Bar Models in Problem Solving
Lesson 5: Comparison Model - Fractions

Australian Curriculum: Mathematics (Years 5 – 7)

ACMNA291: Use efficient mental and written strategies and apply appropriate digital technologies to solve problems (Year 5)

ACMNA126 Solve problems involving addition and subtraction of fractions with the same or related denominators (Year 6)

ACMNA127: Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (Year 6)

ACMNA155 Express one quantity as a fraction of another, with and without the use of digital technologies (Year 7)

Lesson abstract

In this lesson, students solve word problems that give information about fractions of different quantities that represent equal amounts. They use the comparison model for these problems, and solve them by identifying a common unit in the different quantities. Students see worked examples, then practise on several tasks.

Mathematical purpose (for students)

Drawing a comparison bar model requires finding two quantities that are equal.

Mathematical purpose (for teachers)

Through this lesson, students become familiar with using the comparison bar model to solve complex multi-step word problems with fractions. The word problems give information about equality of fractions of two different quantities. For example, a third of one quantity might be half of another. The comparison bar model facilitates an organisation and understanding of the problem, by representing the fractional parts on the bars and identifying the equal parts. Students devise a solution strategy by breaking the quantities into equal 'units'. This 'unitary method' offers a concrete solution method for fraction problems, and also prepares students to identify variables when later solving problems with algebra. Polya's four steps of problem solving are used to structure the solution process.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- comparison model

Lesson Materials

- Slide show *ST4_BarModelsPS_5a_CompFr.pptx*
- Student Sheet 1 – Bar Model Examples 5A (1 per student)
- Student Sheet 2 – Bar Model Examples 5B (1 per student)
- Calculators as needed

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Whole Class Examples

Hand out Student Sheet 1 – Bar Model Examples 5A.

Students should write the solutions to these examples for future reference.

The slide show (*ST4_BarModelsPS_5a_CompFr.pptx*) provides animated solutions that can be integrated with initial instruction and class discussion.

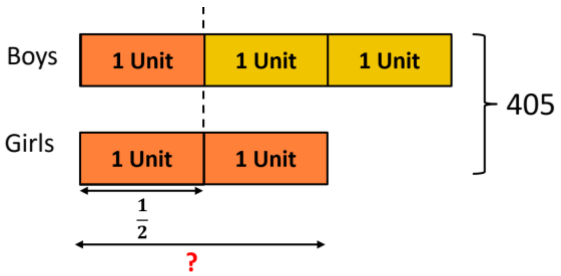
The discussion is organised around Polya's four stages. The first stage is to understand the information given in the problem in a general way. The suggested prompts especially focus on identifying the equal quantities, that are the key to drawing a useful bar model and then to solving the problems.

Bar models are constructed iteratively, and often need to be revised. There are often several ways to draw a useful bar model, and several different paths to a solution that they support.

Example 1

There are 405 children at the Theme Park. If $\frac{1}{3}$ of the boys is equal to $\frac{1}{2}$ of the girls, how many girls are there?

Expected Student Response



5 units = 405 children
1 unit = $405 \div 5 = 81$children
2 units = $2 \times 81 = 162$ children

There are 162 girls.

Discussion organised by Polya's four stages

Read the problem with the class and discuss how to draw and label the bar model. The animated slide show *ST4_BarModelsPS_5a_CompFr.pptx* can be used to support the discussion.

Understand
Encourage students to analyse the quantities and details in the word problem:

- How many children are there in the Theme Park? (ANS: 405 in total)
- What do I have to find? (ANS: How many girls there are)
- What are the important relationships in this problem? What two quantities are equal? (ANS: $\frac{1}{3}$ of the boys is equal to $\frac{1}{2}$ of the girls).

Plan

Use the prompts below to elicit responses from students while drawing the bar model.

- Ask students to select which type of bar model should be used, and to explain their selection. (ANS: The comparison model, because the information in the problem compares parts of two quantities.)
- Ask students what a good first step might be to solve this problem. (ANS: After analysing the problem, start by drawing a bar model and filling in the information from the problem statement).

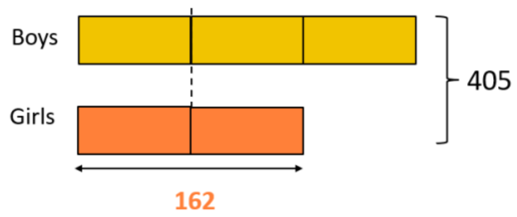
- How is the relationship in the problem represented in the bar model? (ANS: $\frac{1}{3}$ of the boys is equal to $\frac{1}{2}$ of the girls – so the size of that section of each of the bar for boys and the bar for girls should be the same length.)
- How might we show the total number of children on the model, given there will be two bars? (ANS: Show the total at one end, using a bracket to group the two bars together).
- Draw and label the bar model with the students.

Do

- Either work through the problem as a class or allow students some time to work on their solutions independently. Some prompts could include:
  - What might be an appropriate "unit" to define and use? (ANS: The amount which represents both $\frac{1}{3}$ of the boys and $\frac{1}{2}$ of the girls at the theme park. This quantity is common to both bars, and the 'whole' of each bar can be worked out based on multiples of this quantity).
  - What calculations can we do to find the solution? (ANS: Work out the value of 1 unit. From this, we can calculate how many girls there are).
- Points to highlight to students when drawing the model include:
  - It is important when drawing the bar models, to highlight the common amounts in each bar. Pictorially, a vertical line can be drawn to emphasise this, as noticing the common amount between the two bars is key to finding the solution for these problems.
  - The lengths of the bars represent the sizes of the quantities in the problem, but bars do not need to be drawn exactly in proportion.
  - Sometimes you need more than one bar to solve a problem.
  - There is no 'exactly right' way of drawing bars – the aim is to draw a model that helps you to solve the problem.

Check

Encourage students to check the answer against the context, by substituting the values into the bar model.

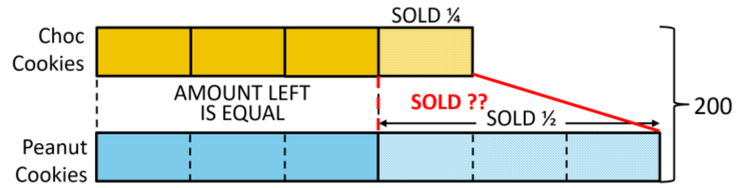


405 – 162 = 243
There are 243 boys.
243 ÷ 3 = 81
$\frac{1}{3}$ of the boys is 81.
162 ÷ 2 = 81
$\frac{1}{2}$ of the girls is 81 which is equal to $\frac{1}{3}$ of the boys.

Example 2

Judy baked 200 cookies, some of which were chocolate cookies and some of which were peanut cookies. After $\frac{1}{4}$ of the chocolate cookies and $\frac{1}{2}$ of the peanut cookies were sold, she had an equal number of chocolate cookies and peanut cookies left. How many cookies were sold altogether?

Expected Student Response



10 units = 200 cookies
1unit = 200 ÷ 10 = 20 cookies
4 units = 4 × 20 = 80 cookies
80 cookies were sold altogether.

Discussion organised by Polya's four stages

Teacher reads the problem with the class and discuss how to draw and label the bar model.

Understand

Encourage students to analyse the quantities and details in the word problem:

- How many cookies did Judy bake? (ANS: 200 cookies in total)
- What two quantities are equal? (ANS: the number of cookies of the two sorts that are left)
- What do I have to find? (ANS: The total number of cookies sold)

Plan

Use the prompts below to elicit responses from students while drawing the bar model.

- Ask students to select which type of bar model should be used, and to explain their selection. (ANS: The comparison model, because information is given that compares parts of the two batches of cookies.)
- How is the relationship in the problem represented in the bar model? (ANS: The amounts left after the sales are equal. When drawing the bars for each cookie type, make these sections equivalent in size. The total length for each bar is worked out from that base).
- How might we show the total number of cookies on the model? (ANS: Show the total at one end, using a bracket to group the two bars together).

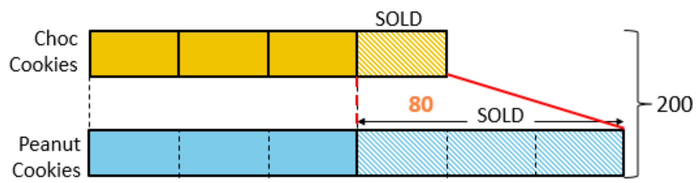
Do

Either work through the problem as a class or allow students some time to work on their solutions independently. Some prompts could include:

- What might be an appropriate 'unit' to define and use? (ANS: The amount which represents ¼ of the cookies. All other quantities can be calculated as multiples of this amount).
- What calculations can we do to find the solution? (ANS: Work out the value of 1 unit. From this, we can calculate how many cookies were sold).

Check

Encourage students to check the answer against all the information in the problem statement, by substituting the values into the bar model.



80 ÷ 4 = 20
20 × 4 = 80 (choc cookies)
$\frac{3}{4}$ of 80 = 60 (choc cookies left)
20 × 6 = 120 (peanut cookies)
$\frac{1}{2}$ of 120 = 60 (peanut cookies left)

Consolidating and Concluding

Further practice

Hand out Student Sheet - Bar Model Examples 5B. Students work individually, in pairs or in groups on selected problems.

Discuss solutions as time permits. Worked solutions are provided in Teacher Sheet - Bar Model Solutions 5B. Animated solutions to Task 1 and Task 2 are in the slide show *ST4_BarModelsPS_5a_CompFr.pptx*.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The comparison model can involve the use of more than two bars. Also, two parts can be equal in quantity.
- The unknown to be found in the word problem can refer to the part or the whole.
- To find the unknown using the comparison model, we can use some or all of the four operations (addition, subtraction, multiplication and division).
- When using the comparison model, it is helpful to indicate the difference in quantities between the bars.
- Creating a unit to represent a particular quantity across all bars in the problem can be helpful.
- It is helpful to highlight the common amounts across the bars. Pictorially, a vertical line can be drawn to emphasise this, as noticing the common amount between the two bars is key to finding the solution for these problems.

Bar Model Examples 5A

Name: ______________

Example 1

There are 405 children at the Theme Park.
If a third of the boys is equal to half of the girls, how many girls are there?

Example 2

Judy baked 200 chocolate cookies, some of which were chocolate cookies and some of which were peanut cookies. After $\frac{1}{4}$ of the chocolate cookies and $\frac{1}{2}$ of the peanut cookies were sold, she had the same number of chocolate cookies and peanut cookies left. How many cookies were sold altogether?

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Bar Model Examples 5B

Name: ______________

Draw bar models to represent the situations below and use them to help solve the problems.

Task 1

In a box, there are 210 green and red apples.
Half of the number of green apples is equal to a quarter of the red apples.
How many more red apples than green apples are there?

Task 2

David spent $\frac{5}{8}$ of his money and Peter spent $\frac{3}{4}$ of his money.
Both had an equal amount of money left.
If Peter had $92 more than David at first, how much money had each of them left?

Task 3

Alice and Brenda each baked muffins.
Alice baked 20 more muffins than Brenda. She sold $\frac{3}{7}$ of her muffins.
Brenda sold $\frac{1}{3}$ of her muffins.
In the end, they were both left with the same number of muffins.

a) How many muffins did Alice bake?
b) How many muffins did Brenda bake?

Task 4

Chris and David had the same hobby of collecting stamps. So far, Chris has collected Australian and Malaysian stamps and David has collected Australian and Singaporean stamps.
$\frac{2}{5}$ of Chris' stamps and $\frac{1}{4}$ of David's stamps were Australian stamps. They both collected the same number of Australian stamps.
Chris collected 45 less stamps than David.
How many stamps in all did each of them collect?

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Bar Model Solutions 5B

Task 1

In a box, there are 210 green and red apples. $\frac{1}{2}$ of the green apples is equal to $\frac{1}{4}$ of the red apples. How many more red apples than green apples are there?

Understand

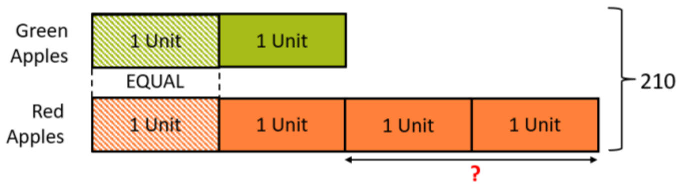
Pose questions like these questions to students, to prompt their in-depth reading of the problem statement:

- How many green and red apples are there in the box? (ANS: 210)
- Are there more green apples or red apples? (ANS: Red)
- What do I have to find? (ANS: How many more red apples than green apples)

Plan

Pose the following questions to students, to prompt their thinking if they require assistance:

- What type of bar model should I draw? (ANS: Comparison model)
- What do I do first? (ANS: Start by drawing two bars and labelling them)
- How is the relationship represented in the bar model? (ANS: By showing the section of the bar representing $\frac{1}{2}$ of the green apples being the same size as the section of the bar that is is equal to $\frac{1}{4}$ of the red apples)
- Draw two bar models and label the model.



Do

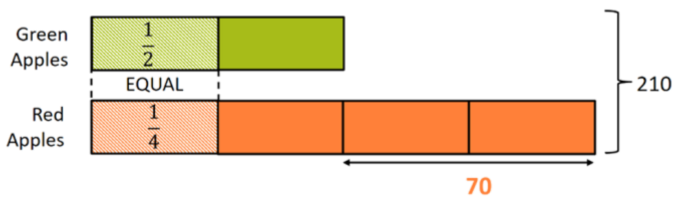
- From the model, find the total number of units.

  6 units = 210 apples
  1 unit = 210 ÷ 6 = 35 apples
  2 units = 2 × 35 = 70 apples

  There are 70 more red apples than green apples.

Check

- *Check the answer by substituting the values into the bar model.*



70 × 2 = 140
There are 140 red apples.
There are 70 green apples.
140 + 70 = 210.
Altogether, there are 210 red and green apples

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Task 2

David spent $\frac{5}{8}$ of his money and Peter spent $\frac{3}{4}$ of his money. Both had an equal amount of money left. If Peter had $92 more than David at first, how much money had each of them left?

Understand

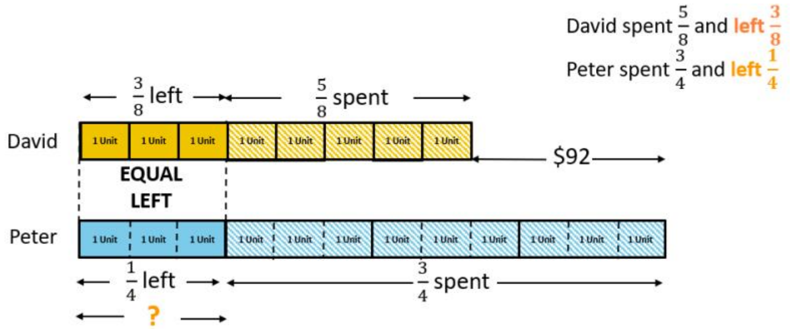
Pose the following questions to students, to prompt their reading of the problem:

- Who has more money at first? (ANS: Peter)
- How much more? (ANS: $92)
- What do I have to find? (ANS: The amount of money David and Peter each had left)

Plan

Pose the following questions to students, to prompt their thinking if they require assistance:

- What type of bar model should I draw? (ANS: Comparison model)
- What do I do first? (ANS: Start by drawing a bar for each person, and labelling them)
- How is the relationship represented in the bar model? (ANS: By showing the parts of the two bars that are equal.)

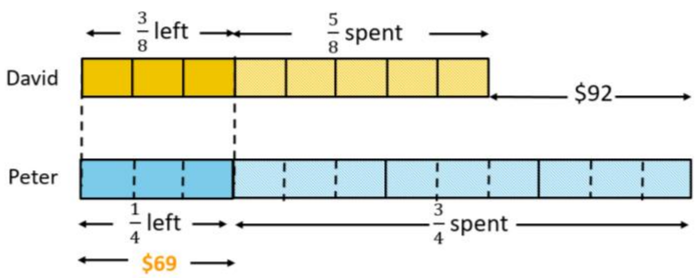


Do

4 units = $92
1 unit = $92 ÷ 4 = $23
3 units = $23 × 3 = $69
Each of them had $69 left.

Check

- Check the answer by substituting the values to see if all the requirements of the original problem have been met.



$\frac{1}{4}$ of Peter's money = $69
Total amount of Peter's money = 4 × $69 = $276

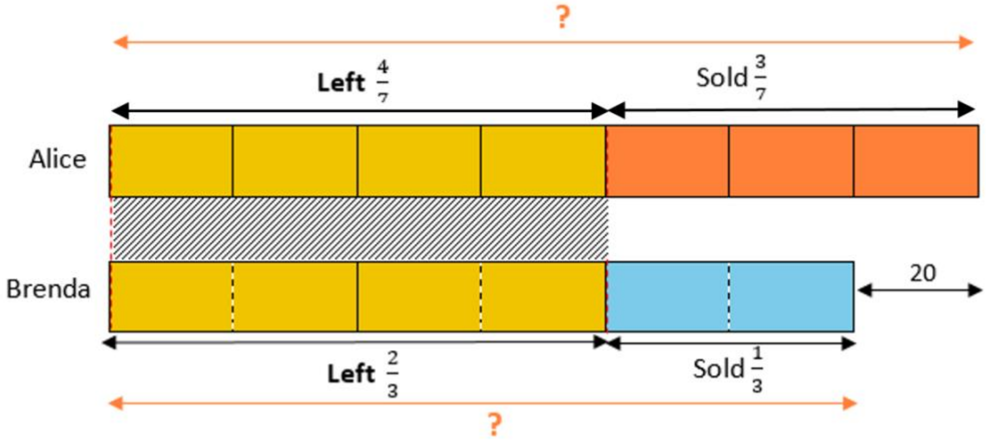
$\frac{3}{8}$ of David's money = $69
$\frac{1}{8}$ of David's money = $69 ÷ 3 = $23
Total amount of David's money = 8 x $23 =$184

276 – 184 = 92
Peter had $92 more than David at first.

Task 3

Alice and Brenda each baked muffins. Alice baked 20 more muffins than Brenda. She sold $\frac{3}{7}$ of her muffins. Brenda sold $\frac{1}{3}$ of her muffins. In the end, they were both left with the same number of muffins.

a. How many muffins did Alice bake?
b. How many muffins did Brenda bake?

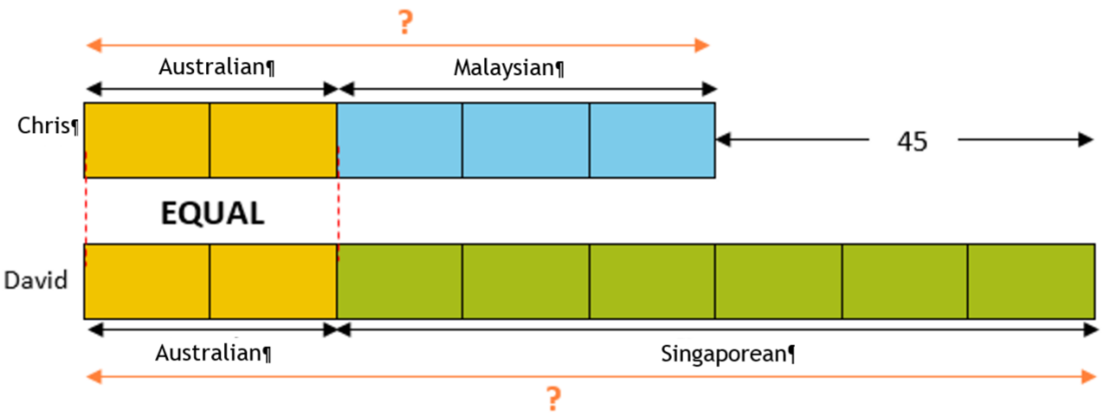


a. 4 sevenths of Alice's muffins is equal to 4 sixths of Brenda's muffins.
This means that 1 seventh of Alice's muffins is one sixth of Brenda's.
So, 7 sevenths of Alice's muffins is equal to 7 sixths of Brenda's.
I unit = 20 muffins
7 units = 7 × 20 = 140 muffins
Alice baked 140 muffins.

b. 1 unit = 20 muffins
6 units = 6 × 20 = 120 muffins
Brenda baked 120 muffins.

Task 4

Chris and David had the same hobby of collecting stamps. So far, Chris managed to collect Australian and Malaysian stamps. David collected Australian and Singaporean stamps. $\frac{2}{5}$ of Chris' stamps and $\frac{1}{4}$ of David's stamps were Australian stamps. They both collected the same number of Australian stamps. Chris collected 45 less stamps than David. How many stamps in all did each of them collect?



3 units = 45
1 unit = 45 ÷ 3 = 15
5 units = 5 × 15 = 75
Chris collected 75 stamps.

1 unit = 15
8 units = 8 × 15 = 120
David collected 120 stamps.