

Unit Overview: Bar Models in Problem Solving

Summary of learning goals

As part of the reSolve Special Topic “The Bar Model Method”, this unit demonstrates how bar models are used to solve word problems about whole numbers and fractions. The bar model method is primarily a problem-solving tool: a way of organising information in a word problem and of visualising the relationships between known and unknown quantities. As students construct bar models, they come to understand the problems deeply, which prepares them to devise solution strategies. As such, the intention of all these lessons is to give students a flexible tool that they can carry with them to help solve problems in all areas of the mathematics curriculum. These lessons present students with multi-step word problems with a variety of mathematical structures, to strengthen students’ skills in tackling problems arising in any real situation.

Four types of bar models are encountered in the unit. The part-whole and comparison (introduced in the *reSolve Bar Model Method Unit 1 “Introduction to Bar Models”*) are used in more complex problems and students see two variants - the stack and change models. The word problems include both additive and multiplicative relationships between quantities and sometimes require combinations of bar models. Whilst some concepts may be unfamiliar to students (for example, fractions of fractions), they can be approached in a highly accessible way with bar models.

To equip students with a further problem solving tool, use of Polya’s four phases of problem solving is embedded in the approach. These lessons can also be a pre-cursor to algebra, when the designated ‘unit’ of the bar model method is replaced by a variable.

Australian Curriculum: Mathematics (Years 6 - 7)

ACMNA123: Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (Year 6)

ACMNA126: Solve problems involving addition and subtraction of fractions with the same or related denominators (Year 6)

ACMNA127: Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (Year 6)

ACMNA154: Multiply and divide fractions and decimals using efficient written strategies and digital technologies (Year 7)

ACMNA155: Express one quantity as a fraction of another, with and without the use of digital technologies (Year 7)

Summary of lessons

Who is this unit for?

This unit is designed for students around Year 6, who understand the situations where whole number operations are used, and have a basic understanding of fractions. The fraction calculations involved are not demanding, because answers are generally obtained by intuitive means supported by diagrams. A few students will require a calculator to assist with the whole number calculations. This is so that their attention can remain on the method of problem

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solving rather than being waylaid by calculation. It is expected that students will have some familiarity with the bar model method, ideally by working through some of the lessons in *reSolve Bar Model Method Unit 1 “Introduction to Bar Models”*. However, the early lessons include some additional scaffolding to support students for whom a bar model is a new idea.

This unit reviews the part-whole and comparison models in Lessons 1 to 5, then moves to more complex applications of these models. Lessons 6 to 8 introduce the stack and change models, again involving both additive and multiplication relationships between quantities.

Lesson 1: Part-whole Model - Division

In this lesson, students study multi-step problems which need a combination of whole number arithmetic operations for their solution. They make two different types of bar models for different division situations. They structure their problem solving by Polya’s four phases. Students then solve problems independently or in groups to consolidate their learning.

Lesson 2: Part-whole Model - Fractions

Students learn how to use the part-whole model to represent fractions in different real-world contexts. Solving the word problems is supported visually by the bar model and structured by Polya’s phases of problem solving. Students study examples and practise with further tasks.

Lesson 3: Part-whole Model - Fractions of Fractions

In this lesson, students encounter problems where the part-whole model is used to represent fractions, and fractions of those fractions. The bar models support an intuitive approach, building understanding of fractions and fraction calculations. Students study examples where either parts, parts of parts or the whole itself are to be found, and practise with similar tasks.

Lesson 4: Comparison Model - Whole Numbers

In this lesson, students use the comparison bar model to help solve multi-step word problems that include information about additive and multiplicative relationships between quantities. They study worked examples and practise with further tasks.

Lesson 5: Comparison Model - Fractions

In this lesson, students solve word problems that give information about fractions of different quantities that represent equal amounts. They use the comparison model for these problems, and solve them by identifying a common unit in the different quantities. Students see worked examples, then practise on several tasks.

Lesson 6: Stack Model - Whole Numbers

This lesson introduces the stack model. Students study problems where they compare two groups of quantities, with either a multiplicative or additive relationship given between individual items. Students work on two examples together, then practise independently or in pairs to consolidate learning.

Lesson 7: Change Model - Whole Numbers

In this lesson, students learn to use the change bar model, a variant on the comparison model. They construct visual representations of complex stories, usually involving multiple quantities that change over time, and use the representations to find a strategy for solving the problems. Students study examples, then practise the techniques on other tasks.

Lesson 8: Change Model - Fractions

In this lesson, students use the change model with word problems involving fractions. By working with the visual model, fraction calculations are replaced by intuitive steps. Students study problems where the quantities in a situation change either by a whole number amount, or by a fraction of the whole. Students participate in building models with classmates, then practise with selected tasks.

Reflection on this sequence

Rationale

This unit adopts several spiral curriculum progression features:

- The unit revisits the key ideas of model drawing taught in *reSolve Bar Model Method Unit 1 “Introduction to Bar Models”*. Greater depth arises from the greater complexity of the information given about the problem situation, and the expanding inter-connected mathematical content.
- Mathematical content knowledge is sequenced carefully so there is increasing complexity of bar model use and content-based mathematical thinking as the unit progresses.
- The lessons within the unit are thoughtfully arranged so that model drawing techniques are reinforced in later lessons. Students develop firm foundations before moving to more complex problems.

reSolve Mathematics is Purposeful

This unit supports a rich interpretation and enactment of the content and proficiencies of the Australian Mathematics Curriculum by offering the bar model as a strategy to help students reduce cognitive load during problem solving and promote visualisation of mathematical relationships. Students explore their mathematical and contextual interpretations of the word problems, and translate these ideas onto the semi-abstract bar models to highlight the quantities and relationships. Polya’s framework for problem solving is reinforced.

reSolve Tasks are Challenging Yet Accessible

The tasks are often challenging as they students work on complicated word problems. However, students are supported to solve these multi-step, multi-part word problems, by drawing upon the bar model as an intermediary visual tool for organising and summarising key information and visualising quantities and relationships.

reSolve Classrooms Have a Knowledge Building Culture

Teachers can promote a knowledge building culture by having students share their mathematical thinking and explore various approaches to solve each problem. Higher order mathematical thinking is encouraged through discussions about the choice and use of bar models to best depict the mathematical relationships. Students can build success through collaborative inquiry by working together to unpack the word problems, presenting their own bar models and critiquing bar models of others.

Further Reading

Ministry of Education [MOE]. (2009). *The Singapore model method*. Singapore: EPB Pan Pacific.

Ng, C.H.J., & Lim, K.H. (2006). *A handbook for teachers in mathematics*. Singapore: Marshall Cavendish Education.

Polya, G. (1945). *How to solve it*. Garden City, New York: Doubleday.

Yeap, B. H. (2011). *Bar modelling - A problem-solving tool: From research to practice (an effective Singapore math strategy)*. Singapore: Marshall Cavendish Education.

