

Bar Models in Problem Solving

Lesson 6: Stack Model - Whole Numbers

Australian Curriculum: Mathematics (Year 6)

ACMNA123: Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (Year 6)

Lesson abstract

This lesson introduces the stack model. Students study problems where they compare two groups of quantities, with either a multiplicative or additive relationship given between individual items. Students work on two examples together, then practise independently or in pairs to consolidate learning.

Mathematical purpose (for students)

The stack model is a variation of the comparison model to use when items in groups are being compared.

Mathematical purpose (for teachers)

In this lesson, students learn to use the stack model (a variant of the comparison model), for multi-step word problems involving whole numbers. The problems outline relationships between groups of items, provide totals for the entire group and require the value of individual items to be found. The problem information is visualised using a stack model, so students can easily identify common 'units' between the two groups. Working with the 'unit' is the key to solving the problems. Through this lesson, students develop an intuitive understanding of the way in which relationships within problems can be used in problem solving. This will stand students in good stead in their future studies of algebra.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- Stack model

Lesson Materials

- Slide show *ST4_BarModelsPS_6a_Stack.pptx*
- [Student Sheet 1 - Bar Model Examples 6A](#) (1 per student)
- [Student Sheet 2 - Bar Model Examples 6B](#) (1 per student)
- Calculators as needed

We value your feedback after these lessons via <https://www.surveymonkey.com/r/G6VGPZ8>



Whole Class Examples

Hand out [Student Sheet 1 - Bar Model Examples 6A](#). Students should write the solutions to these examples, for future reference. The slide show (*ST4_BarModelsPS_6a_Stack.pptx*) provides animated solutions to these examples which can be used during initial instruction and class discussion.

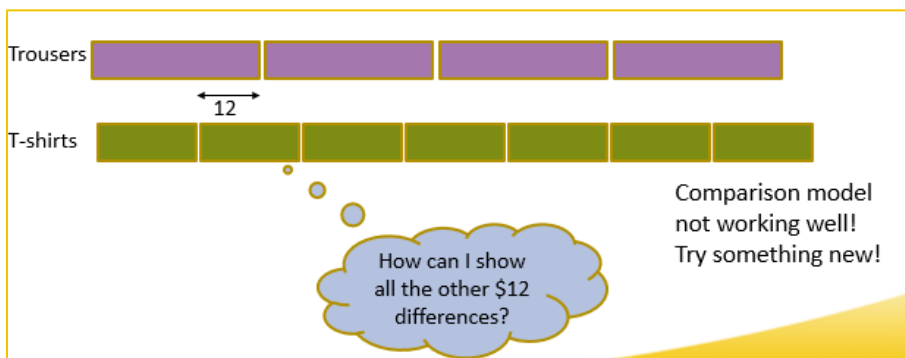
The lesson introduces the stack model. The first example provides an additive comparison (\$12 more) and the second provides a multiplicative comparison (3 times more). Teachers may need to provide support to students in understanding both how to construct the stack bar models, and how to make best use of the strategy of identifying and working with the common 'units'.

The discussion is organised around Polya's four stages. Feel free to move back and forth between the stages. The construction of the bar model can begin at the Understand stage too, recording information as it is gathered. Bar models are constructed iteratively, and often need to be revised.

Example 1

The cost of 4 pairs of trousers and 7 T-shirts is \$147. A pair of trousers costs \$12 more than a T-shirt. How much does a pair of trousers cost?

Allow students some time to draw a useful model. Since the cost of a pair of trousers is compared with the cost of a T-shirt, a comparison model seems a good choice. However, because there are multiple items, drawing a clear comparison model is tricky. Instead, stacking the equal bars allows the comparison (in this case \$12 difference) to be clearly seen. This makes a 'stack model', as illustrated.



Expected Student Response (using stack model)

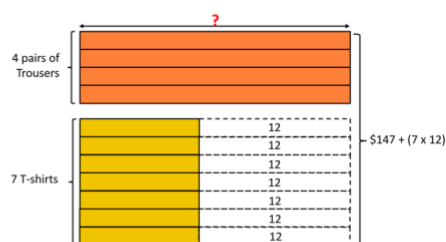
Let 1 unit be the cost of 1 pair of trousers.
These bars are each worth 1 unit:



Difference in price = \$12 per T-shirt
Total difference = $7 \times 12 = \$84$

$(4 + 7)$ units = 11 units
 $11 \text{ units} = \$147 + \$84 = \$231$
 $1 \text{ unit} = \$231 \div 11 = \21

A pair of trousers costs \$21



Discussion organised by Polya's four stages

Read the problem with the class and allow some time for students to draw their own models. Some students may do this successfully, but the stack model to be introduced is easier.

The animated slide show *ST4_BarModelsPS_6a_Stack.pptx* can be used to support the discussion.

Understand

Encourage students to analyse the quantities and information in the problem:

- What is the cost of 4 pairs of trousers and 7 T-shirts? (ANS: \$147)
- Is a pair of trousers more expensive or a T-shirt? (ANS: Trousers are \$12 more expensive than a T-shirt).
- What two quantities are equal? (Cost of pair of trousers is equal to cost of T-shirt + \$12.)
- What do I have to find? (ANS: The cost of 1 pair of trousers)

Plan

This is the first stack bar model that students have encountered. Demonstrate how to draw the stack model and emphasise the benefits of constructing it in the stacked arrangement. The animated slideshow will be useful.

- Ask students how the \$12 difference between the bars can be shown (ANS: Add \$12 to each of the T-shirts to create a common unit with the trousers, then add $7 \times \$12$ to the total to balance this. If students suggest that each pair of trousers can be shown as the cost of a T-shirt plus \$12, this is a viable alternative.)
- The stack bar model enables students to compare the value of items in several groups, when the items in each group are of equal value, but different between groups.

Do

Work through the problem as a group, for this initial task. Some discussion points could include:

- What might be a common unit between the two groups in the model? (ANS: the cost of a pair of trousers, or alternatively the cost of a T-shirt.)
- How many units are in the whole model? (ANS: There are 11 units)
- Work through the calculations with students.

Check

Encourage students to check the answer by substituting the values into the original problem to see that all conditions are met.

Cost of 1 pair of trousers = \$21

Cost of 4 pairs of trousers = $4 \times \$21 = \84

Cost of 1 T-shirt = $\$21 - \$12 = \$9$

Cost of 7 T-shirts = $7 \times \$9 = \63

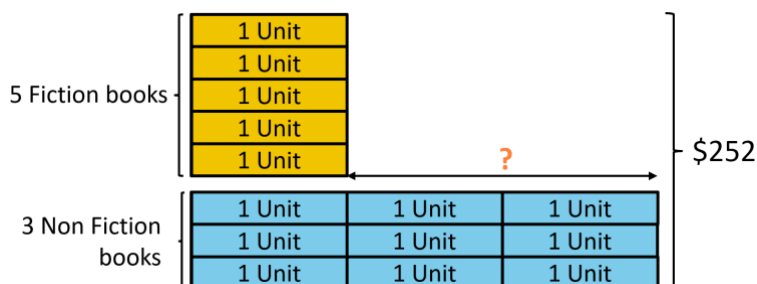
$84 + 63 = 147$

Total cost of 4 pairs of trousers and 7 T-shirts is \$147

Example 2

Helen paid \$252 for 5 fiction books and 3 non-fiction books. Each non-fiction book cost 3 times as much as a fiction book. What was the difference in price between a non-fiction book and a fiction book?

Expected Student Response



$$\begin{aligned}
 14 \text{ units} &= \$252 \\
 1 \text{ unit} &= \$252 \div 14 \\
 &= \$18 \\
 2 \text{ units} &= 2 \times \$18 \\
 &= \$36
 \end{aligned}$$

The difference in price between a fiction book and a non-fiction book was \$36.

Discussion organised by Polya's four stages

Read the problem with the class and discuss how to draw and label the bar model.

Understand

Encourage students to analyse the information in the problem:

- What is the cost of the books? (ANS: \$252 in total)
- Is a fiction book more expensive than a non-fiction book? (ANS: Non-fiction books are more expensive.)
- What do I have to find? (ANS: The difference in price between a fiction book and a non-fiction book.)

Plan

Use prompts to discuss the problem whilst drawing the bar model:

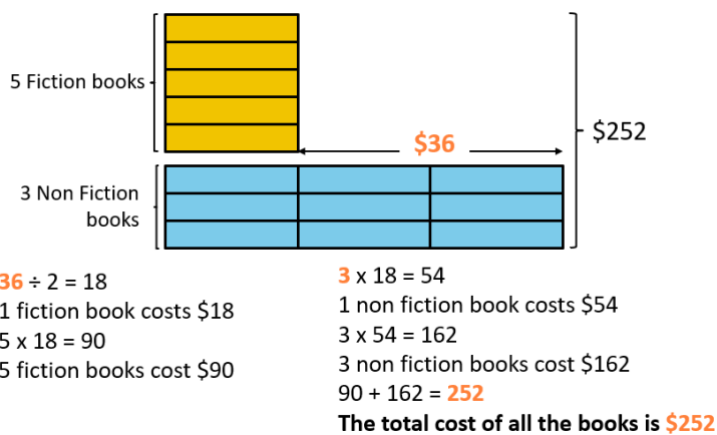
- What kind of model should I draw? (ANS: Either a comparison model or a stack model will work well to visualise this problem. Explain why the Stack Model is appropriate.)
- How do we draw the model? (ANS: Draw 5 stacked bars for fiction, then 3 stacked bars for non-fiction books.)
- How is the relationship between the cost of a fiction book and the cost of a non-fiction book shown? (ANS: The length of the bar for non-fiction is 3 times the length of fiction books.)
- How can we show the total cost of all books? (ANS: At one end, bracket all the bars and label the total cost.)

Do

Either work through the problem as a class or allow students some time to work on their solutions independently. Some prompts could include:

- What might be an appropriate 'unit' to define and use? (ANS: The cost of 1 fiction book.)
- How can we find the solution? (ANS: Work out the value of 1 unit. From this, everything follows.)

Check



Consolidating and Concluding

Further practice

Hand out [Student Sheet 2 - Bar Model Examples 6B](#). Students work on selected problems.

Discuss solutions as time permits. Worked solutions are provided in [Teacher Sheet - Bar Model Solutions 6B](#), and solutions to Task 1 and Task 2 are also included in the slide show *ST4_BarModelsPS_6a_Stack.pptx*.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The stack model involves the use of many bars, stacked to show a comparison.
- Creating a unit to represent a particular quantity across all bars in the problem can be helpful.

Example 1

The cost of 4 pairs of trousers and 7 T-shirts is \$147. A pair of trouser costs \$12 more than a T-shirt. How much does a pair of trouser cost?

Example 2

Helen paid \$252 for 5 fiction books and 3 non-fiction books. Each non-fiction book cost 3 times as much as a fiction book. What was the difference in price between a non-fiction book and a fiction book?

Draw bar models to represent the situations below and use them to help you solve the problems.

Draw bar models to represent the situations below and use them to help you solve the problems.

Task 1

Mary paid \$287 for 2 handbags and 5 blouses. The average cost of a handbag was \$42 more than the average cost of a blouse. How much did she pay for the 5 blouses?

Task 2

Bill paid \$1148 for 2 printers and 3 hard disks. The price of a printer was 2 times as much as the price of a hard disk. How much did Bill pay for one printer?

Task 3

Jean packed apples into two types of box - A Boxes and B Boxes. There were 15 more apples in each Box A than in each Box B. When she completed her packing, there were 6 identical A Boxes and 8 identical B Boxes. In total, Jean packed 860 apples. How many apples were there in all the B boxes?

Task 4

Abby, Ben and Cathy kept their pictures in albums. Abby had 3 albums, Ben had 2 and Cathy had only one album. Each of Abby's albums contained twice as many picture cards as each of Ben's. Cathy's album contained 6 more picture cards than Ben kept in each album. Altogether, they had 195 picture cards. How many picture cards are there in each of Abby's albums?

Task 1

Mary paid \$287 for 2 handbags and 5 blouses. On average, a handbag cost \$42 more than a blouse. How much did she pay for the 5 blouses?

Understand

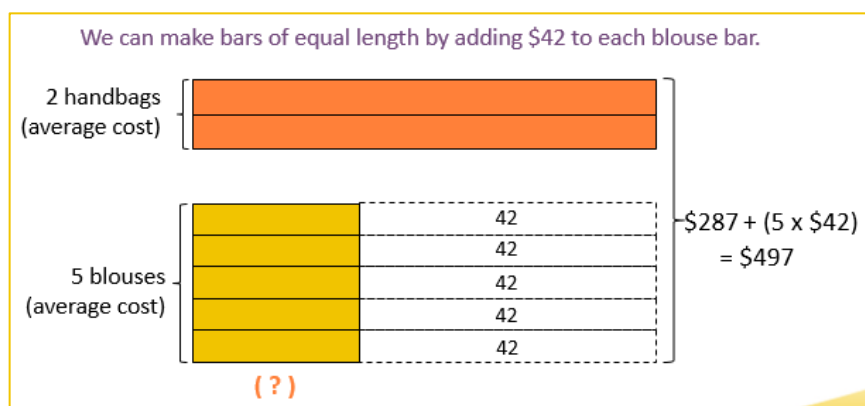
Discuss the problem with students, using the following prompts as needed to guide their thinking:

- What is the cost of the handbags and the blouses? (ANS: \$287)
- Is a handbag more expensive than a blouse? (ANS: Handbag is more expensive.)
- What two quantities are equal? (Ans: average cost of handbag is equal to average cost of blouse + \$42)
- What do I have to find? (ANS: The cost of 5 blouses)

Plan

- Draw and label a bar model.
- How can we show 2 handbags and 5 blouses?

Do



Let 1 unit be the average cost of a handbag.

$$(2 + 5) \text{ units} = \$287 + (5 \times \$42)$$

$$7 \text{ units} = \$497$$

$$1 \text{ unit} = \$497 \div 7 = \$71$$

$$71 - 42 = 29$$

$$1 \text{ blouse costs } \$29$$

$$29 \times 5 = 145$$

$$\mathbf{5 \text{ blouses cost } \$145}$$

Check

Check the answer by substituting the values into the original problem.

$$145 \div 5 = 29$$

$$1 \text{ blouse cost } \$29$$

$$29 + 42 = 71$$

$$1 \text{ handbag cost } \$71$$

$$2 \times 71 = 142$$

$$2 \text{ handbags cost } \$142$$

$$5 \text{ blouses cost } \$145$$

$$145 + 142 = 287$$

$$\text{Their total cost is } \$287.$$

Task 2

Bill paid \$1148 for 2 printers and 3 hard disks. The price of a printer was 2 times as much as the price of a hard disk. How much did Bill pay for one printer?

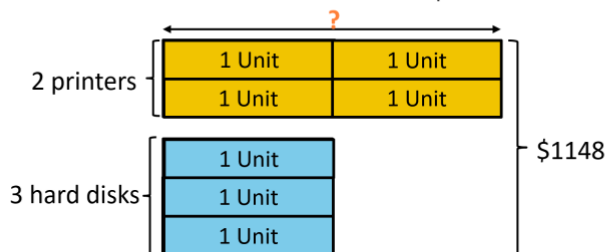
Understand:

Discuss the problem with students, using the following prompts as needed to guide their thinking:

- What is the cost of 2 printers and 3 hard disks? (ANS: \$1148)
- Is a printer more expensive or a hard disk? (ANS: A printer)
- What two quantities are equal? (ANS: the price of a printer is equal to the price of two hard disks.)
- What do I have to find? (ANS: The cost of a printer)

Plan:

- Draw and label a bar model.
- How do we draw the model to show 2 printers and 3 hard disks?



Do:

- From the model, we see that there are 7 rectangular bars to show 2 printers and 3 hard disks, with a total cost of \$1148.
- Let the cost of 1 hard disk be 1 unit.

$$7 \text{ units} = 1148$$

$$1 \text{ unit} = 1148 \div 7 = 164$$

$$2 \text{ units} = 2 \times 164 = 328$$

Bill paid \$328 for one printer.

Check:

$$328 \div 2 = 164$$

$$3 \times 164 = 492$$

$$3 \text{ hard disks cost } \$492$$

$$2 \times 328 = 656$$

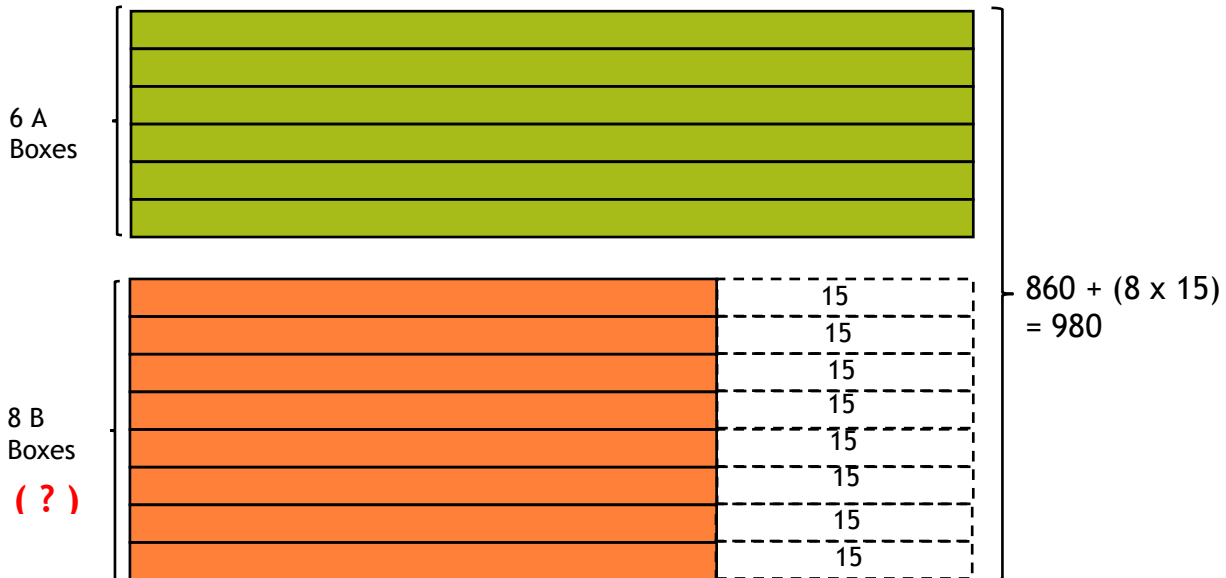
$$2 \text{ printers cost } \$656$$

$$492 + 656 = 1148$$

The total cost for 2 printers and 3 hard disks is \$1148.

Task 3

Jean packed apples into two types of box - A Boxes and B Boxes. There were 15 more apples in each Box A than in each Box B. When she completed her packing, there were 6 identical A Boxes and 8 identical B Boxes. In total, Jean packed 860 apples. How many apples were there in all the B boxes?



$$15 \times 8 = 120$$

$$860 + 120 = 980$$

$$14 \text{ units of Box A type} = 980$$

$$1 \text{ unit of Box A} = 980 \div 14 = 70$$

$$70 - 15 = 55$$

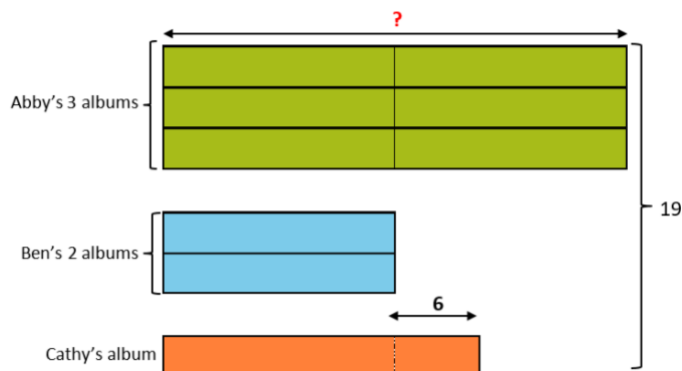
There were 55 apples in one B Box.

$$8 \times 55 = 440$$

Altogether, there were 440 apples in all the 8 Boxes of B.

Task 4

Abby, Ben and Cathy kept their pictures in albums. Abby had 3 albums, Ben had 2 and Cathy had only one album. Each of Abby's albums contained twice as many picture cards as each of Ben's. Cathy's album contained 6 more picture cards than Ben kept in each album. Altogether, they had 195 picture cards. How many picture cards are there in each of Abby's albums?



Let 1 unit be equal to 1 of Ben's albums.

$$195 - 6 = 189$$

$$9 \text{ units} = 189$$

$$1 \text{ unit} = 189 \div 9 = 21$$

$$2 \text{ units} = 2 \times 21 = 42$$

There are 42 picture cards in each of Abby's album.