Bar Models in Problem Solving
Lesson 4: Comparison Model - Whole Numbers

Australian Curriculum: Mathematics (Year 6)

ACMNA123: Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (Year 6)

Lesson abstract

In this lesson, students use the comparison bar model to help solve multi-step word problems that include information about additive and multiplicative relationships between quantities. They study worked examples and practise with further tasks.

Mathematical purpose (for students)

Bar models show comparisons between quantities in a problem.

Mathematical purpose (for teachers)

In this lesson students encounter multi-step word problems where the information given involves multiplicative and additive relationships. They learn to represent these different types of relationships with multiple bars, and to create an overall comparison bar model to help solve the problem. All four operations can be involved in the solution, but calculation is not the focus here. Examining the bar model assists students to find a strategy to solve the problem. The concept of a general term (unit) to represent a quantity is also used throughout the solutions in this lesson, introducing students to early algebraic concepts in an accessible and understandable way. The lesson solutions are structured around Polya's four stages of problem solving.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- Comparison model

Lesson Materials

- Slide show Slide show *ST4_BarModelsPS_4a_CompWN.pptx*
- Student Sheet 1 – Bar Model Examples 4A (1 per student)
- Student Sheet 2 – Bar Model Examples 4B (1 per student)
- Calculators as needed

Whole Class Examples

Hand out Student Sheet 1 – Bar Model Examples 4A.

Students should write the solutions to these examples, for future reference. The slide show (*ST4_BarModelsPS_4a_CompWN.pptx*) provides animated solutions to these examples which can be used during initial instruction and class discussion.

In the following examples, multi-step whole number word problems are solved by comparing the multiple quantities in the problem (more than two) and using the comparison bar model to create a visual representation of the scenario as a means to understanding the relationships in the problem. Teachers should highlight these relationships and how they translate in the construction of the comparison bar model, also noting were to mark the difference between quantities in the model. For more information on the comparison bar model, please refer to the Teacher's Guide *ST4_BarModelMethod_TeachersGuide.pdf*.

The discussion is organised around Polya's four stages. Feel free to move back and forth between the stages. The construction of the bar model can begin at the Understand stage too, recording information as it is gathered. Bar models are constructed iteratively, and often need to be revised.

Example 1

Alex, Ben and Calvin have 440 marbles altogether. Ben has 60 more marbles than Alex. Calvin has 3 times as many marbles as Ben.

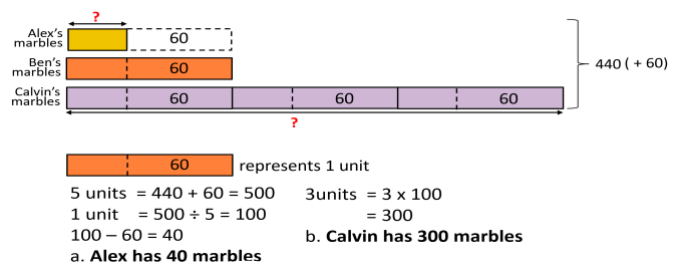
a. How many marbles does Alex have?
b. How many marbles does Calvin have?

Expected Student Response

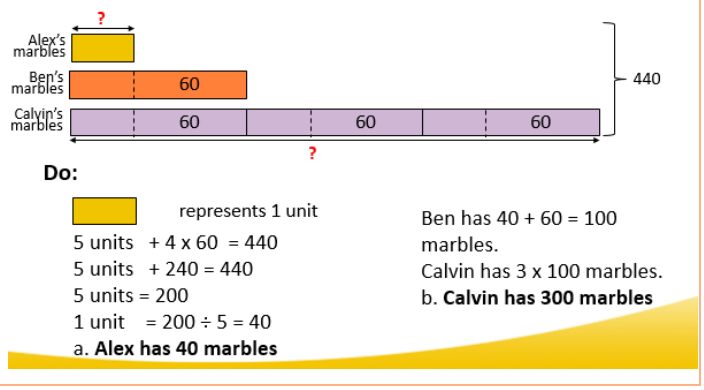
Two solutions with different 'units' are provided on the slideshow for this problem. It is important that students know that there are different correct ways to solve problems.

The first solution on the slideshow (below) uses Ben's number of marbles as the unit. The second solution uses Alex's number of marbles as the unit. The second solution maybe a little easier.

SOLUTION 1



SOLUTION 2



Discussion organised by Polya's four stages

Use the following enabling prompts for students, as students work as a class or in pairs to solve this problem.

Understand

- Encourage students to analyse the information in the word problem:
  - How many marbles do Alex, Ben and Calvin have altogether (ANS: 440)?
  - How many more marbles does Ben have than Alex? (ANS: 60).
  - Do we already know how many marbles any of the boys has? (ANS: No)
- What are the important relationships in this problem which will help us to solve it? (ANS: The number of marbles Ben and Alex has is related additively – Ben has 60 more than Alex. The number of marbles of Calvin and Ben are related multiplicatively – Calvin has 3 times as many marbles as Ben).

- What do we have to find? (ANS: How many marbles each boy has).

Plan

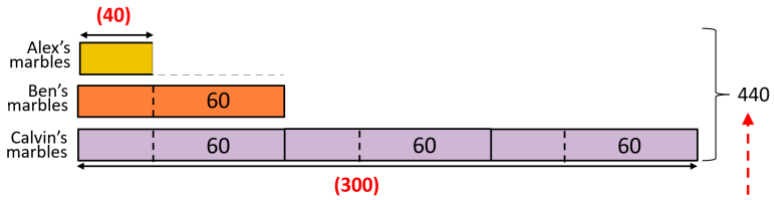
- Ask students to select which type of bar model should be used, and to explain their selection. (ANS: the comparison model because the information compares the number of marbles Alex, Ben and Calvin have).
- Draw and label the bar model with the students. During this process, discuss how the quantities are related and how they can be represented in the bar model.
    - How do we show the difference between Alex's marbles and Ben's marbles in the bar model? (ANS: either draw a horizontal arrow from the end of Alex's bar to the end of Ben's bar, and mark the length as 60 marbles, or add 60 marbles to Alex's bar and add 60 marbles to the total of all marbles).
    - How do we represent Calvin's marbles? (ANS: Make Calvin's bar 3 times as long as Ben's.)

Do

- Either work through the problem as a class or allow students some time to work on their solutions independently. Some prompts could include:
    - What might be an appropriate "unit" to define and use? (ANS: Solution 1 uses the number of marbles Ben has, because both Alex and Calvin's quantities can be related to it easily; Solution 2 uses the number of Alex's marbles. Both are good choices.)
    - What calculations can we do to find the solution? (ANS: Work out the value of 1 unit. From this, we can calculate both Alex and Calvin's number of marbles)
- Points to highlight to students when drawing the model include:
    - It is important to highlight the common amounts in each bar. A vertical line can be drawn to emphasise these equalities.
    - The lengths of the bars represent the sizes of the quantities in the problem, but bars do not need to be drawn exactly in proportion.
    - Sometimes you need more than one bar to solve a problem.
    - There is no 'exactly right' way of drawing bars – the aim is to draw a model that helps you to solve the problem.

Check

Check if the answer is correct



If Alex has **40** marbles,
Ben will have (40 + 60 = 100) marbles.
Calvin has **300** marbles.
40 + 100 + 300 = 440

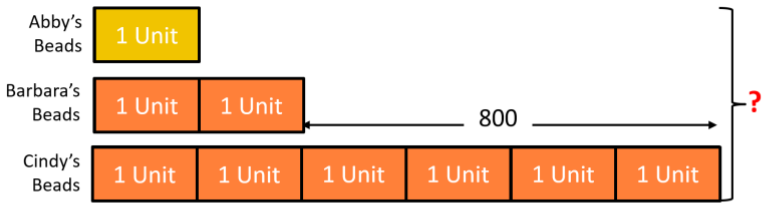
They will have a total of **440** marbles.

Example 2

Abby, Barbara and Cindy collected some beads. Barbara had twice as many beads as Abby and Cindy had three times as many beads as Barbara. Cindy had 800 beads more than Barbara.

a. Who had the least number of beads?
b. Who had the most number of beads?
c. How many beads did the three girls have altogether?

Expected Student Response



From the model:

a) Abby has the least number of beads.

b) Cindy has the most number of beads.

4 units = 800

1 unit = 800 ÷ 4 = 200

9 units = 9 x 200 = 1800

c) They had 1800 beads altogether.

Discussion organised by Polya's four stage

Pose the following enabling prompts for students as the class works together, or as students' independent solutions are reviewed.

Understand

- Encourage students to analyse the information in the problem:
  - What did the three girls collect? (ANS: Beads).
  - How many more beads did Cindy have than Barbara? (ANS: Cindy had 3 times as many beads as Barbara, which equates to 800 beads).
  - What do I have to find? (ANS: Who had the least number of beads, who had the most, and the total number of beads).
- What are the key relationships in the problem, which will help us to draw a bar model and solve the problem? (ANS: The relationship between the number of beads Barbara and Abby have, and the relationship between the number of beads that Barbara and Cindy have).

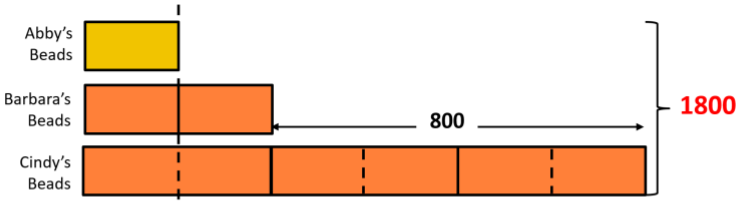
Plan

- Ask students to select which type of bar model should be used, and to explain their selection. (ANS: A comparison model, because the information given compares the number of beads the girls have).
- Discuss how the relationships are represented in the bar model:
  - How do we draw a bar to represent the number of Barbara's beads? (ANS: The bar for Barbara's beads will be twice that length).
  - How do we draw a bar to represent the number of Cindy's beads? (ANS: The bar for Cindy's beads will be three times the length of the bar representing Barbara's beads).
  - How do we show the difference in number of beads between Cindy and Barbara? (ANS: Draw a horizontal arrow running between the end of Barbara's bar to the end of Cindy's bar.)

Do

- What is an appropriate "unit" to define and why? (ANS: The number of beads which Abbey has is a good choice. Other choices are possible.)

Check



1800 ÷ 9 = 200 ; Abby had 200 beads

2 x 200 = **400** ; Barbara had **twice** as many beads as Abby (**200**).

200 x 6 = **1200**; Cindy had **three times** as many beads as Barbara (**400**).

1200 – 400 = **800**; Cindy had 800 beads more than Barbara.

Consolidating and Concluding

Further practice

Hand out Student Sheet 2 - Bar Model Examples 4B. Students work individually, in pairs or in groups on selected problems.

Discuss solutions as time permits. Worked solutions to all tasks are provided in Teacher Sheet - Bar Model Solutions 4B, and solutions to Task 1 and Task 2 are included in the slide show.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The comparison model can involve the use of more than two bars.
- Comparisons can involve addition/subtraction, or multiplication/division. It is very important to observe what type of comparison it is.
- When using the comparison model, it is helpful to indicate the difference (additive or multiplicative) in quantities between the bars.
- Creating a unit to represent a quantity across all bars in the problem can be helpful.
- Constructing the model sometimes needs to be an iterative process. For example, the initial lengths of the bars sometimes might need to be adjusted to usefully show the relationships in the problem.
- Using Polya's four steps of problem solving (Understand, Plan, Do, Check) can help as a structured approach to problem solving. It is worthwhile making sure you understand the problem statement well.

Bar Model Examples 4A

Name: _____

Example 1

Alex, Ben and Calvin have 440 marbles altogether. Ben has 60 more marbles than Alex. Calvin has 3 times as many marbles as Ben.

a. How many marbles does Alex have?
b. How many marbles does Calvin have?

Example 2

Abby, Barbara and Cindy collected some beads. Barbara had twice as many beads as Abby and Cindy had three times as many beads as Barbara. Cindy had 800 beads more than Barbara.

a. Who had the least number of beads?
b. Who had the most number of beads?
c. How many beads did the three girls have altogether?

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Bar Model Examples 4B

Name: _____

Draw bar models to represent the situations below and use them to solve the problems.

Task 1

Anthony, Oliver and Peter collected 1407 used bottles for recycling. Anthony collected half as many bottles as Oliver. Peter collected twice as many bottles as Oliver. How many bottles did Peter collect?

Task 2

A restaurant had some fruit. There were 4 times as many apples as oranges. There were 18 fewer pears than oranges. There were 141 more apples than oranges.

a. How many pears were there?
b. How much fruit was there altogether?

Task 3

Felix, Gordon and Helmer were good friends. They collected some seashells. Felix collected 3 times as many seashells as Gordon. Helmer collected 26 seashells which was 4 less than the number Felix collected.

a. How many more seashells did Helmer collect than Gordon?
b. What was the total number of seashells collected by all three friends?

Task 4

Agnes, Betty and Connie together sewed 325 mittens. Agnes sewed 3 times as many mittens as Betty and Connie sewed 3 times as many mittens as Agnes. How many more mittens must Betty sew to match Agnes' number of mittens?

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Bar Model Solutions 4B

Task 1

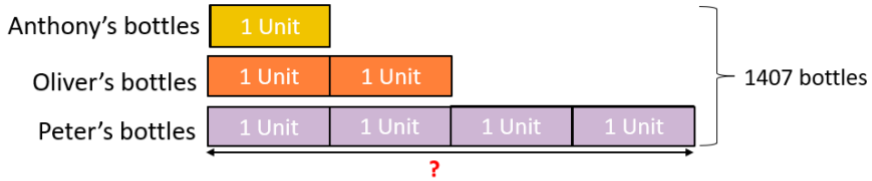
Anthony, Oliver and Peter collected 1407 used bottles for recycling. Anthony collected half as many bottles as Oliver. Peter collected twice as many bottles as Oliver. How many bottles did Peter collect?

Understand

- *How many bottles did Anthony, Oliver and Peter collect for recycling?*
- *Who collected the least number of bottles?*
- *Who collected the most number of bottles?*
- *What do we have to find?*

Plan

- *What type of model is best to use?*
- *How are the relationships represented in the bar model?*
- *Draw and label a model.*



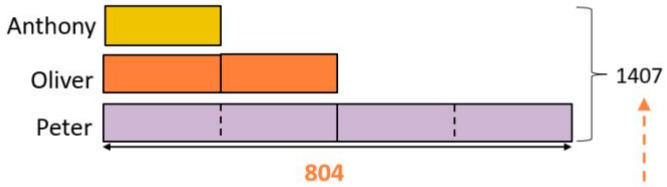
Do:

- *Show the workings*

7 units = 1407 bottles
1 unit = 1407 ÷ 7 = 201 bottles
4 units = 4 x 201 = 804 bottles
Peter collected 804 bottles.

Check

- *Check the answer by substituting it into the problem.*



804 ÷ 2 = 402
Oliver collected 402 bottles. Peter collected twice as many as Oliver.
402 ÷ 2 = 201
Anthony collected 201 bottles. Anthony collected half as many as Oliver.
804 + 402 + 201 =1407
They collected 1407 bottles altogether.

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Task 2

A restaurant had some fruit. There was 4 times as many apples as oranges. There were 18 fewer pears than oranges. There were 141 more apples than oranges.

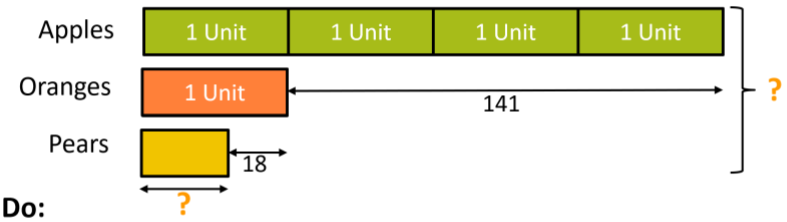
a. How many pears were there?
b. How much fruit was there altogether?

Understand

- *What types of fruit did the restaurant have?*
- *How many more apples than oranges were there?*
- *What do we have to find?*

Plan:

- *What type of bar model would be best to use?*
- *How are the relationships represented in the bar model?*
- *Draw and label a bar model.*



Do:

- *We can define units in the model, to help us solve the problem.*

3 units = 141
1 unit = 141 ÷ 3 = 47

47 – 18 = 29

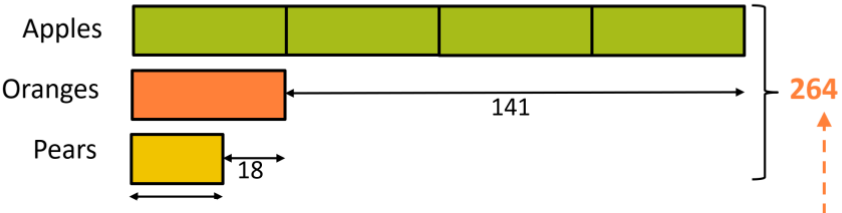
a. There were 29 pears.

5 units = 5 x 47 = 235
235 + 29 = 264

b. There were 264 pieces of fruit altogether.

Check

- *Check the answer by substituting it into the problem.*

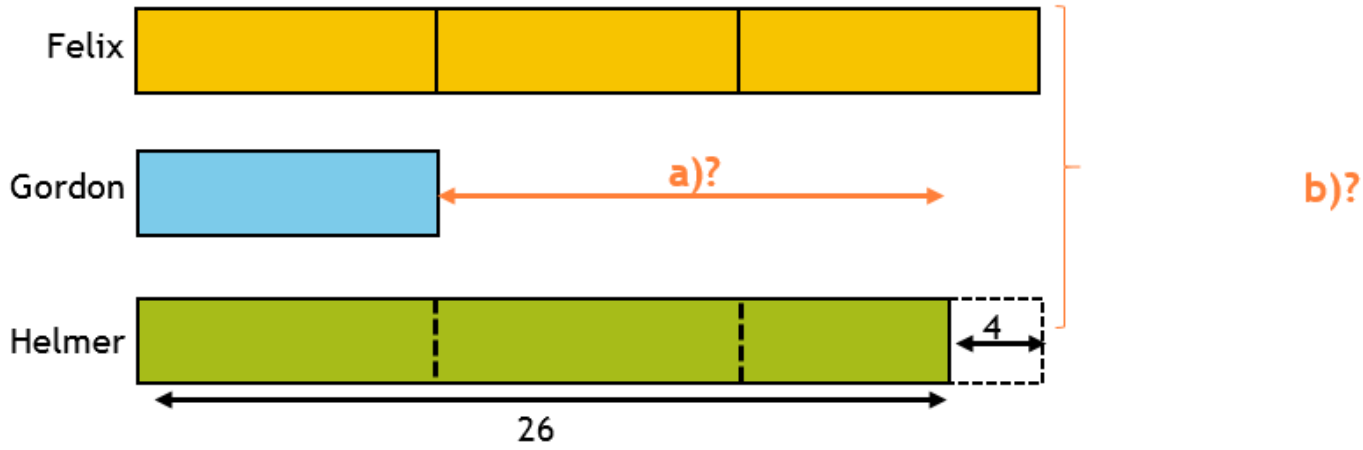


29 + 18 = 47
There were 47 oranges.
47 x 4 = 188
There were 188 apples.
188 – 47 = 141
There were 141 more apples than oranges.
29 + 47 + 188 = 264
The fruit adds to 264 pieces.

Task 3

Felix, Gordon and Helmer were good friends. They collected some seashells. Felix collected 3 times as many seashells as Gordon. Helmer collected 26 seashells, which was 4 less than what Felix collected.

a. How many more seashells did Helmer collect than Gordon?
b. What was the total number of seashells collected by the three friends.



26 + 4 = 30

Felix collected 30 seashells.

3 units = 30

1 unit = 30 ÷ 3 = 10

Gordon collected 10 seashells

26 - 10 = 16

a. Helmer collected 16 more seashells than Gordon

1 unit = 10

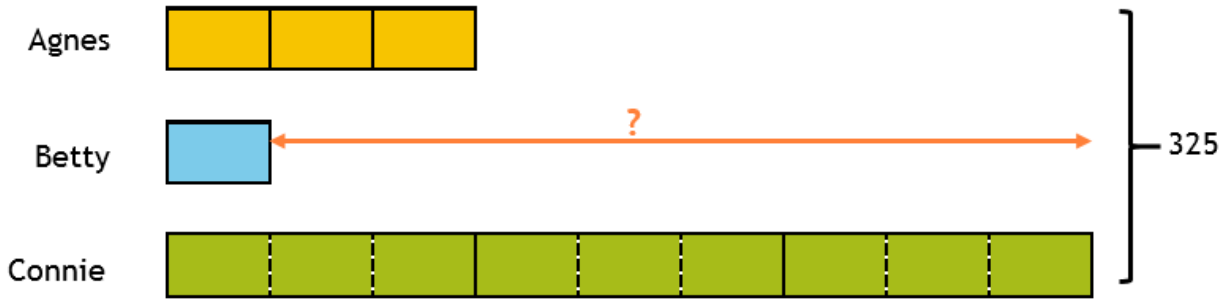
7 units = 7 x 10 = 70

70 - 4 = 66

b. They collected 66 seashells altogether.

Task 4

Agnes, Betty and Connie together sewed 325 mittens. Agnes sewed 3 times as many mittens as Betty, and Connie sewed 3 times as many mittens as Agnes. How many more mittens must Betty sew to match Agnes' number of mittens?



13 units = 325 mittens

1 unit = 325 ÷ 13 = 25 mittens

8 units = 8 x 25 = 200 mittens

Betty must sew 200 more mittens to match the number of Agnes' mittens.