Beginning

GENERALISING: Beginning

JUSTIFYING: Beginning

Teacher Prompts:

- Sort or organise the following according to … (Analysing)
- Alter an aspect of something to see (such and such) effect. If we change this what will happen? (Analysing)
- How can we be sure …? (Justifying)
- Is it just sometimes true, or is it always true?

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Work Sample 2 Rubric

Student Name: Work Sample 2 Reasoning Task: Number Towers Date:

Observation of student's reasoning:

Focused on the position of the '4' & the end result but then seemed confused when they didn't recognise they had refuted a claim.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Refocus student's attention to further develop & test their idea about placement/position of numbers.
E.g. 'sort according to...'
'what changes/stays the same...'

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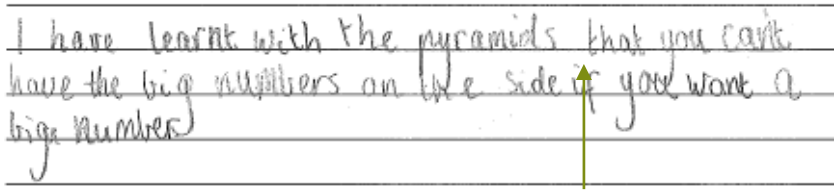
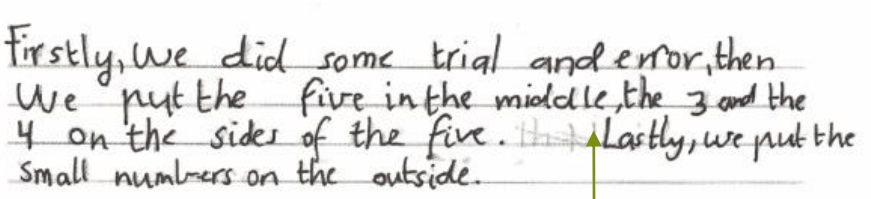
Annotated Work Sample 3





ANALYSING: Developing

GENERALISING: Developing

JUSTIFYING: Developing

Teacher Prompt

Explain why does this (process /procedure/result) work? (Justifying)

Is that … (pattern) always going to work? (Justifying)

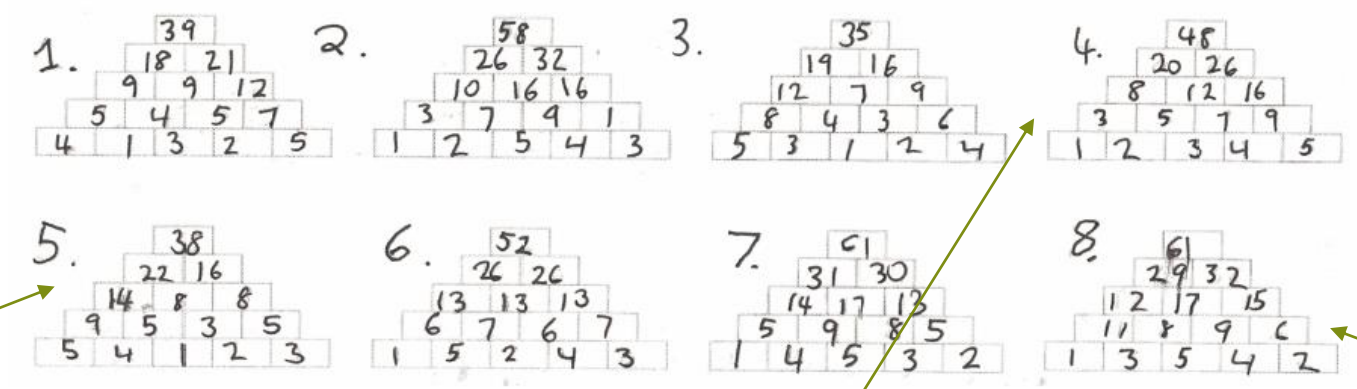
Convince me. (Justifying)

JUSTIFYING: Starting statements in a **logical argument** are correct and accepted by the classroom

This correct first step begins the argument. The next step is to explain why a large total results from placing the large number in the middle.

GENERALISING: **Communicates** a rule about a property using words, diagrams or number sentences.

The student describes that by placing the larger numbers in the 5 in the middle results in a larger number on top.



ANALYSING: Sorts and classifies cases according to a common property.

Orders cases to show what is the same or stays the same and what is different or changes.

The student has used trial and error for number towers 1 to 5. The student then compares each subsequent number tower and according to highest and lowest totals.

JUSTIFYING: **Refutes** a claim by using a counter example.

The student supports their claim about needing large numbers in the middle to make a large number on top by providing an example that shows their reasoning.

JUSTIFYING: **Verifies** truth of statements by using a common property, rule of known facts that confirms each case. *Tests claim by producing another example.*

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Work Sample 3 Rubric

Student Name: WORK SAMPLE 3 Reasoning Task: NUMBER TOWERS Date:

Observation of student's reasoning:

Noticed the importance of the '5' being in the middle to get the highest number & provided/tested with examples.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Next need to explain 'why'.
E.g Convince me it is always going to work.

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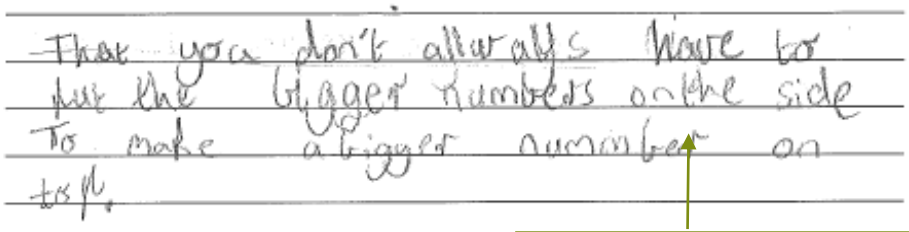
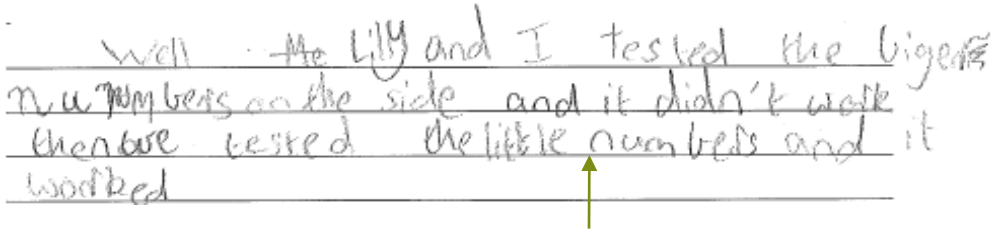
Annotated Work Sample 4


That you don't always have to put the bigger numbers on the side To make a bigger number on top.


Well Me Lily and I tested the bigger numbers on the side and it didn't work then we tested the little numbers and it worked

ANALYSING: Consolidating

GENERALISING: Consolidating

JUSTIFYING: Consolidating

Teacher Prompt

Explain why does this (process/procedure/result) works? (Justifying)

Is that … (pattern) always going to work? (Justifying)

Convince me. (Justifying)

JUSTIFYING: Starting statements in a **logical argument** are correct and accepted by the classroom.

Correctly states first step in the argument. The next

GENERALISING: **Explains what** the rule means using one example.

Identifies the boundary – that is, placing the bigger number on the sides does not work.

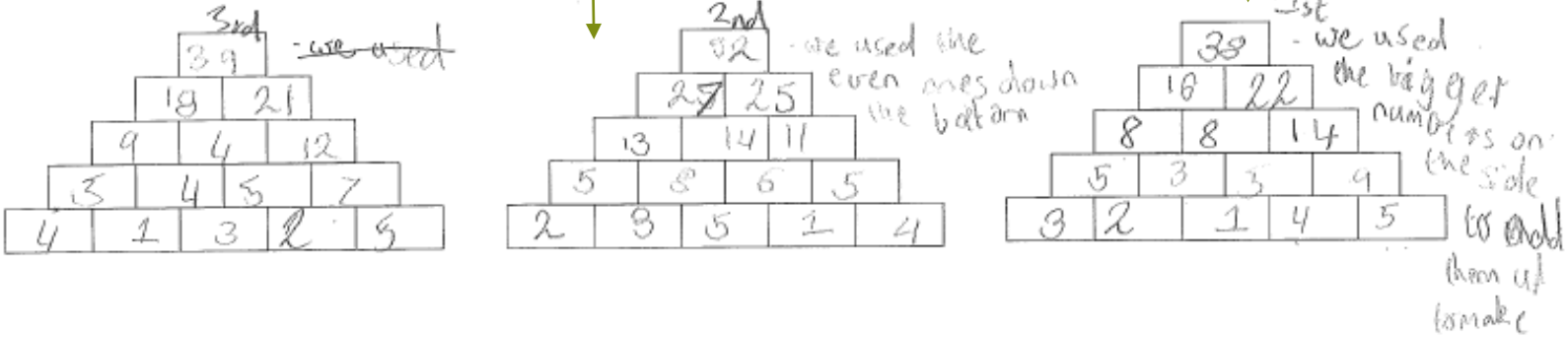
ANALYSING: Sorts and classifies cases according to a common property *(the number at the top).*

Orders cases to show what is the same or stays the same and what is different or changes.

The student has analysed their towers according to the placement of the numbers.

JUSTIFYING: **Refutes** a claim by using a counter example.

Provides an example to explain their thinking.



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Work Sample 4 Rubric

Student Name: WORK SAMPLE 4 Reasoning Task: NUMBER TOWERS Date:

Observation of student's reasoning:

- Sorted / classified towers according to position of numbers.
- Identified larger numbers needing to be in middle with examples

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Next step is to explore 'why' this happens.
E.g - Is that always going to work?
- Convince me.

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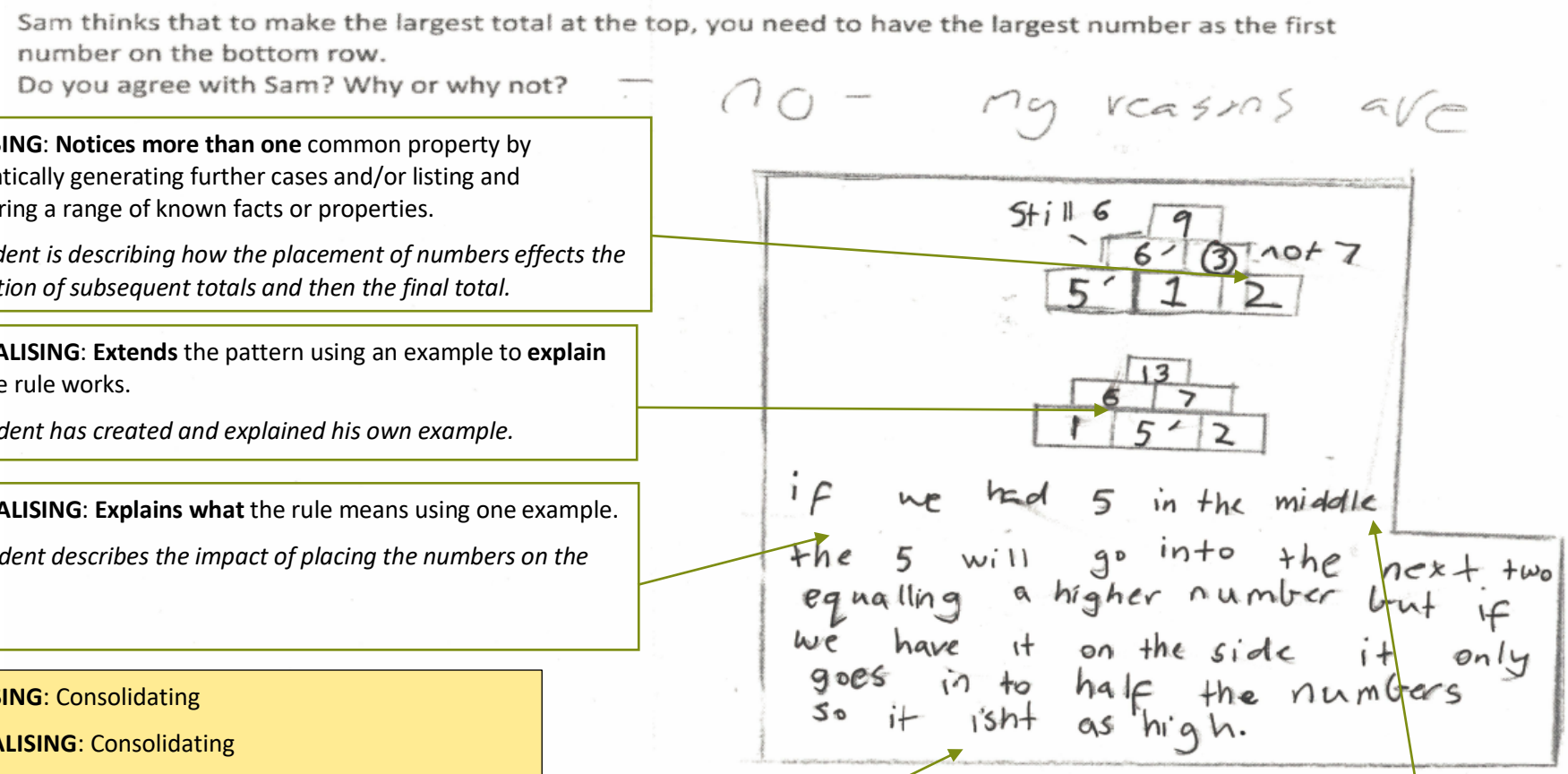
Annotated Work Sample 5



ANALYSING: **Notices more than one** common property by systematically generating further cases and/or listing and considering a range of known facts or properties.

The student is describing how the placement of numbers effects the distribution of subsequent totals and then the final total.

GENERALISING: **Extends** the pattern using an example to **explain how** the rule works.

The student has created and explained his own example.

GENERALISING: **Explains what** the rule means using one example.

The student describes the impact of placing the numbers on the "side".

ANALYSING: Consolidating

GENERALISING: Consolidating

JUSTIFYING: Consolidating

Teacher Prompt

Is it just sometimes true, or is it always true? (Justifying)

Is that … (pattern) always going to work? (Generalising)

Convince me. (Justifying)

JUSTIFYING: **Extends** the generalisation using logical argument.

Attempts to explain the 'distribution' of numbers as they go up the tower and how it changes the final total of the tower.

JUSTIFYING: Uses a correct **logical argument** that has a complete chain of reasoning to it and uses words such as 'because', 'if…then…', 'therefore', 'and so', that leads to…'

Begins with 'if' and infers 'then' at the start of the second line.

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Work Sample 5 Rubric

Student Name: Work Sample 5 Reasoning Task: Number Towers Date:

Observation of student's reasoning:

Identified how the position of the largest number effects the distribution of remaining totals & attempts to explain it with his own example.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Use prompts to further investigate & develop a watertight argument.

E.g Convince me.

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Annotated Work Sample 6

ANALYSING: **Notices and explores** relationships between numerical structure of patterns
The student describes the "effect" the position of the number on the bottom row has on subsequent totals.

GENERALISING: **Generalised** properties by forming a statement about the relationship between common properties

Clearly articulated conjecture.

GENERALISING: **Applies** the rule to find further examples or cases

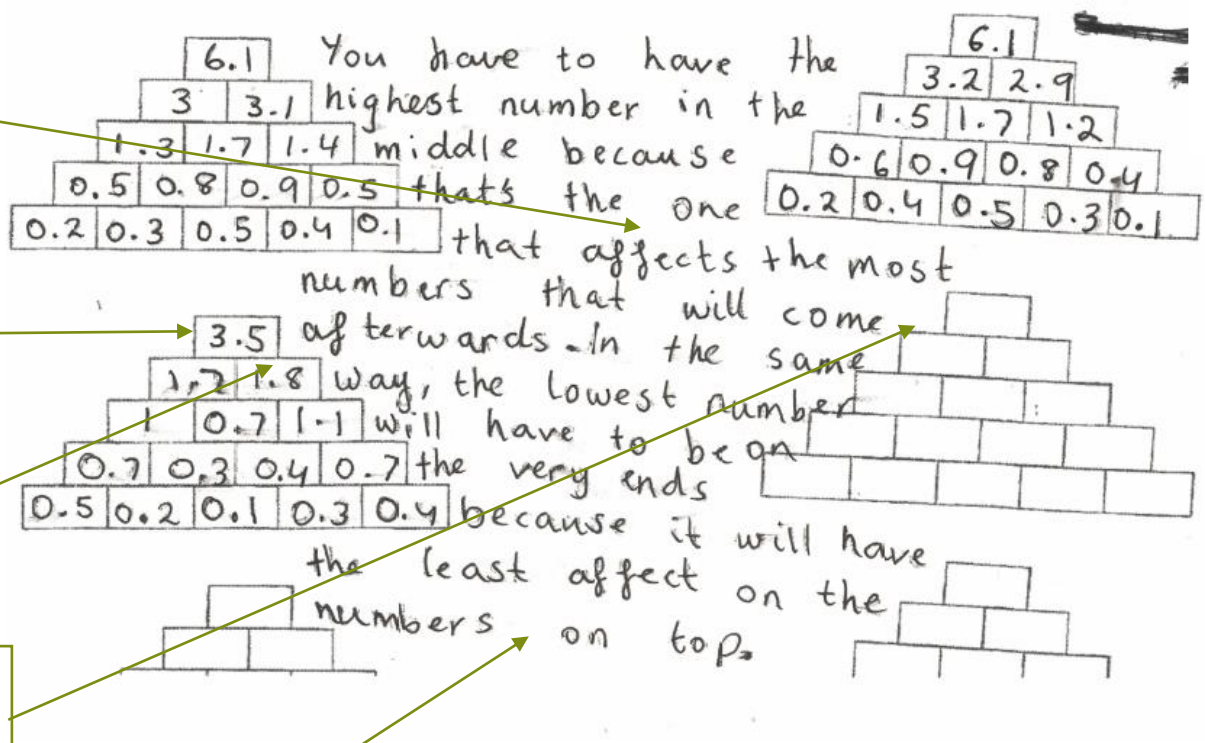
Tests and refines conjecture on further examples.

JUSTIFYING: Uses a correct **logical argument** that has a complete chain of reasoning to it and uses words such as 'because', 'if…then…', 'therefore', 'and so', that leads to…' *language used indicates awareness of causal relationships*

JUSTIFYING: **Extends** the generalisation using logical argument.

Describes "most effect" and "least effect" on creating the largest total possible.

The next step is to develop a watertight argument for all cases.



ANALYSING: Extending

GENERALISING: Extending

JUSTIFYING: Consolidating

Teacher Prompt

Is it just sometimes true, or is it always true? (Justifying)

Convince me.

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re(Solve) MATHS BY INQUIRY

Work Sample 6 Rubric

Student Name: WORK SAMPLE 6 Reasoning Task: NUMBER TOWERS Date:

Observation of student's reasoning:

Clearly explain the "effect" the position of the smallest/largest numbers has on the total.
She tested/refined conjectures.

	Analysing	Generalising	Justifying
Not Evident	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture).	• **Does not justify**.
Beginning	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule for the pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • **Argument** is not coherent or does not include all steps.
Developing	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms and records other cases or examples.	• **Attempts to verify** by testing cases and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
Consolidating	• **Systematically searches** for examples, extends pattern or analyses structure to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
Extending	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols (including algebraic symbols) and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

Use prompts to develop water tight argument for all cases.

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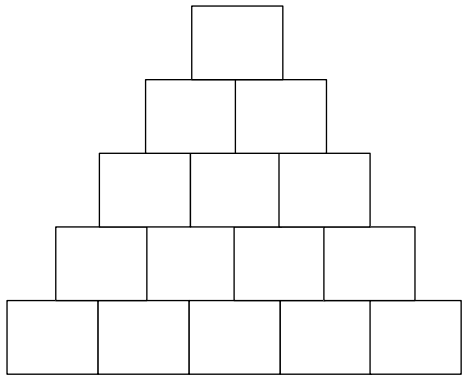
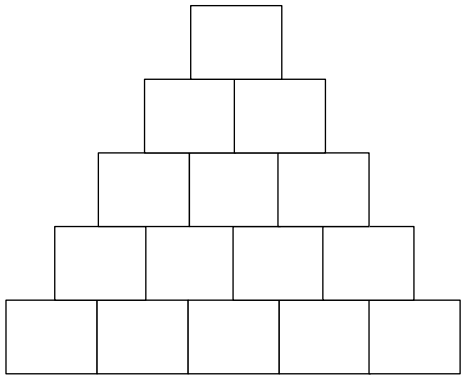
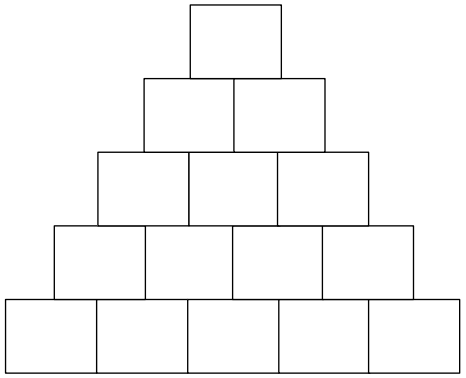
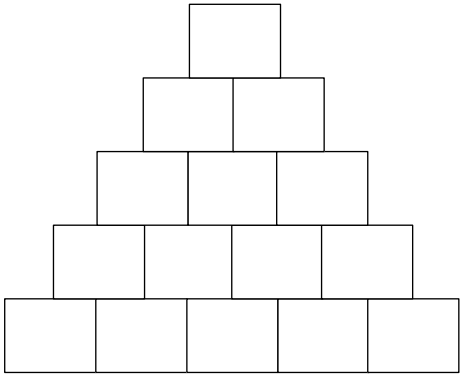
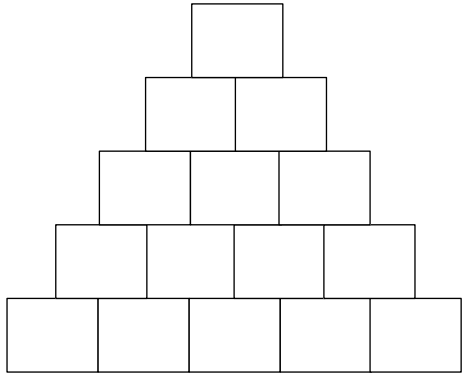
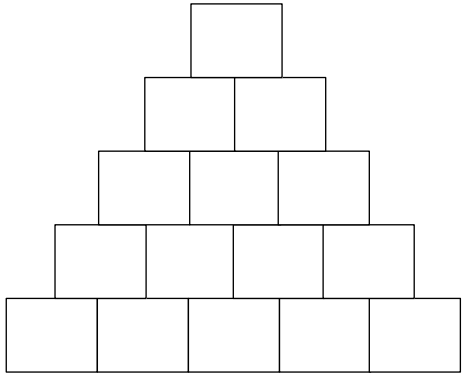
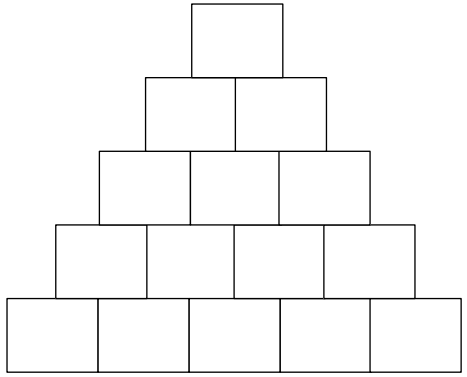
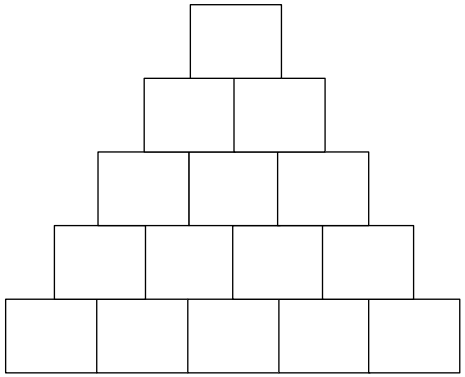
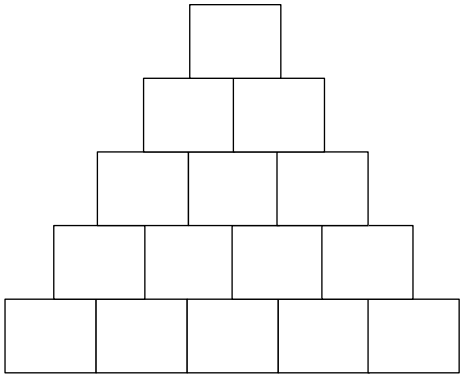
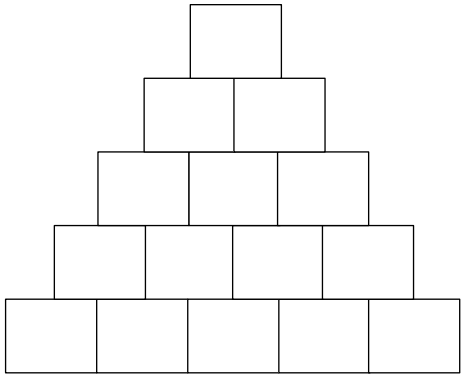
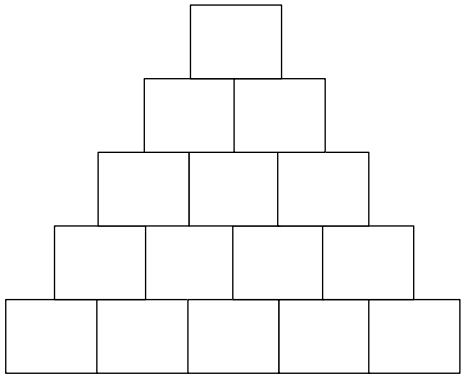
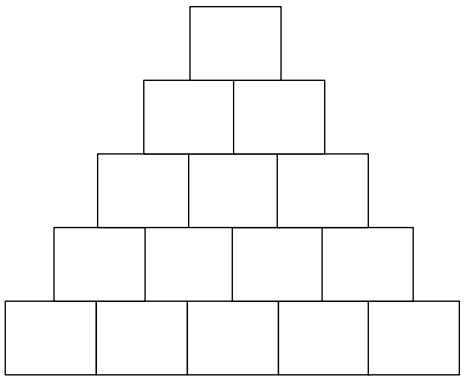
Number Towers Template

Name:_____

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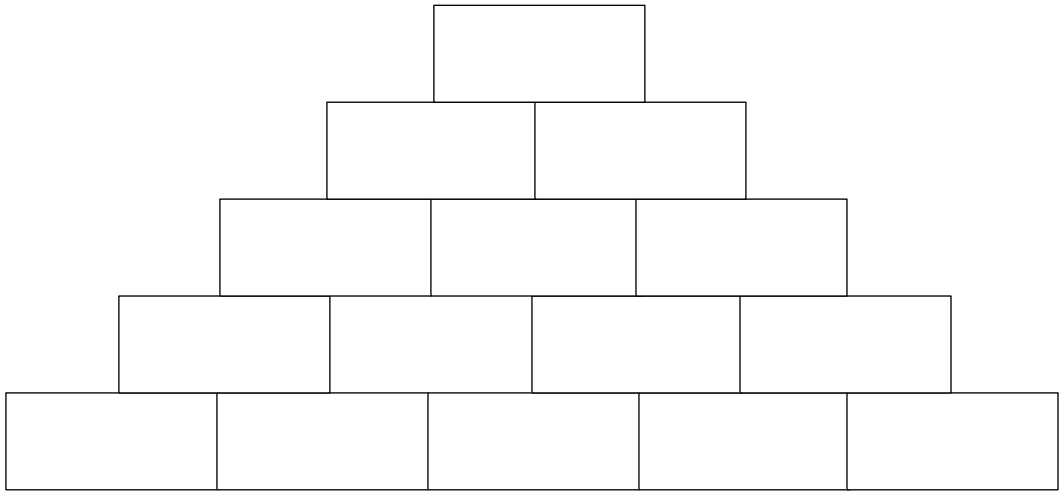
Largest Number Tower

Name:_____



To make the largest total on top I thought that…

_____

_____

_____

_____

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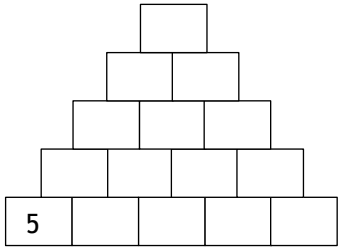
Jane's Conjecture

Name:_____

Jane is making a Number Tower with 1, 2, 3, 4 and 5 on the bottom row.
Jane thinks that to make the largest total at the top, you need to put the largest number as the first number on the bottom row.
Do you agree with Jane?
Why or why not?



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Student Name: **Reasoning Task:** **Date:**

Observation of student's reasoning:

	ANALYSING	GENERALISING	JUSTIFYING
NOT EVIDENT	• **Does not notice** common property or pattern.	• **Does not communicate** a common property or rule (conjecture) for a pattern.	• **Does not justify**.
BEGINNING	• **Recalls** random known facts or **attempts to sort** examples or **repeats** patterns.	• **Attempts to communicate** a common property or rule (conjecture) for a pattern.	• **Describes** what they did and **recognises** what is correct or incorrect. • A**rgument** is not coherent or does not include all steps.
DEVELOPING	• **Notices** a common property, or **sorts and orders** cases, or **repeats and extends** patterns. • **Describes** the property or pattern.	• **Generalises: communicates** a rule (conjecture) using mathematical terms, and records other cases or examples.	• **Attempts to verify** by testing cases, and **detects and corrects** errors or inconsistencies. • Starting statements in a **logical argument** are correct.
CONSOLIDATING	• **Systematically searches** for examples, extends patterns, or analyses structures, to form a conjecture. • Makes **predictions** about other cases.	• **Generalises: communicates** a rule (conjecture) using mathematical symbols and **explains what** the rule means or **explains how** the rule works using examples.	• **Verifies** truth of statements by confirming all cases or **refutes** a claim by using a counter example. • Uses a correct **logical argument.**
EXTENDING	• **Notices and explores** relationships between properties.	• **Generalises** cases, patterns or properties using mathematical symbols and **applies** the rule. • **Compares** different expressions for the same pattern or property to show equivalence.	• Uses a **watertight logical argument**. • **Verifies** that the generalisation holds for *all* cases using logical argument.

Comments (feedback, reasoning prompts for further development):

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