THE BAR MODEL METHOD

Bar Models in Problem Solving
Lesson 1: Part-whole Model - Division

Australian Curriculum: Mathematics (Year 6)

ACMNA123: Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (Year 6)

Lesson abstract

In this lesson, students study multi-step problems which need a combination of whole number arithmetic operations for their solution. They make two different types of bar models for different division situations. They structure their problem solving by Polya's four phases. Students then solve problems independently or in groups to consolidate their learning.

Mathematical purpose (for students)

Bar models help us plan how to solve word problems.

Mathematical purpose (for teachers)

This lesson exemplifies how the part-whole bar model can be used to solve multi-step word problems in whole numbers. All problems involve division, with the different types of division (partition and quotition) being reflected in the different appearance of the relevant bar models.

The lesson models how students can structure their problem solving by using Polya's four phases of Understand, Plan Do, Check. Bar models assist students to organise the information in a problem, and to visualise the relationships between the quantities. This supports students' decisions on the sequence in which arithmetic operations are applied, and which operations are needed. For this reason, building the model is presented as the main activity of the 'Plan' phase.

Lesson Length	60 minutes approximately
Vocabulary Encountered • part-whole model	Lesson Materials • Slide show *ST4_BarModelsPS_1a_PartWhWN.pptx* • Student Sheet 1 – Bar Model Examples 1A (1 per student) • Student Sheet 2 – Bar Model Examples 1B (1 per student) • Calculators as required

We value your feedback after these lessons via https://www.surveymonkey.com/r/G6VGPZ8

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Preliminary information

The lesson plan suggests two whole class examples, followed by four tasks for increasingly independent practice. Balance whole class work and independent practice depending on the students' prior experience with the bar model method. Students who have not seen bar models before may need an introductory lesson using some of the examples from the *reSolve Bar Model Method Unit 1 "Introduction to Bar Models"*.

Whole class examples (Student Sheet 1 – Bar Model Examples 1A) and further practice tasks for students (Student Sheet 2 – Bar Model Examples 1B) are provided. Solutions to the Tasks can be found in Teacher Sheet - Bar Model Solutions 1B.

The animated slideshow (*ST4_BarModelsPS_1a_PartWhWN.pptx*) provides solutions to the whole class examples, and two of the practice tasks. The slideshow demonstrates how a bar model is built up, piece by piece.

This lesson mainly uses the part-whole bar model - one long bar representing the total quantity, made up of several shorter bars that represent the parts.

All the examples in this unit require at least two arithmetic operations to complete, one of which is division.

Polya's four stages

The lessons in this unit are structured around Polya's problem-solving phases of 'Understand, Plan, Do, Check'. This is intended to provide students with a structured framework to apply when solving problems.

- The *Understand* phase is for looking at the information in the question: grasping the story of what happened, finding out what quantities are involved, clarifying details. Students might need to return to this phase when questions arise later.
- The *Plan* phase is when students draw the bar model. The thinking that goes into making a useful bar model generally highlights what calculations will need to be done to find the answer.
- The *Do* phase is used here to refer to doing the calculations.
- The *Check* phase is important – emphasise how all the conditions of the problem statement must be satisfied.

The four phases do not need to be followed strictly in a linear manner, for example:

- Plan and Do often happen at the same time.
- Students may go back through the phases. For example, students who are stuck during the Do phase need to review whether they have missed key information.
- Some students will want to draw the bar model right from the start, as this helps them to understand the question. Other students will wait until the Plan phase to draw their model.

For further information on Polya's phases, refer to the reSolve *Bar Model Method Teacher's Guide*.

Whole Class Examples

Hand out Student Sheet 1 – Bar Model Examples 1A. Students should write the solutions to these examples, for future reference.

Throughout the examples, emphasise how drawing the bar model helps students to organise the information, and how it can then be used to plan the mathematical solution.

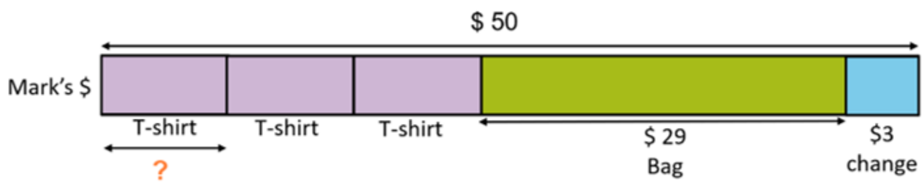
Example 1

This example requires splitting the whole bar ($50) into five parts, three of which are equal. The solution involves subtraction and division.

Mark bought 3 identical T-shirts and a bag. He paid $50 to the cashier and received $3 change. The bag cost $29. What was the cost of a T-shirt?

Solution

Cost of 3 T-shirt units = 50 - 3 - 29
= 18
Cost of 1 T-shirt = 18 ÷ 3
= 6
The cost of one T-shirt was $6



Discussion organised by Polya's four stages

Understand
The purpose here is to understand the information in the problem.
Encourage students to analyse the quantities and details in the problem (For example, cost of T-shirt, cost of bag, amount paid to the cashier and the change). The following prompts may assist whilst discussing this:

- What are the items Mark bought? (ANS: 3 T-shirts and 1 bag).
- What does "identical" T-shirts mean? (ANS: exactly the same, each T-shirt costs the same amount).
- What are all the quantities/values mentioned in the problem? (ANS: cost of T-shirt, cost of bag, amount paid to the cashier and the change).
- How much did the bag cost? (ANS: $29).
- What do I have to find? (ANS: The cost of a T-shirt).
- How are the "cost of T-shirt", the "cost of bag" and the "change" related to the "amount paid to the cashier"? (ANS: all of them add up to form the total, which is the amount paid to the cashier).

Plan
Discuss how these quantities are related and how they can be represented using bar models. The following discussion points may assist:

- How do we represent these quantities using the bar model? (ANS: draw bars representing the quantities).
- What type of bar model (i.e. part-whole model or comparison model) is suitable here? Why? (ANS: This problem requires the use of the part-whole model because the amount paid to the cashier can be taken as the "whole" whilst the cost of T-shirt, cost of bag and the change can be the various "parts" that make up the whole. The comparison model is not appropriate here because we are not comparing the difference between two or more quantities here but rather finding three missing "parts" within the whole. If the students are not familiar with the two types of bar models taught in Unit 1 of this Special Topic (The Bar Model Method), the discussion can focus on the part-whole model only so as not to confuse students).
- If the part-whole model is used, which are the "parts" and which is the "whole". Why? (ANS: amount paid to the cashier can be taken as the "whole" whilst the three costs of T-shirts, cost of bag and the change can be the various "parts" that make up the whole).
- Do we use the bars to represent the number of items (T-shirts, bag) or the cost? Why? (ANS: the cost; because the quantities represented should be in the same units; i.e. dollars).
- How do we represent the *identical* T-shirts and the bag within the full bar? (ANS: 3 equal parts within the bar representing the cost of each T-shirt, 1 part within the bar representing the cost of the bag; the parts for the cost of each T-shirt and the cost of the bag need not be the same size. All the parts should join up to form the full complex bar which is the whole).

Do
Work through the problem as a group or allow students some time to work on this independently before discussing. Some discussion points could include:

- Draw the parts within the bar and label the parts to show the part-whole relationship between $50, the cost of each T-shirt and the cost of the bag.
- What operations (addition, subtraction,…) do we use here?
- How much did Mark pay for the bag and 3 T-shirts? (ANS: $50 - $3 = $47)
- How should we represent the working that follows from the bar model?

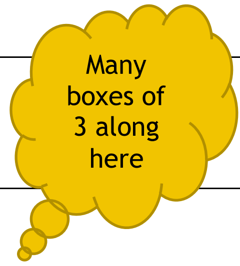
Check

- Substitute the cost of a T-shirt into the problem conditions and work out the total to see if it is $50.
  1 T-shirt cost **$6**
  3 T-shirts cost $6 x 3 = $18
  18 + 29 + 3 = 50
  Mark had **$50** at first.

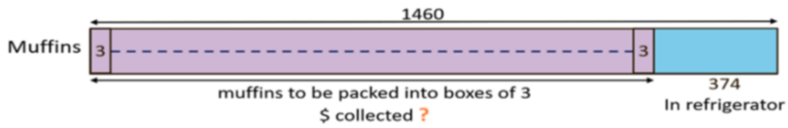
Example 2

This example is more complicated than Example 1 because the number of parts is unknown, so the bar cannot be divided into the right number of parts. It is important to appreciate that the bar model is just a sketch intended to help visualise the structure of the problem.

A bakery baked 1460 muffins and 374 of them were put in the refrigerator.
The other muffins were packed into boxes of 3 muffins each.
Each box of muffins was sold at $10.
How much money did the bakery collect from the sale of the muffins?



Solution



1460 – 374 = 1086
1086 muffins are packed into boxes of 3

1086 ÷ 3 = 362
There are 362 boxes of muffins sold.

362 x 10 = 3620
The bakery collected $3620 from the sale of the muffins.

Discussion organised by Polya's four stages

Understand
Encourage students to analyse the quantities and details given in the word problem. For example, number of muffins, number of boxes of muffins, cost of each box of muffins. Do not do calculations at this point – just note the relevant information. Some points for discussion could include:

- Explain the problem in your own words.
- Highlight the quantities/values to be looked at in the problem (ANS: number of muffins, number of boxes of muffins, cost of each box of muffins)
- How many muffins did the bakery bake altogether at first? (ANS: 1460 muffins)
- What are the 'other' muffins, and how could we find out how many there are? (ANS: The muffins not in the refrigerator, so [1460 – 374] muffins)
- How much did each box of muffins cost? (ANS: $10)
- What do I have to find? (ANS: the amount of money collected from the sale of the boxes of muffins.)

Plan
The slideshow can be used to demonstrate how a bar model might be built up.
Use prompts to discuss the problem whilst drawing the bar model:

- In the part-whole bar, which quantities make the "parts" and which is the "whole"? Why? (ANS: the total number of muffins can be taken as the "whole" whilst the muffins put in the refrigerator and each box of muffins would form the various "parts")
- How do we represent number of muffins, number of boxes of muffins and cost of each box of muffins using the bar model? (ANS: draw a part-whole model representing the quantities; use the number of muffins as the reference quantity; the number of boxes cannot be exactly drawn because it is not known).

Do
Either work through the problem as a class or allow students some time to work on their solutions independently. Some prompts could include:

- Draw the bars and label them to show the part-whole relationship between the total number of muffins, the muffins packed into boxes $50 and the cost of each box of muffins.
- How many muffins were packed into boxes of 3? (ANS: 1460 – 374 = 1086)
- From the model, calculate the number of muffins to be packed into boxes of 3.
- How should we represent the working that follows from the bar model?

Check
This is an important part of problem solving. Students should check that all the conditions in the problem are satisfied by the proposed solution.

Consolidating and Concluding

Further practice

Hand out Student Sheet 2 – Bar Model Examples 1B. Students work individually, in pairs or in groups on selected problems.

As with the worked examples, an important feature of these problems is that some involve partition and some involve quotition division. In Tasks 1 and 3 (quotition division), the size of parts is known (packs of 10, trays of 12) but the number of parts is not known. In this case, the bar model used to solve the problem can only be a sketch. In Tasks 2 and 4 (partition division), the number of parts is known (30 jars, 13 bags), but the size of parts is not known. These bar models are easier to draw.

Discuss solutions as time permits. Key points to highlight include:

- identify the quantities precisely
- highlight the part-whole relationships
- discuss how the bar model helps decide on the operations needed to solve the tasks.

Worked solutions are provided in Teacher Sheet - Bar Model Solutions 1B, and solutions to Task 1 and Task 2 are also included in the slide show *ST4_BarModelsPS_1a_PartWhWN.pptx*.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The part-whole bar model can involve the use of many parts within the bar representing the whole.
- The bar model is a sketch to help solve the problem. The length of the bars represent the size of the quantities in the problem, but they do not need to be drawn exactly in proportion.
- It is always important to mark which parts are of equal size.
- To find the unknown, we may have to have several steps and use a combination of the four arithmetic operations (addition, subtraction, multiplication and division).
- Constructing the model sometimes needs to be an iterative process. For example, the initial lengths of the bars sometimes might need to be adjusted to usefully show the relationships in the problem.
- Polya's four phases of problem solving (Understand, Plan, Do, Check) provides a structured approach to problem solving.

Bar Model Examples 1A

Name: ______________

Example 1

Mark bought 3 identical T-shirts and a bag.

He paid $50 to the cashier and received $3 change.

The bag cost $29.

What was the cost of a T-shirt?

Example 2

A bakery baked 1460 muffins and 374 of them were put in the refrigerator.

The other muffins were packed into boxes of 3 muffins each.

Each box of muffins was sold at $10.

How much money did the bakery collect from the sale of the muffins?

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Bar Model Examples 1B

Name: ______________

Draw bar models to represent the situations below and use them to solve the problems.

Task 1

A fruit seller had 900 apples.
He threw away 60 rotten apples and sold the rest at 10 for $6.
How much money would he receive after selling all the apples?

Task 2

Mary bought 9 tins of cookies.
Three tins each contained 16 almond cookies.
The other 6 tins each contained 28 chocolate chip cookies.
Mary then packed equal numbers of cookies into 30 glass jars.
Each jar can hold one or both types of cookies.
There were 6 cookies left.
How many cookies were there in each glass jar?

Task 3

Farmer Jane collected 350 eggs.
38 eggs were broken.
The rest of the eggs were put into trays of 12.
She sold each tray for $3.
How much money would she receive after selling all the eggs?

Task 4

Sam bought 3 large and 4 small packets of candies.
The large packets each contained 12 candies.
The small packets each contained 8 candies.
He then packed them into 13 bags that all had the same number of candies.
Then there were 3 candies left.
How many candies were there in each bag?

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Bar Model Solutions 1B

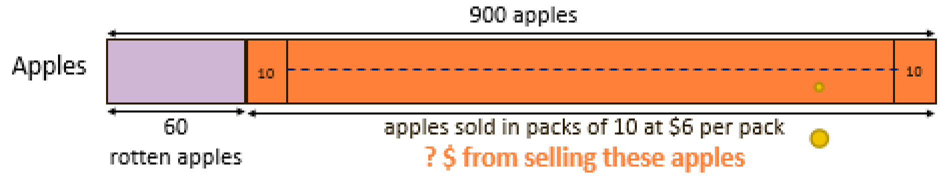
Task 1

A fruit seller had 900 apples. He threw away 60 rotten apples and sold the rest at 10 for $6. How much money would he receive after selling all the apples?

Understand (this is not the time for calculations, but for seeing what information is available)

- *How many apples were to be sold? (ANS: the good apples only) NO CALCULATIONS AT THIS PHASE*
- *How many apples were sold for $6 (ANS: 10)*
- *What do I have to find? (ANS: $ received from sale of all good apples)*

Plan: *Draw a model and label it.*



Do:

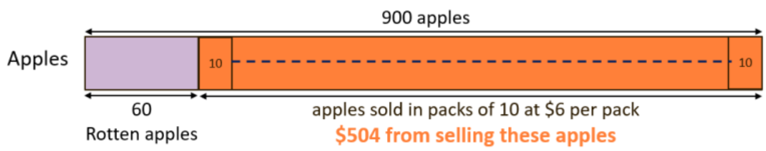
900 – 60 = 840
840 apples are in packs of 10 to be sold.

840 ÷ 10 = 84
There are 84 packs of apples to be sold at $6 each.

84 x 6 = 504
The fruit seller received $504 from selling the apples.

Check

Check the answer by substituting it into the original question.



504 ÷ 6 = 84

There were 84 sets of 10 apples sold at $6 per set.

84 x 10 = 840

840 apples were sold.

840 + **60** = 900

The fruit seller had **900** apples at first.

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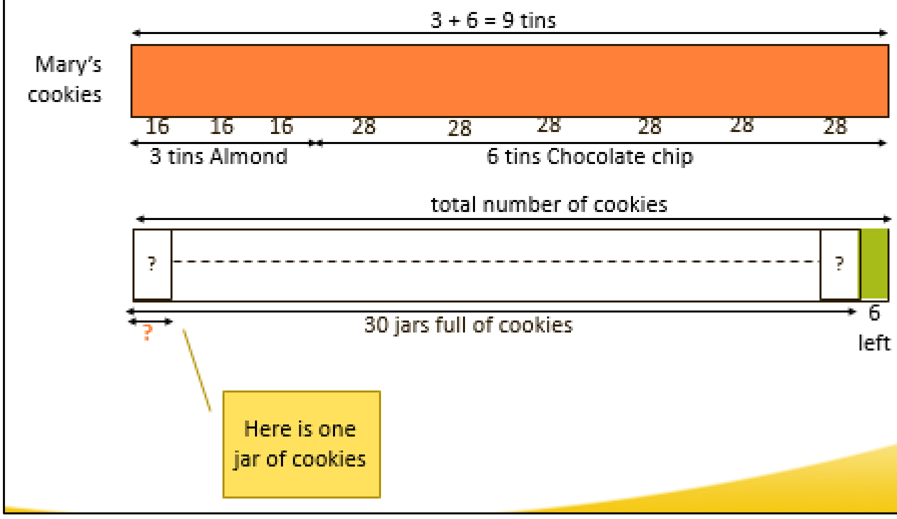
Task 2

Mary bought 9 tins of cookies.
Three tins each contained 16 almond cookies.
The other 6 tins each contained 28 chocolate chip cookies.
Mary then packed equal numbers of cookies into 30 glass jars. Each jar can hold one or both types of cookies.
There were 6 cookies left.
How many cookies were there in each glass jar?

Plan: *Draw a model and label it.*



Do

3 x 16 = 48

There were 48 almond cookies

6 x 28 = 168

There were 168 chocolate chip cookies

48 + 168 = 216

Total number of cookies Mary bought was 216

216 - 6 = 210

210 cookies were put in 30 glass jars equally.

210 ÷ 30 = 7

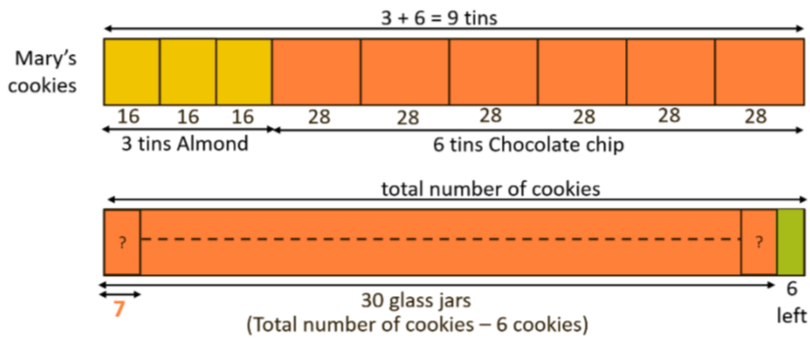
There were 7 cookies in each jar.

Check

Check the answer by substituting it into the original question and seeing that all the conditions are met.

7 x 30 = 210
210 cookies were put into 30 glass jars
210 + 6 = 216
There were 216 cookies altogether.

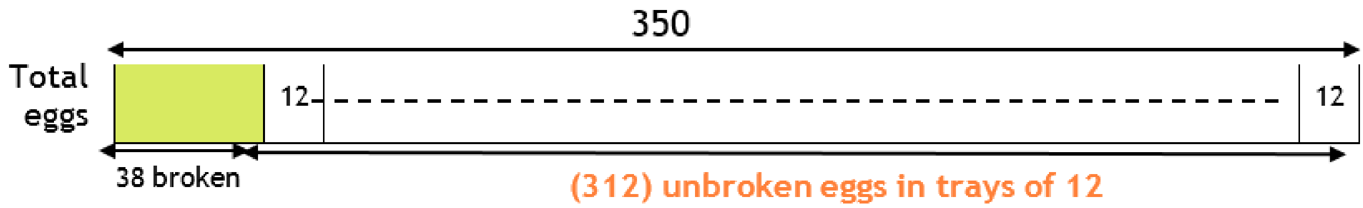
216 - (6 x 28) = 48
There were 48 almond cookies.
48 ÷ 16 = 3
There were 3 tins of almond cookies.



Task 3

Farmer Jane collected 350 eggs. 38 eggs were broken and the rest of the eggs were put into trays of 12. She sold each tray for $3. How much money would Farmer Jane receive after selling all the eggs?

Do:



Do

350 - 38 = 312

312 eggs were not broken.

312 ÷ 12 = 26

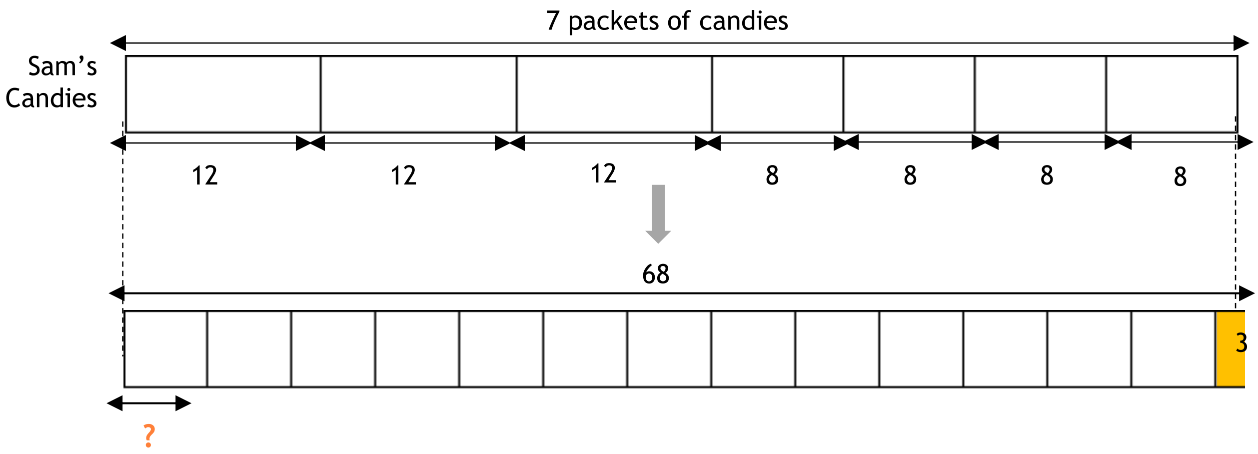
There were 26 trays to be sold.

26 x 3 = 78

Farmer Jane received $78 after selling all the eggs.

Task 4

Sam bought 3 large and 4 small packets of candies. The large packets each contained 12 candies and the small packets each contained 8 candies. He then packed them into 13 bags that all had the same number of candies. Then there were 3 candies left. How many candies were there in each bag?



At first,

12 x 3 = 36

8 x 4 = 32

36 + 32 = 68

There were 68 candies altogether.

68 - 3 = 65

65 candies were put equally into 13 bags.

65 ÷ 13 = 5

There were 5 candies in each bag.