Bar Models in Problem Solving
Lesson 2: Part-whole Model - Fractions

Australian Curriculum: Mathematics (Years 5 to 6)

ACMNA103 Investigating strategies to solve problems involving subtraction of fractions with the same denominator (Year 5)

ACMNA126 Solve problems involving addition and subtraction of fractions with the same or related denominators (Year 6)

Lesson abstract

Students learn how to use the part-whole model to represent fractions in different real-world contexts. Solving the word problems is supported visually by the bar model and structured by Polya's phases of problem solving. Students study examples and practise with further tasks.

Mathematical purpose (for students)

Fractions of different wholes can be shown on one bar model.

Mathematical purpose (for teachers)

This lesson illustrates how the part-whole model can assist when solving multi-step fraction word problems. The problems involve finding the whole or parts, when fractional parts and/or the corresponding quantities are given. Making the models facilitates organisation and understanding of the problem, which enables students to solve otherwise complex problems. The concept of a general term (unit) to represent a quantity is used throughout the solutions in this lesson, introducing students to early algebraic concepts in an accessible, informal and understandable way. In several problems, students deal simultaneously with fractions that reference different wholes.

It is intended that this lesson provides students with skills for problem solving, that they will continue using in their regular classwork.

Lesson Length 60 minutes approximately

Vocabulary Encountered

Lesson Materials

- Slide show *ST4_BarModelsPS_2a_PartWhFr.pptx*
- Student Sheet 1 – Bar Model Examples 2A (1 per student)
- Student Sheet 2 – Bar Model Examples 2B (1 per student)
- Calculators as desired

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Background

In this lesson, students solve multi-step fraction word problems which involve finding the whole or parts of the whole, given information on one or more fractional parts and their corresponding quantities. Showing the whole bar divided into multiple 'units' of equal size enables students to readily conceptualise the problem at hand. Teachers should provide guidance on how the division of the bar is related to the fractions in the problem.

Polya's problem-solving methodology of 'Understand, Plan, Do, Check' is used throughout the introductory examples, to provide students with a structured framework to apply when solving problems. Note that the four steps do not need to be followed strictly in a linear manner. For further information on Polya's problem solving methodology, please refer to the reSolve *Bar Model Method Teacher's Guide*.

The lesson plan suggests two whole class examples (Student Sheet 1 - Bar Model Examples 2A) and further practice tasks for students (Student Sheet 2 - Bar Model Examples 2B). Solutions to the Tasks can be found in Teacher Sheet - Bar Model Solutions 2B.

The slide show (*ST4_BarModelsPS_2a_PartWhFr.pptx*) provides animated solutions to the whole class examples, and two of the practice tasks. Teachers should balance whole class work and independent practice depending on the students' prior experience with the bar model method.

Whole Class Examples

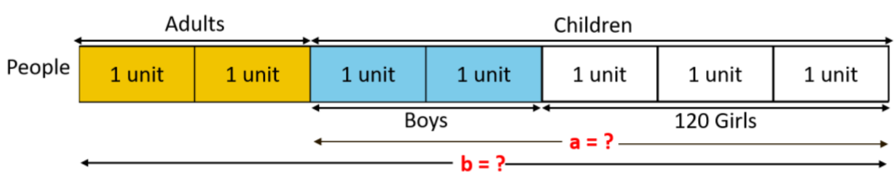
Hand out Student Sheet 1 - Bar Model Examples 2A. Students should write the solutions to these examples, for future reference.

Example 1

> At a children's show, $\frac{2}{7}$ of the people attending were adults.
>
> $\frac{2}{5}$ of the children were boys.
>
> There were 120 girls attending the performance.
>
> (a) How many children were there at the children's show?
>
> (b) How many people were there at the children's show?

Expected Student Response

3 units = 120 people
1 unit = 120 ÷ 3
= 40 people
5 units = 5 x 40
= 200 people



a) There were 200 children at the children's show.

7 units = 7 x 40
= 280

b) There were 280 people at the children's show.

Discussion organised by Polya's four stages

Understand

Analyse the quantities and details given in the problem with the students (for example, number of adults, number of boys and number of girls), and draw connections between the fractional parts and their respective quantities (e.g. number of boys is the same as $\frac{2}{5}$ of the children). The following questions points may be useful:

- Who attended the show? (ANS: adults, boys and girls)
- What are all the quantities/values mentioned in the problem? (ANS: number of adults, number of boys and number of girls)
- What do the fractions in the given problem refer to?
- Out of the people who attended the show, what fraction were adults? (ANS: $\frac{2}{7}$)
- What do I have to find? (ANS: a. Number of children; b. number of people)

Plan

Discuss how these quantities are related (the number of girls, boys and adults form the total) and how they can be represented using the bar model.

- How do we represent these quantities using the bar model? (ANS: draw a long bar representing the total set of people)
- How do we represent the number of children and adults within the full bar? (ANS: 2 of 7 parts will represent the fraction of adults, the remaining 5 parts will represent the fraction of children)
- When we draw the part-whole model, how many parts should we first divide the whole bar into? (ANS: 7 units, because $\frac{2}{7}$ of the people are adults)
- How many units represent the fraction of children? (ANS: 5 units out of the 7, the remaining $\frac{5}{7}$ of the people watching the performance must be children). (NOTE: the units are clearly marked on the slideshow.)
- How many units represent the fraction of children that are boys? (ANS: 2 units out of the 5 units)
- How many units can be used to represent 120 girls attending the performance? (ANS: 3 units)

Do

Either work through the problem as a class or allow students some time to work on their solutions independently. Some prompts could include:

- How do we calculate the number of people in 1 unit? (ANS: Use the information that 120 girls attended the performance. We know 120 girls are represented by 3 units, so we can calculate the number of people in 1 unit from that).
- Why do we need to do this? (ANS: to allow us to calculate the number of people in total, the number of boys and the number of adults, as they each are represented by multiples of 1 unit).
- After we find the number of people in 1 unit, how do we answer question (a)? (ANS: Multiply the number of people in 1 unit by 5)
- How do we find the answer for question (b)? (ANS: Multiply the number of people in 1 unit by 7).

Check

- Check the answer by substituting it into the problem and working out the total to see if it matches the given initial quantities.

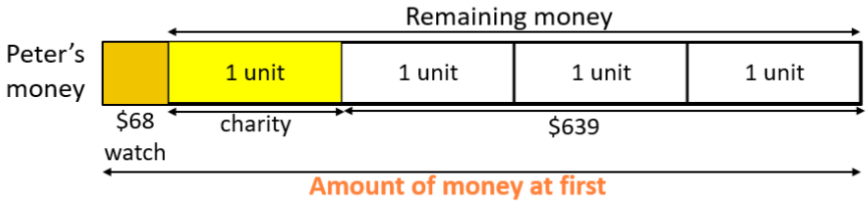
  $\frac{3}{7} \times \mathbf{280} = 120$ or $\frac{3}{5} \times \mathbf{200} = 120$

  There were **120** girls at the children's show.

Example 2

Peter spent \$68 on a watch. He donated $\frac{1}{4}$ of his remaining money to charity and had \$639 left. How much money did Peter have at first?

Expected Student Response



3 units = 639 (The key point has been to divide the remaining money, not the whole amount, into 4 parts)

1 unit = 639 ÷ 3

= 213

4 units = 4 x 213

= 852

68 + 852 = 920

Peter had \$920 at first.

Discussion organised by Polya's four stages

Understand

Students read the problem. Analyse the quantities given in the word problem. Some discussion points could include:

- Can you explain the problem in your own words?
- Highlight the quantities/values given in the problem (cost of watch, remaining money, amount finally left with, amount Peter had at first).
- What do I have to find? (ANS: the amount of money Peter had at first).

Plan

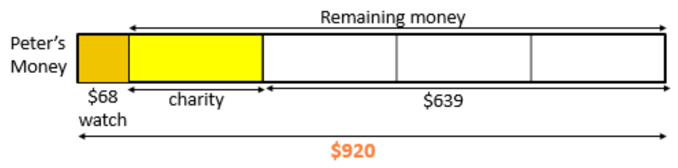
- How do we represent cost of watch, remaining money, amount finally left with, amount Peter had at first using the bar model? (ANS: draw part-whole model representing the quantities)
- It is important to note that the remaining amount must be further sectioned into parts (amount to charity and amount left).
- How do we represent the cost of watch and the remaining money using the bar? (ANS: two parts, one smaller part for the cost of watch, one bigger part for the remaining money).
- Why is the amount of remaining money taking up the larger part? (ANS: the amount finally left is much larger in value than the cost of watch, which gives us a clue regarding which part is largest).
- How do we divide the bar representing the remaining money? (ANS: into 4 parts).
- Why? (ANS: $\frac{1}{4}$ of the remaining money is donated to charity).
- What quantity does this refer to? (ANS: the amount donated to charity).

Do

- Draw the bars and label them to show the part-whole relationship between the cost of watch, remaining money, amount finally left with, amount Peter had at first.
- How many units can be used to represent \$639? (ANS: 3 of the 4 remainder units)
- How should we represent the working that follows from the bar model?

Check

- Check the answer by substituting it into the original problem to confirm that all the conditions of the question are met.



920 - 68 = 852
Amount left after spending on watch was $852

$\frac{1}{4} \times 852 = 213$ (amount given to charity)

852 – 213 = 639
Amount left after spending on watch and charity was $639.

Consolidating and Concluding

Further practice

Hand out Student Sheet 2 - Bar Model Examples 2B. Students work individually, in pairs or in groups on selected problems.

Discuss solutions as time permits. Key points to highlight include:

- Identify the quantities and the fraction of the whole that each quantity refers to. Stress which whole each fraction references.
- Highlight the part-whole relationships.
- Identifying the number of parts to divide the bar into, highlighting the relationship of that with the fractions in the problem.
- Highlight the value of defining units to enable solving the problems.

Worked solutions are provided in Teacher Sheet - Bar Model Solutions 2B, and solutions to Task 1 and Task 2 are also included in the slide show *ST4_ProblemSolving_2a_Fractions.pptx*.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The part-whole model is particularly useful as a visual tool for illustrating fraction word problems,
- The bar model contains information about the fraction, its size and the relationship between the fraction and the various wholes in the model and the problem.
- A fraction is always a measurement in relation to a whole. It is important to be clear about what the whole is intended to be, for any fraction.
- Defining units can be helpful when solving problems using bar models.

Bar Model Examples 2A

Name: ______________

Example 1

At a children's show, $\frac{2}{7}$ of the people attending were adults. $\frac{2}{5}$ of the children were boys. There were 120 girls attending the performance.

(a) How many children were there at the children's show?

(b) How many people were there at the children's show?

Example 2

Peter spent $68 on a watch. He donated $\frac{1}{4}$ of his remaining money to charity and had $639 left. How much money did Peter have at first?

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Bar Model Examples 2B

Name: ______________

Draw bar models to represent the situations below and use them to solve the problems.

Task 1

Nancy was reading a book. On the first day, she read $\frac{2}{7}$ of the total number of pages in the book. On the second day, she read another 40 pages and was left with $\frac{9}{14}$ of the total number of pages in the book to read. What was the total number of pages in the book?

Task 2

Henry gave $520 of his weekly salary to his parents and spent $\frac{2}{5}$ of the remaining salary on transport. He spent another $290 on food. The amount of money he had left was $\frac{1}{2}$ of the amount he spent on transport. How much was his weekly salary?

Task 3

After Denise spent $\frac{1}{4}$ of her salary on a handbag and $150 on a pair of shoes, she was left with $\frac{5}{8}$ of her salary. What was the amount of Denise's salary?

Task 4

Every month, Ben gave his parents $730 and spent $\frac{1}{2}$ of the amount of the remaining salary on transport. He spent another $120 on food. The amount of money he had left was $\frac{2}{3}$ of the amount of money spent on transport. How much was Ben's salary?

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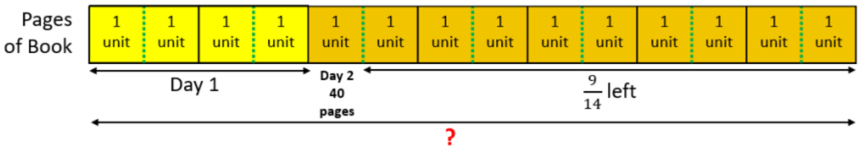
Task 1

Nancy was reading a book. On the first day, she read $\frac{2}{7}$ of the total number of pages in the book. On the second day, she read another 40 pages and was left with $\frac{9}{14}$ of the total number of pages in the book to read. What was the total number of pages in the book?

Plan

- *How many units can the total number of pages in the book be divided into to start with?* ***(ANS: 7 parts)***
- *How many of those units can be used to represent the number of pages read in Day 1?* ***(2 parts)***
- *What can we do, to enable us to represent $\frac{9}{14}$ of the total pages on the model?* ***(Divide the total number of pages - that is, the whole bar - into 14 units rather than 7 parts. That is, each of the 7 parts can be divided into two, to create smaller units)***
- *How many units can be used to represent 40 pages of the book?* ***(1 small unit - why is this?)***
- *Draw a model*



Do

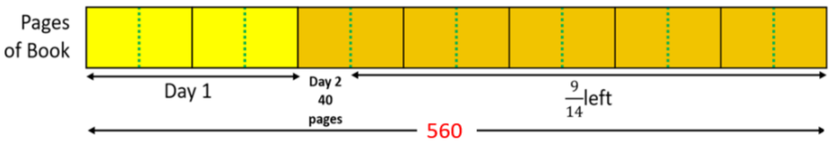
1 unit = 40

14 units = 14 x 40

= 560

The total number of pages in the book was 560.

Check



$\frac{2}{7}$ X 560 = 160

Nancy read 160 pages on Day 1

560 - 160 = 400

After Day 1, there were 400 pages to read.

400 – 40 = 360

After Day 2, there were 360 pages left.

$\frac{360}{560} = \frac{9}{14}$

After Day 2, there were $\frac{9}{14}$ of the total number of pages left.

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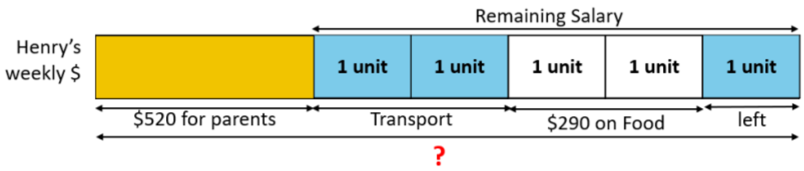
Task 2

Henry gave \$520 of his weekly salary to his parents and spent $\frac{2}{5}$ of the remaining salary on transport. He spent another \$290 on food. The amount of money he had left was $\frac{1}{2}$ of the amount he spent on transport. How much was his weekly salary?

Plan

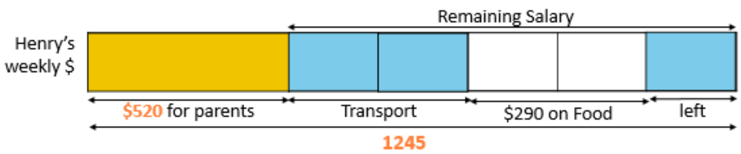
- *Draw a model with the long bar representing Henry's full salary.*
- *How do we represent " $\frac{2}{5}$ of the remaining salary" in the bar model?*
  (ANS: Divide the remaining salary into 5 equal units)
- *How many units can be used to represent the amount of money he had left?*
  (ANS: 1 of the 5 equal units in the remaining salary)



Do

2 units = \$290
1 unit = \$290 ÷ 2
= \$145
5 units = 5 x \$145
= \$725
\$520 + \$725 = \$1 245
Henry's weekly salary is \$1 245.

Check



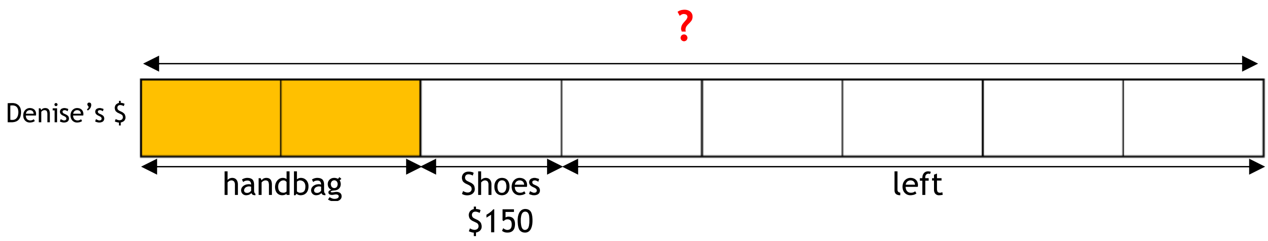
PARENTS
\$1 245– \$520 = \$725 (left after giving money to parents)
TRANSPORT
$\frac{2}{5}$ of 725 = 290 (spent on transport)
725 – 290 = 435
\$435 left after paying for transport
FOOD
435 – 290 = 145
\$145 left after paying for food

Yes – the amount left (\$145) is half of what was spent on transport (\$290)

Task 3

After Denise spent $\frac{1}{4}$ of her salary on a handbag and $150 on a pair of shoes, she was left with $\frac{5}{8}$ of her salary. What was Denise's salary?

To draw the bar model, note that Denise spent $\frac{2}{8}$ of her salary on the handbag, and divide the bar representing her salary into 8 equal parts.



1 unit = $150

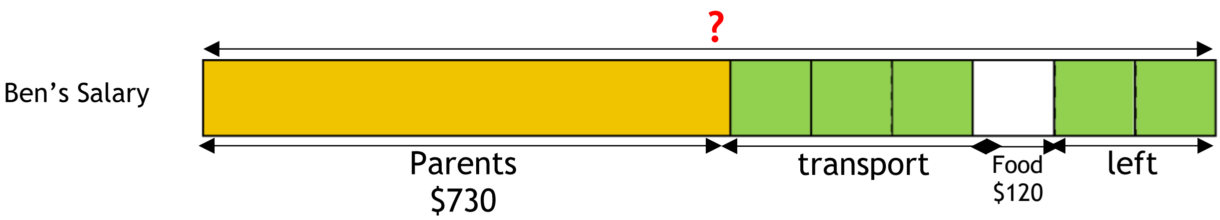
8 units = $150 X 8

= $1200

Denise's salary was $1200.

Task 4

Every month, Ben gave his parents $730 and spent $\frac{1}{2}$ of the amount of the remaining salary on transport. He spent another $120 on food. The amount of money he had left was $\frac{2}{3}$ of the amount of money spent on transport. How much was Ben's salary?



Amount spent on transport = Amount spent on food + Amount finally left (because the transport was half)

Amount spent on transport = Amount spent on food + $\frac{2}{3}$ of Amount spent on transport

Amount spent on food = $\frac{1}{3}$ of Amount spent on transport

Amount spent on transport = $120 x 3 = $360

Amount finally left = $120 x 2 = $240

Ben's salary = $730 + $360 + $120 + $240 = $1450

Alternatively:

1 unit = $120

6 units = $120 × 6

= $720

$720 + $730 = $1450

Ben's salary was $1450.