Bar Models in Problem Solving
Lesson 3: Part-whole Model - Fractions of Fractions

Australian Curriculum: Mathematics (Years 6 - 7)

ACMNA126: Solve problems involving addition and subtraction of fractions with the same or related denominators (Year 6)
ACMNA127: Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (Year 6)
ACMNA154 Multiply and divide fractions and decimals using efficient written strategies and digital technologies (Year 7)

Lesson abstract

In this lesson, students encounter problems where the part-whole model is used to represent fractions, and fractions of those fractions. The bar models support an intuitive approach, building understanding of fractions and fraction calculations. Students study examples where either parts, parts of parts or the whole itself are to be found, and practise with similar tasks.

Mathematical purpose (for students)

Bar models can help us to understand problems involving fractions of fractions.

Mathematical purpose (for teachers)

This lesson involves students working with fractions within a whole and fractions of those fractions, in an intuitive and accessible way. The bar model enables the complex concept of a fraction of a fraction to be readily represented visually, promoting students' understanding and enabling them to break down the scenario to determine the values of the various parts. A key idea is that a fraction is always only understood if the whole is known. The fractions in these problem statements reference different wholes (e.g. an amount of money before and after an event). This is the central idea for understanding multiplication of fractions intuitively.

The concept of a general term (unit) to represent a quantity is also used throughout the solutions in this lesson, introducing students to early algebraic concepts in an accessible, informal and understandable way. Polya's four steps of problem solving are used to structure the solution process.

Lesson Length 60 minutes approximately

Vocabulary Encountered

Lesson Materials

- Slide show Slide show *ST4_BarModelsPS_3a_PartWhFrFr.pptx*
- Student Sheet 1 -Bar Model Examples 3A (1 per student)
- Student Sheet 2 –Bar Model Examples 3B (1 per student)

We value your feedback after these lessons via https://www.surveymonkey.com/r/G6VGPZ8

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Whole Class Examples

Hand out Student Sheet 1 – Bar Model Examples 3A.

Students should write the solutions to these examples, for future reference.

The slide show (*ST4_BarModelsPS_3a_PartWhFrFr.pptx*) provides animated solutions to these examples.

The problems in this lesson are solved by finding a fraction of a quantity (i.e. a part of a whole), and then finding a fraction of this fraction (for example, a half of a third). Finding a fraction of a fraction can be done by fraction multiplication, but in this lesson, it is not necessary to know this. Instead the calculation is done 'visually' using the bar model. Because the concept of a fraction of a fraction may be unfamiliar to some students, additional discussion on this aspect of the problems will be helpful, whilst working through the examples. The part-whole model is used to represent these scenarios.

Use Polya's 4-step problem solving process (Understand, Plan, Do, Check) to guide students. The prompts given are sample scaffolding questions suitable to each of the 4 steps. Note that students may move back and forth between the steps as they monitor their process. For example, students who are stuck during the "Do" step can be guided back to the "Understand" step again to see if they have missed out key information given in the problem statement.

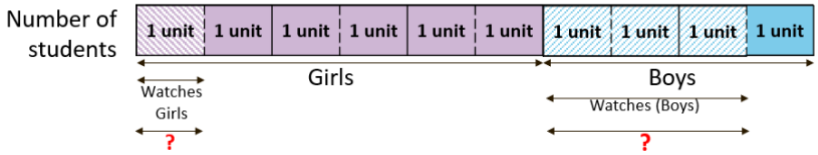
Some of the suggested questions in the "understand" phase are designed to highlight how the various fractions in each problem refer to different wholes.

Checking the answers could be done by multiplying fractions. However, it is expected that most students will not calculate, but will refer to the bar models, or to special numerical cases.

Example 1

In a class, $\frac{3}{5}$ of the students are girls and the rest are boys. $\frac{1}{6}$ of the girls and $\frac{3}{4}$ of the boys wear watches. What fraction of the students wear watches?

Expected Student Response



1 unit = Number of girls who wear watches
3 units = Number of boys who wear watches
10 units = Total number of students

$$\frac{1+3}{10} = \frac{4}{10} = \frac{2}{5}$$

$\frac{2}{5}$ of the students wear watches.

Discussion organised by Polya's four stages

Read the problem with the class and discusses how to draw and label the bar model.

The animated slide show Slide show *ST4_BarModelsPS_3a_PartWhFrFr.pptx* builds up the bar model step-by-step. Showing it can be integrated with students' suggestions, to support the discussion.

Understand
Encourage students to analyse the information in the problem and describe the story:

- What fraction of the students were boys? (ANS: $\frac{2}{5}$).

- What four parts of the class are involved in the story? (ANS: Boys who do and do not wear watches, girls who do and do not wear watches).
- What (if anything) does the problem say about the fraction and the number of boys who wear watches

Plan

Some points to discuss could include:

- Ask students to select which type of bar model should be used & why? (ANS: A part-whole model, because the number of students in the class can be taken as the "whole" whilst the number of girls and the number of boys can be the various "parts" that make up the whole initially. These initial parts are later split.)
- Draw and label a bar model with the students. It is likely that students will not have often have encountered situations involving fractions of fractions. Emphasise how to represent the initial division of boys' and girls' bars, and then the further division of each of those categories.
    - How do we represent $\frac{3}{5}$ in the bar model? (ANS: By dividing the whole bar into 5 equal sections, then shading 3 of them.)
    - We need to look at just the girls to find $\frac{1}{6}$ of them, and just the boys to find $\frac{3}{4}$ of them.
    - Are there further divisions which will need to be made? (ANS: Yes, we will need to subdivide the bar representing the number of girls up to show $\frac{1}{6}$ of them – this can be done by halving each of the three parts already there. This also works for the boys' bar.)

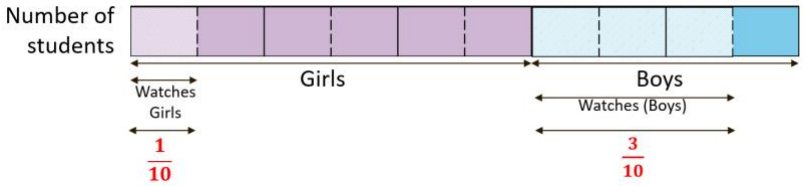
Do

Work through the problem as a group. Some discussion points could include:

- Fortunately, we now have a common size of division in the whole bar for both girls and boys, to help us work out the answers. We'll call these common parts the 'units'.
- What is the value of 1 unit in this problem? (ANS: $\frac{1}{10}$ of the number of students).
- How many units of girls wear watches, and how many units of boys wear watches?

Check

Encourage students to check the answer against the context. Some students might multiply fractions. Others might only be able to check with whole numbers, by using a convenient class size (e.g. 20).



As seen in the model, $\frac{1}{10}$ of students are girls with watches and $\frac{3}{10}$ of the students are boys with watches. Total fraction of students with watches will be $\frac{4}{10}$ or $\frac{2}{5}$.

CHECK WITH A SPECIAL CASE

If there are 20 students in the class, there will be 12 girls and 8 boys.

2 girls will wear watches.

6 boys will wear watches

All together 8 out of 20 will wear watches. This is 2 fifths.

Example 2

Helen had some money.

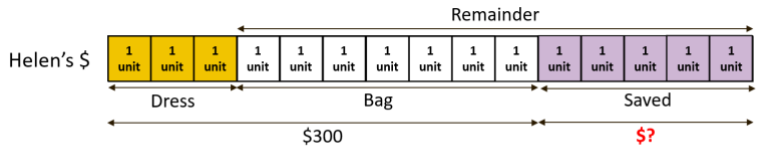
She spent $\frac{1}{5}$ of it on a dress and saved $\frac{5}{12}$ of the remaining money in the bank.

She spent the rest of the money on a bag.

She spent $300 on the dress and the bag.

How much money did she save in the bank?

Expected Student Response



Show your working.

10 units = 300
1 unit = 300 ÷ 10
= 30
5 units = 5 x 30
= 150
Helen saved $150.

Discussion organised by Polya's four stages

Read the problem with the class and discuss how to draw and label the bar model.

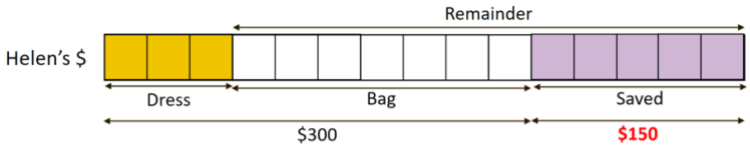
Understand
Encourage students to analyse the information in the problem (for example, the cost of the dress and bag, the fractions in the question and what they represent). The key idea is that the two fractions in the problem statement relate to different wholes.

- Tell the story of the problem situation in your own words.
- What three parts has Helen's money been split into? (ANS: money for dress, bag and saving)
- Did Helen save 5 twelfths of her money? (ANS: No, she saved less than that.)
- Did the bag cost 7 twelfths of Helen's money? (No, it cost less than that.)
- What do I have to find? (ANS: How much money Helen saved).

Plan & Do
Work through drawing the model and solving the problem as a group. Some discussion points could include:

- What type of bar model would be best to use? (ANS: a part-whole model because the total amount of money Helen has can be taken as the 'whole' whilst the money spent on a dress and bag and the remaining amount of money saved form the various 'parts'.)
- How many parts should the bar initially be divided into? (ANS: 5)
- How do we further divide the parts up, to enable us to show $\frac{5}{12}$ of the remaining money as savings? (ANS: If we divide each of the 4 'remaining' parts into thirds, there will be 12 smaller parts. Five of these represent the savings).
- How can we show the cost of the bag and dress on the bar model? (ANS: It is convenient to show the bag and dress units are next to each other, so a single cost can be shown between them. Redrawing the bar model might be useful.)
- What can we define 1 unit as being, in this problem? (ANS: $\frac{1}{15}$ of Helen's money, as we also divide the cost of a dress into thirds, to enable the entire model to be divided into common units).
- What amount do we know from the question, which could help us to work out the value of 1 unit? (ANS: $300 for the cost of a dress and a bag).

Check
Encourage students to check the answer by substituting it into the problem:



5 units = 150 (savings)
1 unit = 150 ÷ 5 = 30
3 units = 30 x 3 = 90
Dress cost $90
$\frac{90}{450} = \frac{1}{5}$
Cost of dress is $\frac{1}{5}$ of Helen's money.

Cost of bag: 7 units = 30 x 7 = $210
210 + 90 = 300
Cost of bag and dress was $300.

Consolidation and Concluding

Further Practice

Hand out Student Sheet 2 –Bar Model Examples 3B. Students work individually, in pairs or in groups on selected problems.

Discuss solutions as time permits. Animated worked solutions to both examples and Task 1 and Task 2 are included in the slide show *ST4_BarModelsPS_3a_PartWhFrFr.pptx*. Worked solutions to all Tasks are provided in Teacher Sheet - Bar Model Solutions 3B. Key points to highlight in discussion include:

- Highlight the part-whole relationships.
- Note what 'whole' each fraction is referencing.
- Note that the parts of a whole can be divided into further smaller parts (smaller units). It is important to ensure that at any point in time, all the units in a bar are of equivalent value, so when we divide larger units into smaller ones, the entire bar needs to be divided in this way.
- Task 1 can be done by multiplying fractions, but it is recommended that it be done by looking at the bar model.

Conclusion

Summarise the learning points for the lesson, asking students to add their own observations:

- The part-whole model can involve the use of many parts (more than two parts) within the whole; some of the parts may not be equal in quantity.
- It is always important to know which 'whole' a fraction refers to.
- Defining units can be helpful when solving the problems using bar models.
- When problems involve fractions of fractions, dividing the bar into larger units then dividing those further into smaller units enables us to model the problem.

Bar Model Examples 3A

Name: _____

Example 1

In a class, $\frac{3}{5}$ of the students are girls and the rest are boys.

$\frac{1}{6}$ of the girls and $\frac{3}{4}$ of the boys wear watches.

What fraction of the students wear watches?

Example 2

Helen had some money.

She spent $\frac{1}{5}$ of it on a dress and saved $\frac{5}{12}$ of the remaining money.

She spent the rest of the money on a bag.

She spent $300 on the dress and the bag.

How much money did she save?

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Bar Model Examples 3B

Name: _____

Draw bar models to represent the situations below and use them to solve the problems.

Task 1

In a basket, $\frac{2}{5}$ of the apples are red and the rest are green apples.
$\frac{3}{8}$ of the red apples and $\frac{1}{2}$ of the green apples are rotten.
What fraction of the apples are rotten?

Task 2

Robert gave $\frac{2}{7}$ of his marbles to his brother and $\frac{3}{10}$ of the remainder to his classmate.
He had 70 marbles left.
How many marbles did he have at first?

Task 3

Jane had some coloured pencils.
$\frac{1}{3}$ of them were green, $\frac{1}{9}$ of them were blue and another $\frac{1}{3}$ of them were yellow.
There were 8 remaining red pencils.
How many pencils did Jane have?

Task 4

There were coloured bean bags in a box.
$\frac{1}{6}$ of them were red bean bags. Of the remainder, $\frac{7}{15}$ were blue bean bags.
The rest were 32 purple bean bags.
How many bean bags were there in the box altogether?

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Bar Model Solutions 3B

Task 1

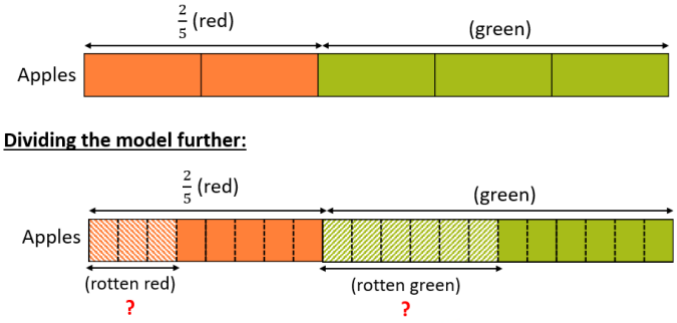
In a basket, $\frac{2}{5}$ of the apples are red and the rest are green apples. $\frac{3}{8}$ of the red apples and $\frac{1}{2}$ of the green apples are rotten. What fraction of the apples are rotten?

Understand - ask questions like this so that students think about the problem situation (no calculation needed)

- *What fraction of the apples were green? (ANS: $\frac{3}{5}$)*
- *There are four kinds of apples in the problem. What are they?*
- *Are there more good apples or rotten apples?*
- *Are there more good red apples or rotten red apples?*
- *What do I have to find?* ***(ANS: Fraction of apples that were rotten)***

Plan

- Draw and label a bar model.



Do

- To show $\frac{3}{8}$ of the red apples are rotten, we divided each of the larger units into 4, so the total number of red apples was divided into 8 equal parts.
- To show $\frac{1}{2}$ of the green apples are rotten, we divided each larger unit into 4 (the same as red apples), so the total number of green apples was divided into 12 equal parts.

From the bar model,

Fraction of red rotten apples = 3 out of 20 equal total parts.
Fraction of green rotten apples = 6 out of 20 equal total parts.

3 + 6 = 9
9 out of 20 parts are rotten apples.
$\frac{9}{20}$ **of the apples are rotten.**

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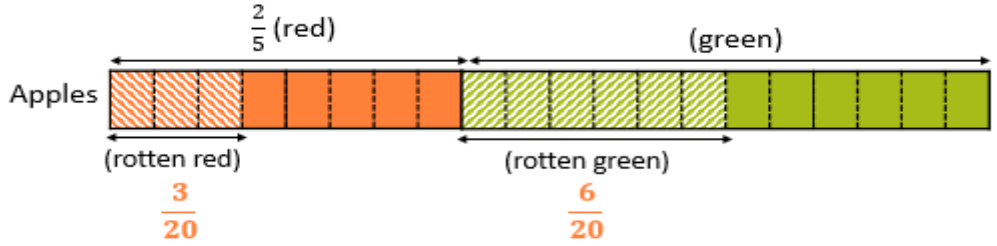
Check



Look at the bar model to work out the fractions of fractions

$\frac{3}{8}$ of $\frac{2}{5} = \frac{3}{20}$
$\frac{3}{20}$ of the apples that were rotten were red ones.

$\frac{1}{2}$ of $\frac{3}{5} = \frac{6}{20}$
$\frac{6}{20}$ of the apples were rotten green ones.

$\frac{9}{20} = \frac{3}{20} + \frac{6}{20}$
$\frac{9}{20}$ of apples were rotten.

Task 2

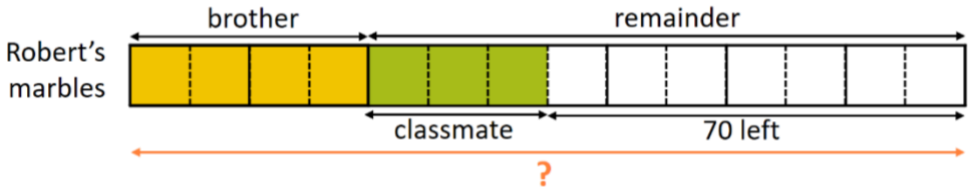
Robert gave $\frac{2}{7}$ of his marbles to his brother and $\frac{3}{10}$ of the remainder to his classmate.
He had 70 marbles left.
How many marbles did he have at first?

Understand

- *What three parts has the 'whole' (Robert's marbles) been split into?*)
- *Does the classmate get more than 70 marbles or less? How do you know?*
- 2/7 is less than 3/10. Does this tell us that the brother got less marbles than the classmate?)
- What do I have to find? (ANS: The number of marbles Robert has at first)

Plan

- Draw and label a bar model to represent the part-whole situations.
- Which part shows the $\frac{2}{7}$ of marbles which was given to Robert's brother?



Do

To show $\frac{3}{10}$ of the remaining marbles, we divide the remaining marbles into 10 equal units. 3 of those units show the number of marbles which were given to his classmate.

7 units = 70 marbles
1 unit = 70 ÷ 7 = 10 marbles
14 units = 14 x 10 = 140 marbles
Robert had 140 marbles at first.

Check

Check the answer by substituting the answer of **140 marbles** back into the question.

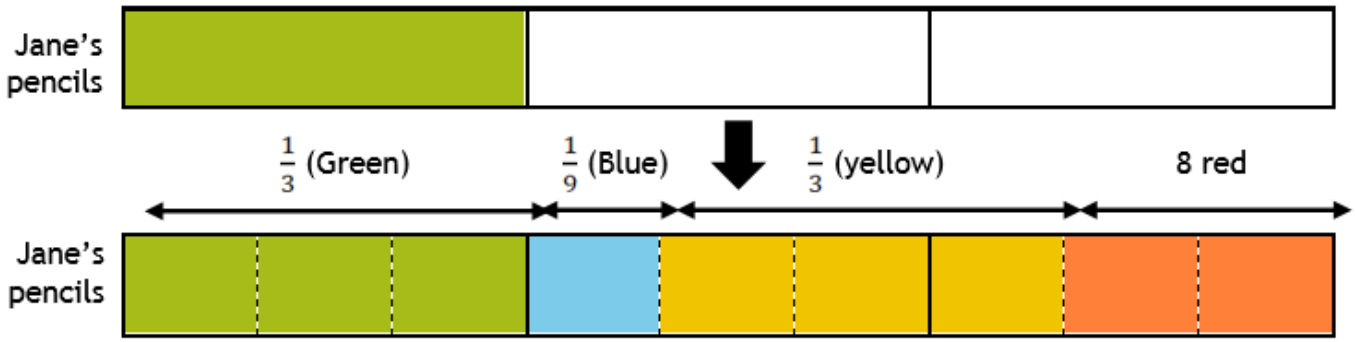
If Robert had 140 marbles to start with:

- he gave 2/7 of 140 marbles to his brother (so 40 marbles)
- he had 100 left
- then he gave 3/10 of 100 to his classmate – he gave him 30
- then he had 100 – 30 = 70 left.

Check complete.

Task 3

Jane had some coloured pencils. $\frac{1}{3}$ of them were green, $\frac{1}{9}$ of them were blue and another $\frac{1}{3}$ of them were yellow. There were 8 remaining red pencils. How many pencils did Jane have?



2 units = 8

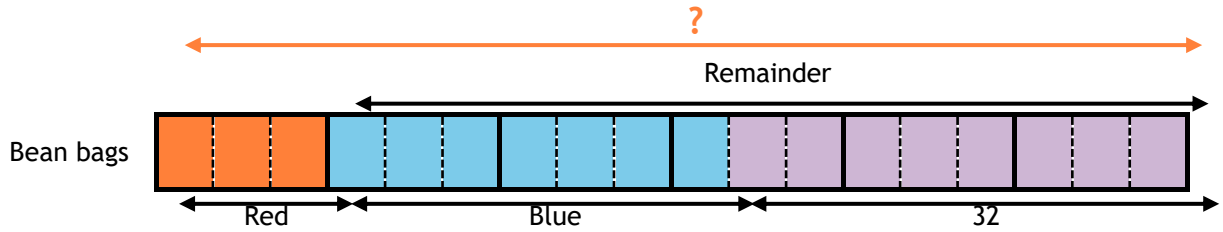
1 unit = 8 ÷ 2 = 4

9 units = 9 x 4 = 36

Jane had 36 coloured pencils.

Task 4

There were coloured bean bags in a box. $\frac{1}{6}$ of them were red bean bags. Of the remainder, $\frac{7}{15}$ were blue bean bags. The rest were 32 purple bean bags. How many bean bags were there in the box altogether?



8 units = 32

1 unit = 32 ÷ 8

= 4

18 units = 18 x 4

= 72

There were 72 bean bags altogether.