Bar Models in Problem Solving
Lesson 7: Change Model - Whole Numbers

Australian Curriculum: Mathematics (Year 6)

ACMNA123: Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (Year 6)

Lesson abstract

In this lesson, students learn to use the change bar model, a variant on the comparison model. They construct visual representations of complex stories, usually involving multiple quantities that change over time, and use the representations to find a strategy for solving the problems. Students study examples, then practise the techniques on other tasks.

Mathematical purpose (for students)

Change models help us keep track of 'before' and 'after' information.

Mathematical purpose (for teachers)

This lesson introduces the change model using examples involving whole numbers. The change model is useful to represent challenging multi-step word problems involving multiple quantities, which change in relation to each other (e.g. before – after situations). A change model generally consists of four bars, being two linked comparison models. There are multiple pieces of information in the problem statement, so organising all the information in a visual representation is especially useful. Relationships can be additive or multiplicative. Using the change model, and often by identifying 'units' that are common across the bars, students can find the sequence of arithmetic operations that solve the problem. Looking at variants of the basic bar models emphasises that the bar model method is not a prescription, but a tool to be used flexibly as the problem situation demands. Polya's four steps of problem solving are used to structure the solution process.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- Change model

Lesson Materials

- Slide show *ST4_BarModelsPS_7a_ChangeWN.pptx*
- Student Sheet 1 – Bar Model Examples 7A (1 per student)
- Student Sheet 2 – Bar Model Examples 7B (1 per student)
- Calculators as needed

We value your feedback after these lessons via https://www.surveymonkey.com/r/G6VGPZ8

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Background

Hand out Student Sheet 1 – Bar Model Examples 7A.

Students should write the solutions to these examples, for future reference.

The slide show (*ST4_BarModelsPS_7a_ChangeWN.pptx*) provides animated solutions to these two examples which can be used during initial instruction and class discussion.

In the problems for this lesson, quantities change over time and the changes are recorded using a change bar model. Usually a change model consists of two comparison models, one for 'before' and one for 'after'.

The discussion is organised around Polya's four stages. The first "Understand" stage is intended to make sure students absorb the information given in the problem statement.

Whole Class Examples

Example 1

Charlotte had 132 sweets and chocolates. After 38 sweets were given away, the number of sweets is the same as the number of chocolates. How many sweets were there at first?

Expected Student Response (using change model)

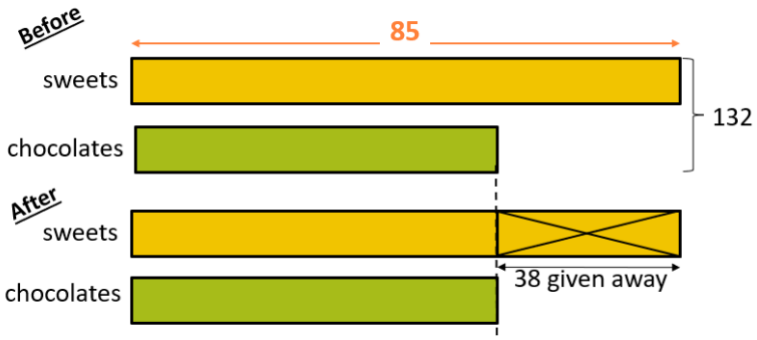
Call the number of sweets/chocolates afterwards 1 unit

2 units = 132 – 38 = 94 sweets/chocolates

1 unit = 94 ÷ 2 = 47 sweets/chocolates

47 + 38 = 85 sweets

There were 85 sweets at first.



Discussion organised by Polya's four stages

Explain that the aim of the first example is to introduce a new type of bar model in a simple situation.
The animated slide show ST4_BarModelsPS_7a_ChangeWN.pptx can be used to support the discussion.

Read the problem with the class and discuss how to solve it. This problem can be done without the change model and students might like to try it first just using a comparison model (Do this after the 'Understand" phase below). The comparison model would probably look like the 'after' pair of bars in the solution below.

Understand
Encourage students to analyse the before and after situations in the problem:

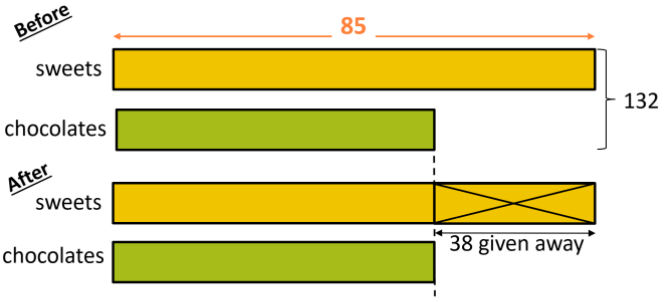
- Does Charlotte have more sweets or chocolates at first? (ANS: Charlotte has more sweets)
- What happened after 38 sweets were given away? What two quantities are equal? (ANS: The number of sweets and chocolates were the same)
- What do we have to find? (ANS: The number of sweets Charlotte has initially)

Plan
This is the first change model in these lessons. Emphasis how to draw the change model and the usefulness of comparing the relationships in the before and after situations. This requires construction of two comparison models – one for the 'before' situation, and one for 'after'.

Do
Work through the problem as a group using the change model, for this initial task. Some discussion points could include:

- Observe the change from one situation (before) to another situation (after). How might this help to find the solution?
- What might be a common 'unit' between all the bars? (ANS: The number of sweets or chocolates in the after situation)
- Work through the calculations with students.

Check
Encourage students to check the answer by seeing that the conditions of the original problem are met.

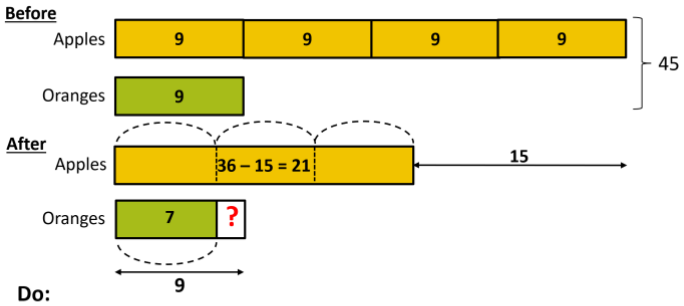


85 – 38 = 47
There will be 47 sweets after 38 were given away.
47 + 47 = 94
94 + 38 = 132
There were 132 sweets and chocolates at first.

Example 2

There were a total of 45 apples and oranges in a basket. There were 4 times as many apples as oranges. After 15 apples and some oranges were removed, there were 3 times as many apples as oranges left. How many oranges were removed from the basket?

Expected Student Response



Do:

- Observe the change from one situation (before) to another situation (after) to find the solution

3 units = 21
1 unit = 21 ÷ 3 = 7

9 – 7 = 2
2 oranges were removed from the basket.

Discussion organised by Polya's four stages

Understand
Encourage students to analyse the before and after situation given in the problem:

- How many apples and oranges are there in the basket altogether at first? (ANS: 45).
- Are there more apples or oranges? (There are more apples, both before and after fruit was removed).
- What happened after 15 apples and some oranges were removed? (ANS: the ratio of apples to oranges changed).
- Note that there is a multiplicative relationship in this problem as well as additive relationships.
- What do we have to find? (ANS: The number of oranges removed from the basket).

Plan

Discuss the before and after situations and how they are related whilst constructing the bar model.

- Distinguish the before and after situations, so students are clear what the bar models should show.
- Construct the two comparison models – one for the 'before' situation, and one for 'after'. The before-situation shows that the number of apples is 4 times as many as the number of oranges. This problem also requires the use of the change model because after 15 apples and some oranges were removed, there were 3 times as many apples as oranges left.).
- Draw and label the model with students.

Do
Some discussion points could include:

- Observe the change from one situation (before) to another situation (after). How might this help to find the solution?
- What might be a common 'unit' between all the bars? (ANS: The number of oranges in the 'before' situation)
- Work through the calculations with students.

Check
Encourage students to check the answer aby substituting it into the problem. The slideshow demonstrates how to tick off each of the conditions in the original problem statement to make sure they are all met.

9 – 2 = 7 After 2 were removed, there were 7 oranges left. 3 x 7 = 21 There were 3 times as many apples as oranges after the fruit was removed.	21 + 15 = 36 (apples) 36 + 9 = 45 There were 45 apples and oranges at first.

Consolidating and Concluding

Further practice

Hand out Student Sheet 2 - Bar Model Examples 7B. Students work individually, in pairs or in groups on selected problems.

Discuss solutions as time permits. Worked solutions are provided in Teacher Sheet - Bar Model Solutions 7B, and solutions to Task 1 and Task 2 are also included in the slide show *ST4_ProblemSolving_7a_WNChange.pptx*.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The change model shows the relationship between the new value of a quantity (after situation) and its original value (before situation).
- A change model is often made up of multiple comparison models; one comparison model to represent a 'before' situation, and another to represent the changed values of the 'after' situation.

Bar Model Examples 7A

Name: _____

Example 1

Charlotte had 132 sweets and chocolates. After 38 sweets were given away, the number of sweets is the same as the number of chocolates. How many sweets were there at first?

Example 2

There were a total of 45 apples and oranges in a basket. There were 4 times as many apples as oranges. After 15 apples and some oranges were removed, there were 3 times as many apples as oranges left. How many oranges were removed from the basket?

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Bar Model Examples 7B

Name: _____

Draw bar models to represent the situations below and use them to help you solve the problems.

Task 1

Emily had $2400 and Sarah has $1400. After Emily had given Sarah some money, they had an equal amount of money. How much money did Emily give Sarah?

Task 2

Alex and Ben had an equal number of cookies at first. Alex then threw away 37 of his cookies which were burnt while Ben bought another 15 cookies. In the end, Ben had three times as many as cookies as Alex. How many cookies did Alex have at first?

Task 3

James and Zenda bought 81 marbles altogether. James bought 15 more than Zenda. After Zenda gave 8 marbles to James, how many more marbles does James have than Zenda?

Task 4

Mrs Ng gave Peggy and Wendy an equal number of funfair coupons. Peggy then gave 6 of her coupons to Wendy. Wendy wanted even more so she asked Mrs Ng for 16 more coupons. In the end, Wendy had three times as many funfair coupons as Peggy. How many funfair coupons did Peggy have at first?

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Bar Model Solutions 7B

Task 1

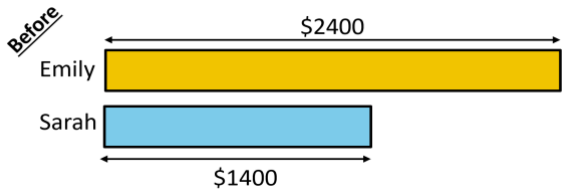
Emily had $2400 and Sarah has $1400. After Emily had given Sarah some money, they had an equal amount of money. How much money did Emily give Sarah?

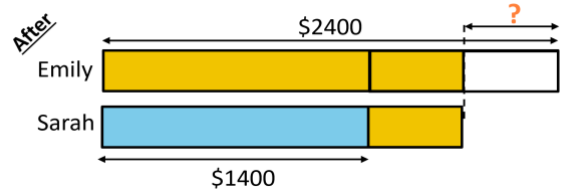
Understand

- Who had more money at first? (ANS: Emily had more money at first).
- What happened after Emily gave Sarah some money? (ANS: Both girls then had the same amount of money).
- What do I have to find? (ANS: The amount of money Emily gave Sarah)

Plan

- What kind of model should we draw? (ANS: A change model, showing comparisons before and after)
- Draw and label the comparison model.





Do

The key point is to notice that Emily needs to give Sarah half of her 'excess'.

2400 – 1400 = 1000
The difference of amount of Emily's and Sarah's money at first.

1000 ÷ 2 = 500

After Emily gave Sarah $50, they had equal amounts of money.

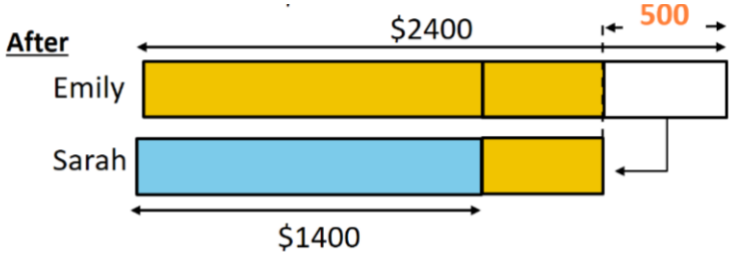
Check

Check the answer by substituting it into the original problem.

2400 – 500 = 1900

1400 + 500 = 1900

Both Emily and Sarah have equal amounts at the end.



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Task 2

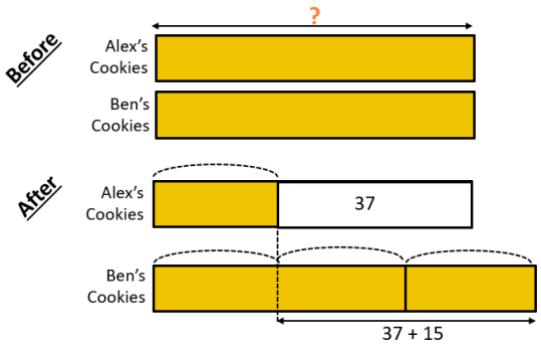
Alex and Ben had an equal number of cookies at first. Alex then threw away 37 of his cookies which were burnt while Ben bought another 15 cookies. In the end, Ben had three times as many as cookies as Alex. How many cookies did Alex have at first?

Understand

- Who had the most cookies at the start?? (ANS: The had the same amount.)
- Who had the most cookies at the end? How did that happen? (ANS: Ben)
- Name some quantities that are equal. Be precise. (ANS: The two batches of cookies at the start; Alex's final number is equal to one third of Ben's final number of cookies.)
- What do I have to find? (ANS: How many cookies Alex had at first).

Plan

- What kind of model should we draw? (ANS: A change model).
- Make sure students are clear about which is the before and which is the after information.
- Draw and label the change model.



Do

Observe the change from one situation (before) to another situation (after) to find the solution

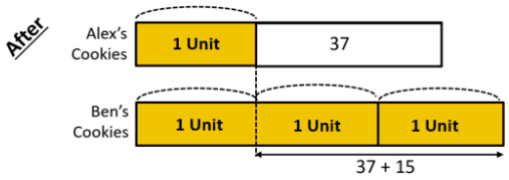
<u>After</u>

2 units = 37 + 15 = 52

1 unit = 52÷ 2 = 26

26 + 37 = 63

Alex had 63 cookies at first.



Check

Check the answer by substituting it into the original problem.

63 – 37 = 26

Alex had 26 cookies left after throwing away some.

63 + 15 = 78

Ben had 78 cookies after buying more.

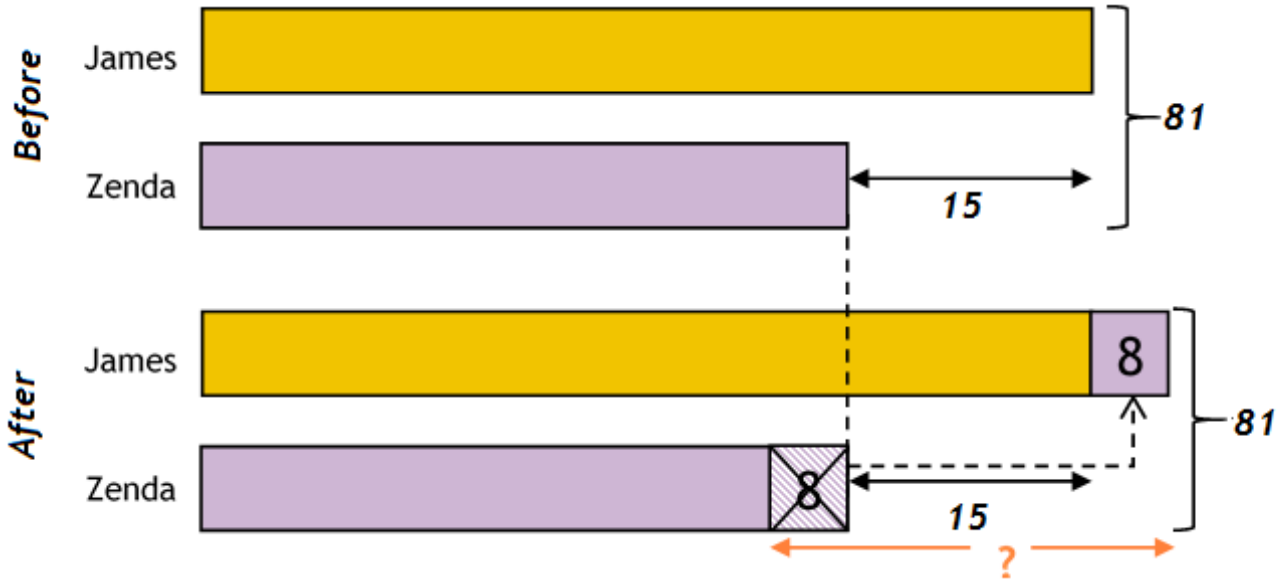
78 ÷ 3 = 26

26 + 37 = 63

Alex had 63 cookies at first.

Task 3

James and Zenda bought 81 marbles altogether. James bought 15 more than Zenda. After Zenda gave 8 marbles to James, how many more marbles does James have than Zenda?



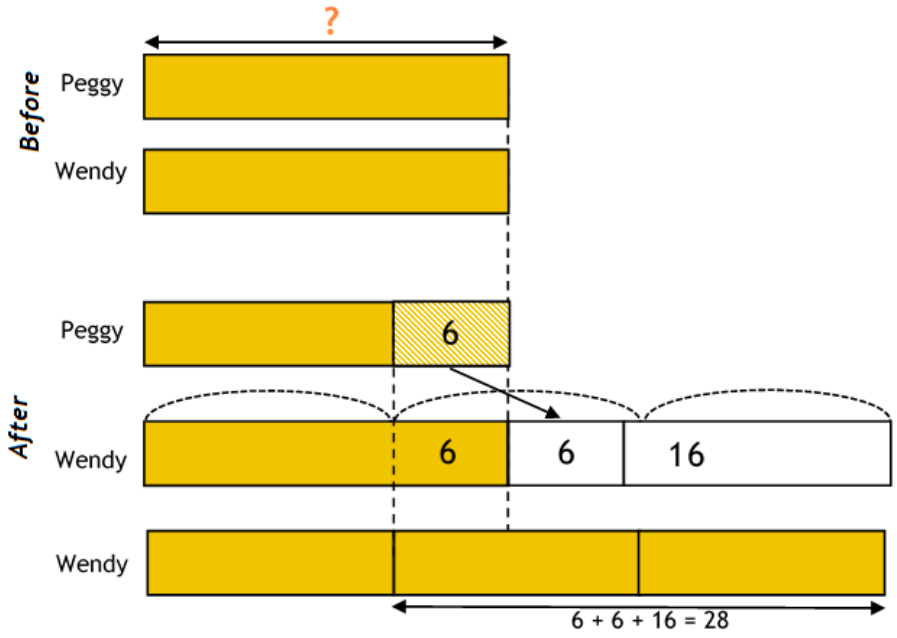
Solution

Look at the 'after' bar model. The difference is 8 + 15 + 8 = 31

After Zenda gave away 8 marbles to James, **James had 31 more marbles than Zenda.**

Task 4

Mrs Ng gave Peggy and Wendy an equal number of funfair coupons. Peggy then gave 6 of her coupons to Wendy. Wendy wanted even more so she asked Mrs Ng for 16 more coupons. In the end, Wendy had three times as many funfair coupons as Peggy. How many funfair coupons did Peggy have at first?



2 units = 6 + 6 + 16 = 28

1 unit = 28 ÷ 2 = 14

14 + 6 = 20

Peggy had 20 funfair coupons at first.