Bar Models in Problem Solving
Lesson 8: Change Model - Fractions

Australian Curriculum: Mathematics (Year 6)

ACMNA123: Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers.

ACMNA126: Solve problems involving addition and subtraction of fractions with the same or related denominators

ACMNA127: Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (Year 6)

Lesson abstract

In this lesson, students use the change model with word problems involving fractions. By working with the visual model, fraction calculations are replaced by intuitive steps. Students study problems where the quantities in a situation change either by a whole number amount, or by a fraction of the whole. Students participate in building models with classmates, then practise with selected tasks.

Mathematical purpose (for students)

Bar models help us find solutions to complex problems.

Mathematical purpose (for teachers)

In this lesson, students learn to solve multi-step problems with both fractions and whole numbers which involve a change in quantities of items between two different (before-after) situations. The word problem situations are visually represented with a change model, which is usually made of two comparison models, showing the 'before' and the 'after' situations. Through this lesson, students learn to analyse multiple aspects of relationships in problems, both additive and multiplicative. Students then identify common 'units' between the groups of bars in the change bar model and use them to solve the problems. This 'unit' technique is clearly demonstrated in the examples and animated solutions provided, and introduces students to some early algebra concepts of generalisation. Additionally, Polya's four steps of problem solving are used to structure the solution process.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- Change model

Lesson Materials

- Slide show *ST4_BarModelsPS_8a_ChangeFr.pptx*
- Student Sheet – Bar Model Examples 8A (1 per student)
- Student Sheet – Bar Model Examples 8B (1 per student, optional)
- Calculators as needed

We value your feedback after these lessons via https://www.surveymonkey.com/r/G6VGPZ8

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Background

Hand out Student Sheet – Bar Model Examples 8A. Students should write the solutions to these examples, for future reference.

The slide show (*ST4_BarModelsPS_8a_ChangeFr.pptx*) provides animated solutions to these examples which can be used during initial instruction and class discussion.

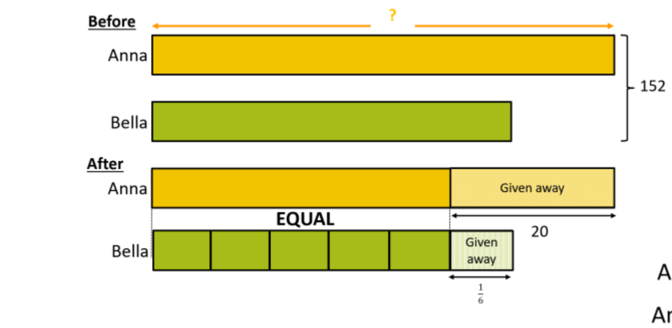
In the following examples, quantities of items change over time by either a whole number amount or a fractional amount and are analysed using the change bar model as a visual aid. There is a lot of information in these problems, so it is good to have it well organised by the bar model. Remember that bar models often need to be redrawn as the problem is better understood.

Whole Class Examples

Example 1

Anna and Bella had 152 sweets altogether. After Anna gave away 20 of her sweets and Bella gave away $\frac{1}{6}$ of her sweets, they had the same number of sweets left. How many sweets did Anna have at first?

Expected Student Response

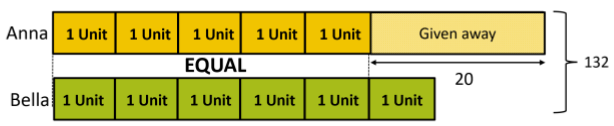


Anna gave away 20 sweets. Bella gave away $\frac{1}{6}$ of her sweets and had $\frac{5}{6}$ of them left.

Anna's leftover sweets = $\frac{5}{6}$ of Bella's leftover sweets.

When Anna gave away 20 sweets, there were 152 – 20 = 132 leftover sweets.

Let us call $\frac{1}{6}$ of Bella's sweets "1 unit" & label all of the equal units in the model.



11 units = 152 – 20 = 132 sweets
1 unit = 132 ÷ 11 = 12 sweets
5 units = 5 x 12 = 60 sweets
Anna had 60 sweets after giving away 20 sweets.

60 + 20 = 80
Anna had 80 sweets at first.

Discussion organised by Polya's four stages

Read the problem with the class and discuss how to draw and label the bar model as students work through it. The animated slide show *ST4_BarModelsPS_8a_ChangeFr.pptx* can be used to support the discussion.

Understand
Encourage students to analyse the before – after situation given in the word problem:

- Tell the story in your own words.
- How many sweets did the girls have at first? (ANS: 152 sweets).
- How many sweets in total were left, after Anna gave away some sweets? (ANS: 152-20 = 132 sweets).
- Do we know how many sweets Bella gave away, or will we have to work it out? (ANS: We don't know this without working it out).
- What do we have to find? (ANS: How many sweets Anna had at first).

Plan
Emphasise how to draw the change model and represent the fractions. Discuss the importance of comparing the relationships in the before and after situations whilst the model is being constructed. Key points to discuss include:

- What kind of model should we draw? (ANS: A change model is suitable – before and after the girls give sweets It will be made of two comparison models.).
- Draw and label the change model with students.

Do
Work through the problem as a group, for this initial task. Some discussion points could include:

- Observe the change from one situation (before) to another situation (after). How might this help to find the solution?
- What might be a common 'unit' between all the bars? (ANS: $\frac{1}{6}$ of Bella's sweets)
- Work through the calculations with students.

Check
Encourage students to check the answer by substituting it into the original problem.

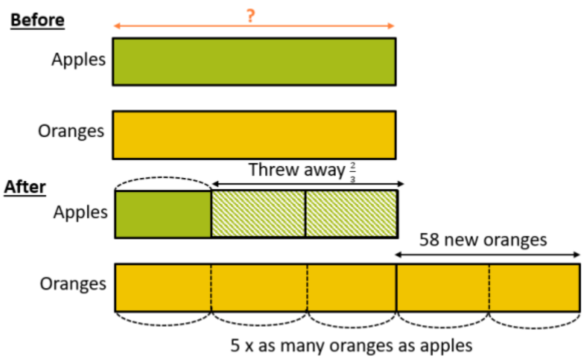
We found Anna had 80 sweets at first
80 – 20 = 60
Anna had 60 sweets left which is the same as $\frac{5}{6}$ of the sweets Bella had left.

$\frac{1}{6}$ of Bella's sweets = 60 ÷ 5 = 12 sweets
All $\frac{6}{6}$ of Bella's sweets = 6 x 12 = 72 sweets
80 + 72 = 152 (total)

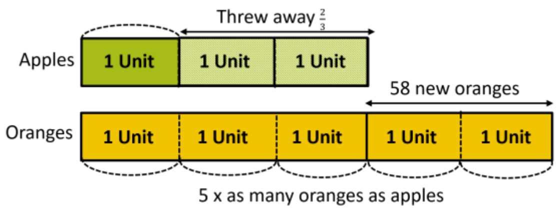
Example 2

James had an equal number of apples and oranges at first. He threw away $\frac{2}{3}$ of apples as they were rotten, then bought another 58 oranges. In the end, he had five times as many oranges as apples. How many apples did he have at first?

Expected Student Response



Let us call the number of apples left at the end "1 unit" & label all of the equal units in the model.



We can see from the bar model:
2 units = 58 pieces of fruit
1 unit = 58 ÷ 2 = 29 pieces of fruit
3 units = 3 x 29 = 87 pieces of fruit
James had 87 apples at first.

Discussion organised by Polya's four stages

read the problem with the class and discus how to draw and label the bar model as you work through the stages.

Understand
Encourage students to analyse the before and after situation given in the problem:

- What is the relationship between the number of apples and oranges at first? (ANS: They are the same).

- What happened when James threw away the rotten apples and bought more oranges? (ANS: there were less apples than before and more oranges than before – the numbers are no longer equal).
- What do we have to find? (ANS: The number of apples James had at first).

Plan
Discuss the situations before and after the throwing away and buying, and how they are related whilst constructing the bar model.

- What kind of model should we draw? (A change model, as the quantities of fruit change over time).
- Show information given before and after the amount of fruit changes (ANS: This requires construction of two comparison models – one for the 'before' situation, and one for 'after'. The before-situation shows that the number of apples and oranges being equal. This problem also requires the use of the change model, because the 'after' situation shows that James threw away $\frac{2}{3}$ of the rotten apples and bought another 58 pears).
- Draw and label a change model with students.

Do

- Observe the change from one situation (before) to another situation (after) to find the solution

Check
Encourage students to check the answer by seeing that all requirements of the original problem are met.

$\frac{1}{3}$ x 87 = 29
29 apples were left after the rotten apples were thrown away.

29 x 5 = 145
There were 145 oranges after some were added.
145 – 87 = 58
James bought 58 extra oranges.

Consolidating and Concluding

Further practice

Hand out Student Sheet 2 - Bar Model Examples 8B. Students work individually, in pairs or in groups on selected problems.

Discuss solutions as time permits. Worked solutions are provided in Teacher Sheet - Bar Model Solutions 8B, and solutions to Task 1 and Task 2 are also included in the slide show *ST4_ProblemSolving_8a_FractChange.pptx*.

Conclusion

Summarise the learning points for the lesson and for the sequence of lessons and invite students to add their own observations:

- A change model is usually made up of two comparison models; one comparison model to represent a 'before' situation, and one comparison model to represent changed values in an 'after' situation.
- Using a bar model is a good way to organise the data in a problem. You can write important information in a place where it will be useful.
- Using a bar model is a good way to visualise the mathematical relationships between quantities in a problem. You can highlight differences and you can also show ratios (e.g. how many times as big).
- By drawing bar models, you can solve complex word problems which involve a lot of information.

Bar Model Examples 8A

Name: ______________

Example 1

Anna and Bella had 152 sweets altogether. After Anna gave away 20 of her sweets and Bella gave away $\frac{1}{6}$ of her sweets, they had the same number of sweets left. How many sweets did Anna have at first?

Example 2

James had an equal number of apples and oranges at first. He threw away $\frac{2}{3}$ of apples as they were rotten, and bought another 58 oranges. In the end, he had five times as many oranges as apples. How many apples did he have at first?

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Bar Model Examples 8B

Name: ______________

Draw bar models to represent the situations below and use them to help you solve the problems.

Task 1

John had 360 stickers and Mark had some stickers.
Then John gave away a third of his stickers.
Then Mark gave away 63 of his stickers.
Then they had the same number of stickers left.
How many stickers did Mark have at first?

Task 2

Amelia and Eva had 199 muffins altogether.
Amelia sold 3 fifths of her muffins and Eva sold 37 muffins.
Amelia then had twice as many muffins as Eva.
How many muffins did Amelia have left?

Task 3

Daniel and Edward earned some money.
Daniel spent 2 fifths of his money to buy some books.
Edward spent the same amount of money on a pair of shoes.
He had 2 thirds of his money left.
Daniel then spent a third of his remaining money to go out.
Then Edward had $50 more than Daniel.
How much money did the boys have at first?

Task 4

A fruit seller packed apples and pears into two baskets.
Both baskets had the same number of pears.
Basket A had 5/7 apples and 2/7 pears.
Basket A had equal numbers of apples and pears.

Then the fruit seller took some apples from Basket B and put them into Basket A.
The number of apples in Basket A was then twice what it had been at the start.
Now Basket A has 20 more apples than Basket B.

How many apples and pears did the fruit seller put into each basket at the start?

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Bar Model Solutions 8B

Task 1

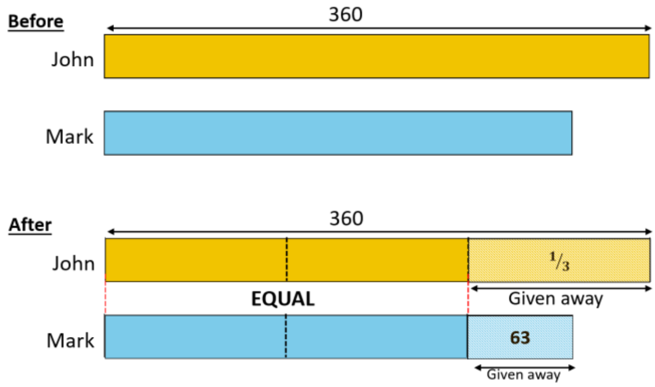
John had 360 stickers and Mark had some stickers. When John gave away $\frac{1}{3}$ of his stickers and Mark gave away 63 of his stickers, they had the same number of stickers left. How many stickers did Mark have at first?

Understand

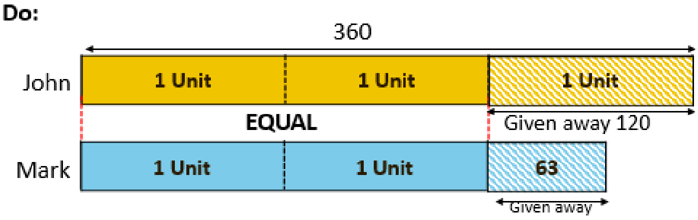
- Do John and Mark have the same number of stickers at first?
- What happened after the boys gave away some stickers?
- What do we have to find?

Plan

- We need to show the information given before and after the number of stickers changed.
- What kind of model should we draw?
- Draw and label a change model.



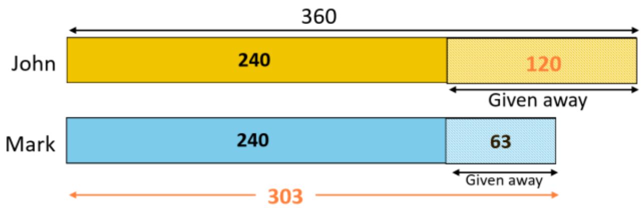
Do:



- Label all the equal lengths on the model as '1 unit'.
- John gave away $\frac{1}{3}$ of his stickers. This is 120 stickers. It is also 1 unit.

Mark started with 2 units plus 63 stickers.
2 units = 2 x 120 = 240 stickers
240 + 63 = 303 stickers
Mark had 303 stickers at first.

Check



303 – 63 = 240
Mark had 240 stickers left.

120 x 2 = 240
John had 240 stickers left.

They had the **same number of stickers left.**

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Task 2

Amelia and Eva had 199 muffins altogether. Amelia sold $\frac{3}{5}$ of her muffins and Eva sold 37 muffins. Amelia then had twice as many muffins as Eva. How many muffins did Amelia have left?

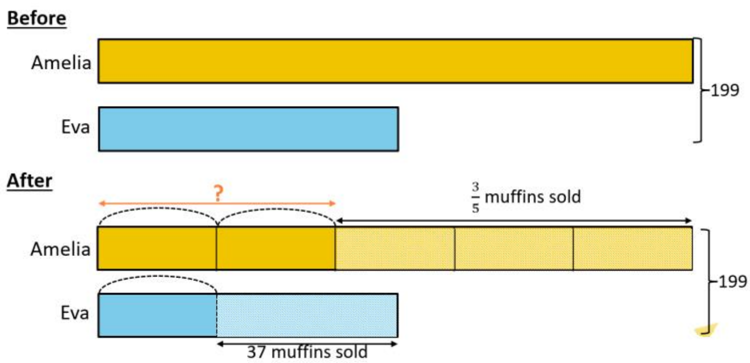
(A complexity in this problem is that the information that Eva sold 37 muffins cannot be used until later information (Amelia has twice as many) has been illustrated on the bar model.

Understand

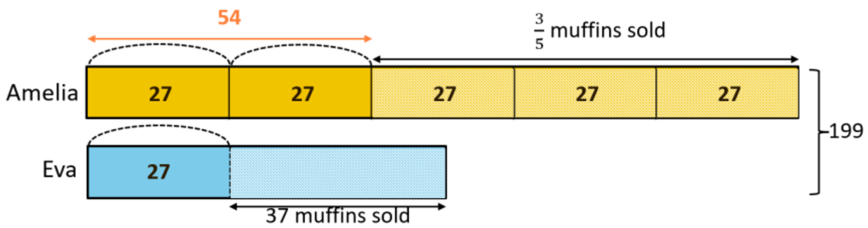
- How many muffins did Amelia and Eva altogether?
- What happened after Amelia sold $\frac{3}{5}$ of her muffins and Eva sold 37muffins?
- What do I have to find?

Plan

- We need to show information given before and after 37 muffins were sold.
- What kind of model should we draw?
- Draw and label a change model.



Do



- Let's call the number of muffins that Eva had left "1 Unit" and label all of the equal units on the model.

6 units = 199 – 37 = 162 muffins
1 unit = 27 muffins

2 units = 2 x 27 = 54 muffins
Amelia had 54 muffins left after she sold $\frac{3}{5}$ of her muffins

Check

54 ÷ 2 = 27
27 x 5 = 135
Amelia had 135 muffins at first.
27 + 37 =64
Eva had 64 muffins at first
135 + 64 = 199
They had a total of 199 muffins at first.

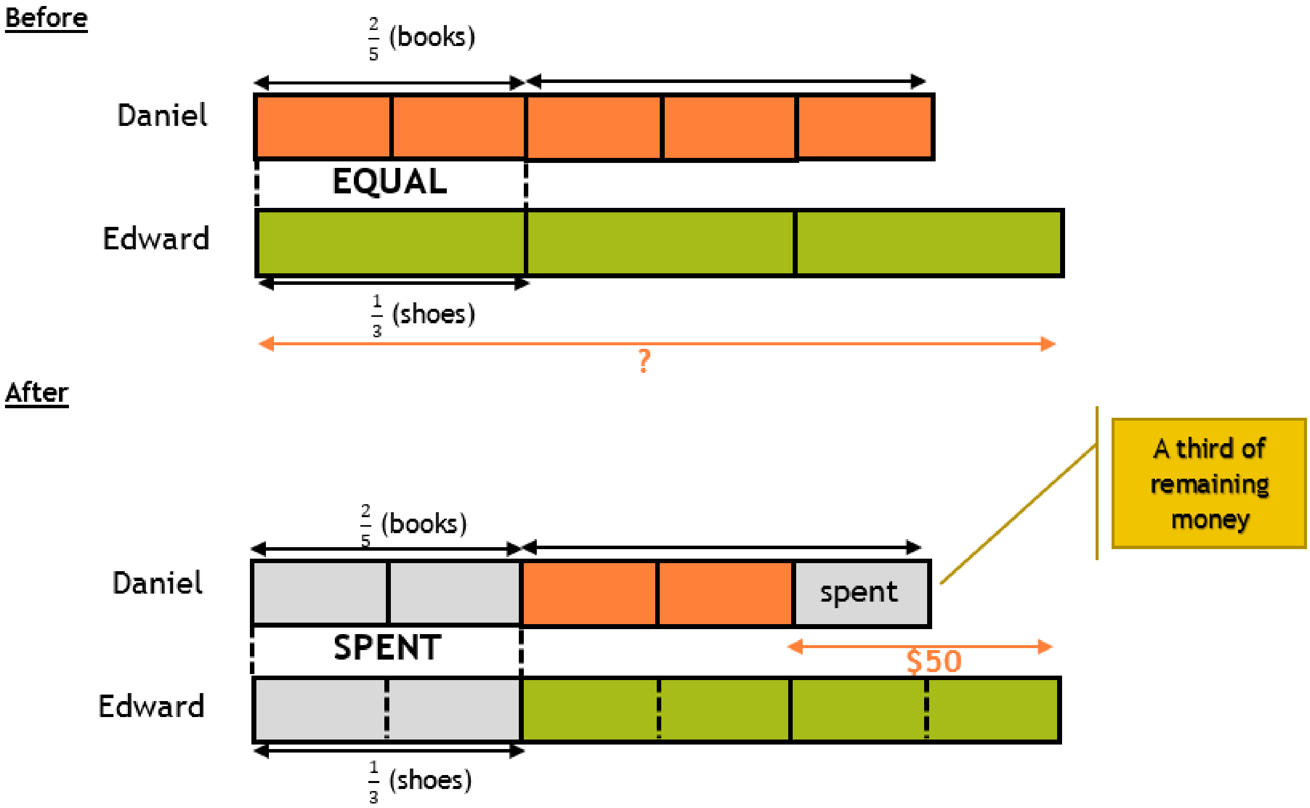
Task 3

Daniel and Edward earned some money. Daniel spent $\frac{2}{5}$ of his money to buy some books.

Edward spent the same amount of money on a pair of shoes. He had $\frac{2}{3}$ of his money left.

Daniel then spent a third of his remaining money to go out. Then Edward had $50 more than Daniel.

How much money did the boys have at first?



Draw the bar models for before and after Daniel spent the third of his remaining money.

This shows that $50 is 2 units (in other words, 2 fifths of Daniel's original money, and one third of Edwards/ original money.

Daniel earned $125 and spent $50 on books and $25 going out and then had $50 left.

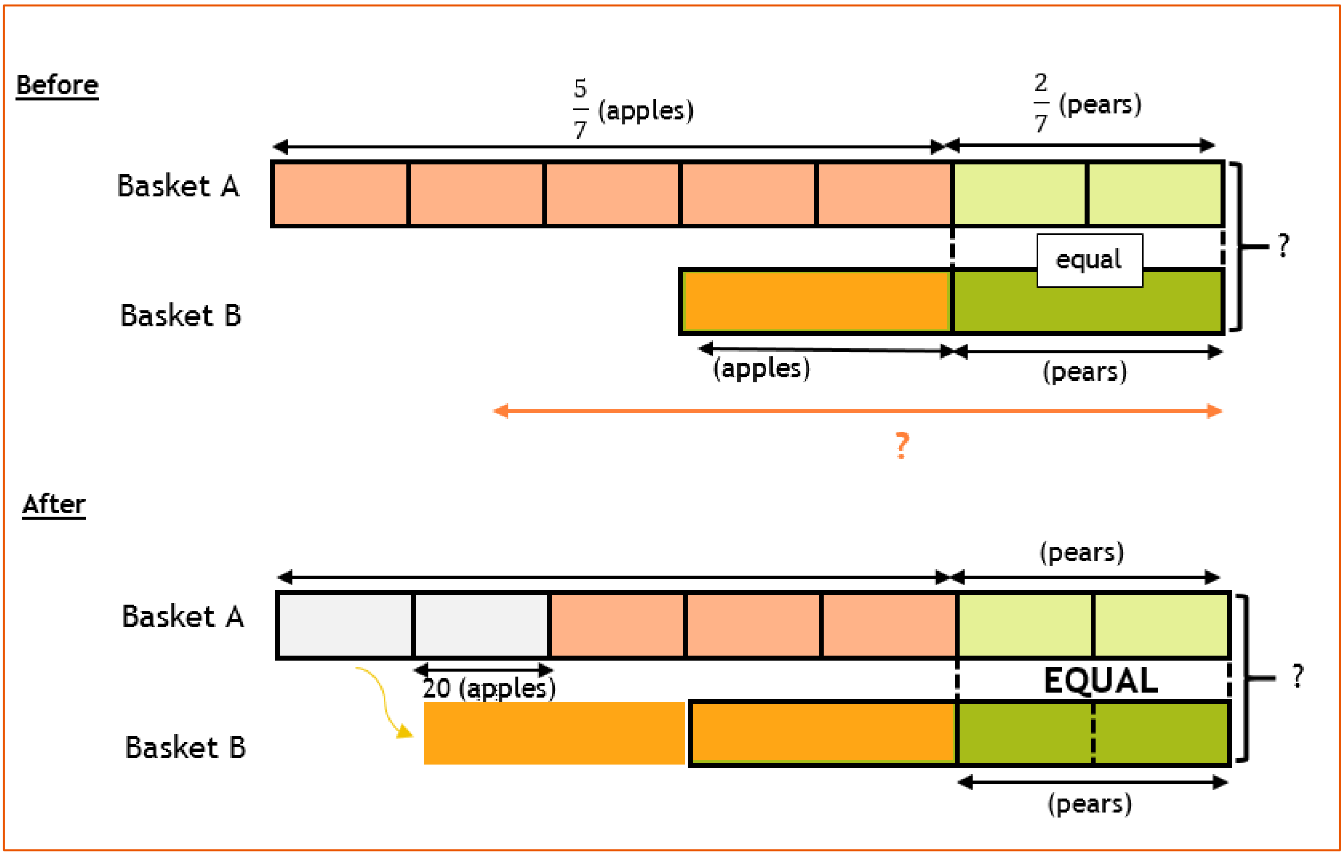
Edward spent $50 on shoes, and at the end had $100 left.   This is $50 more than Daniel had left.

Task 4

A fruit seller packed apples and pears into two baskets.
Both baskets had the same number of pears.
Basket A had 5/7 apples and 2/7 pears.
Basket A had equal numbers of apples and pears.

Then the fruit seller took some apples from Basket B and put them into Basket A.
The number of apples in Basket A was then twice what it had been at the start.
Now Basket A has 20 more apples than Basket B.

How many apples and pears did the fruit seller put into each basket at the start?



After all the changes are included to produce the 'after' bar model, the bar model shows that 20 apples is equal to one seventh of the amount of fruit initially in Basket A.

At the start, Basket A had 100 apples and 40 pears. Basket B had 40 apples and 40 pears.

The fruit seller moved 40 apples to Basket B leaving 100 – 40 = 60 apples in Basket A and making 40 + 40 = 80 apples in Basket B. This confirms that the difference was 80 – 60 = 20.

The total amount of fruit was 140 apples and 80 pears.