re(Solve) MATHS BY INQUIRY

Lesson 7: Complex Motion

Australian Curriculum: Mathematics — Year 5

ACMMG114: Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries.
ACMSP120: Describe and interpret different data sets in context.

Australian Curriculum: Mathematics — Year 6

ACMSP147: Interpret and compare a range of data displays, including side-by-side column graphs for two categorical variables.

Australian Curriculum: Science — Years 5 & 6

ACSIS090: Construct and use a range of representations, including tables and graphs, to represent and describe observations, patterns or relationships in data using digital technologies as appropriate.

- Identifying patterns in data and developing explanations that fit these patterns.

ACSIS218: Compare data with predictions and use as evidence in developing explanations.

- Sharing ideas as to whether observations match predictions, and discussing possible reasons for predictions being incorrect.

Lesson abstract

Students investigate the motion of a ball in two dimensions, as it travels across a sloping table. They predict the path of the ball, before tracing its path on butcher's paper. Then they change the angle of the launcher so that the ball hits a specified target on the table. Class discussion focuses on the shape of the path of the ball and the forces which affect it, and the changes in speed which are occurring.

Mathematical purpose (for students)

To trace the path of a ball launched onto a sloping table and consider the effect of forces on its motion.

Mathematical purpose (for teachers)

This capstone lesson is an opportunity for revising and assessing students' grasp of concepts developed earlier in this unit: relating speed, acceleration and deceleration, gravity and friction. Students are introduced to the idea that two-dimensional motion can be thought of as a combination of two one dimensional motions. They use this idea to apply their knowledge from previous lessons to a new and complex situation. Students also make links between the motion of a ball and the shape of its path.

Lesson Length 90 minutes approximately

Vocabulary Encountered

- Arc
- Parabola (optional)
- Symmetry

Materials — see Teachers' Guide Appendix B for full details

- 5 propped up tables with butcher's paper, launching track & ball
- Marker pens, scissors, tape, water, tissues
- 5 copies of this paper target
- Workbook *ST3_Motion_Y5&6_7a_Workbook.pdf* (1 per student)
- Student copies of potential projects from *ST3_Motion_Projects.pdf.*

We value your feedback after these lessons via https://www.surveymonkey.com/r/JJCGHVX

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

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Background

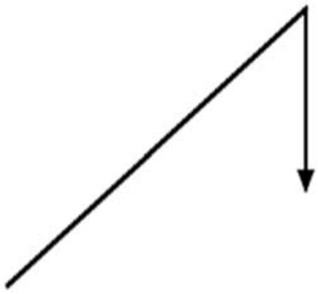
Motion in two dimensions

In previous lessons the objects have been constrained to move in one dimension only (by running along a track, or being dropped from a height, or being pulled by string). This makes it easier to observe and understand what is happening to the object. In this lesson students will use the insights they have gained in previous lessons to investigate motion in two dimensions.

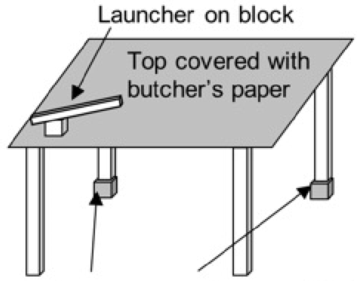
Students will be familiar with throwing a ball. In this lesson, we will slow down this motion by launching the ball onto a sloping table instead of throwing it. Students will observe the motion, mark the path of the ball, and suggest reasons for what they observe.

Students may expect the path of the ball to be an arc, or they may have other predictions. In earlier times it was thought that a ball travelled in a straight line and when it had reached as far up as it could go, it fell directly down to earth — a naive interpretation that some of your students will share.

Path of a thrown ball (parabolic arc)



Medieval idea of the path (pre-Galileo)



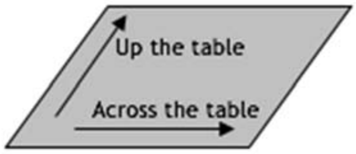
Sloping table experiment set-up

Relating two-dimensional movement to previous lessons

When a ball is thrown from one person to another, we can usually see that it travels both horizontally and vertically, i.e. it moves across the room and at the same time moves up in the air and then down again. This movement can be thought of as a combination of one dimensional movements.

- In the vertical direction, the ball is being pulled down by gravity, and gets slower and slower until it stops moving up at all, then starts moving down again, getting faster and faster until it reaches the ground. This is similar to the ball in Lesson 4: Rolling Uphill, which moved up the ramp, decelerating, until it stopped, then moved back down the ramp, accelerating.
- In the horizontal direction, the ball moves at a fairly constant speed, only decelerating slightly due to air resistance and (in the case of our experiment on the sloping table) friction. This is similar to a ball rolling on a flat track, like in the second activity in Lesson 4.

The lesson activities prompt students to make these connections. Since the experiment is on a shallow sloping table, it is recommended to use the following phrases to descibe the direction of the motion: “up the table”/”down the table”, and “across the table”.



Relating the path of the ball to its speed

Students will trace the path of the ball onto butcher’s paper, but this path does not give them any direct information about the speed of the ball (see note below). Students will use their observations of the speed of the ball, combined with their knowledge of deceleration/acceleration on a slope, to suggest that the ball is moving fast when it leaves the launching track, decelerating until it reaches its peak, and then accelerating again until it falls off the table.

The mathematical name for the shape of the path (parabola) is not necessary; the shape is an arc, which is sufficient at this level. Most students will either know or be able to see that the arcs produced are (almost) symmetrical in shape. The shape is not quite symmetrical: the second 'half' is usually slightly steeper (the ball doesn't travel as far horizontally in this section, because friction has been gradually reducing its speed since it left the launching track). Some assymetry may also be caused by uneven tilting of the table or the floor!

Note: the path of the ball could be thought of as a graph (with the x-axis being distance 'across the table' and the y-axis being distance 'up the table'), but it is a very different graph from the ones used in previous lessons, as it does not include time. The even passage of time can be seen approximately in the x-axis because the ball is moving at a fairly constant speed 'across the table', but it is not recommended that this approach be used with students.

Practical considerations

It is best to use large tables so that students can easily see a variety of paths.

Experimenting beforehand with your set-up enables you to guide students and streamline the lesson.

For students to see the arcs and their symmetry, the initial speed of the ball needs to be adjusted so that most of the paths form full arcs on the table (see pictures below) [Workbook step 8]. The instructions to students are a bit vague in order to not 'give away' the idea that the path is an arc to anyone reading ahead. You may need to clarify that they are trying to get lots of different arcs all fitting on the table. Some broad guidelines:

- Students are first asked to find a launching track direction that allows the ball's path to 'cover as much of the table as possible'. They may find (by experimenting) that a launching track direction of around 45° (very approximately) uses more of the table than a very high or very low angle. Alternatively, they may reason that the direction that points straight to the top right corner (the diagonal) will cover most of the table.
- Students will then experiment with different release heights. Releasing the ball higher up the launching track means it has more speed when it hits the table, and gets further before reaching its peak. If the ball starts off too fast it will run off the table before reaching a peak and moving back down the table. If it starts off too slow it will not get very far across the table.

A good speed: lots of arcs using most of the space on the table	Too fast: ball falls off the table before completing its arc	Too slow: lots of arcs but they are small and will be harder to see

In the next activity *Hitting the Target* students adjust the launching angle to see the many possible paths that the ball can take. These paths need to be viewed as a set, noting how paths are similar or different, when just one variable is changed.

This section can be extended by allowing students to also adjust the initial speed of the ball (release height) to hit the target when it is anywhere in the top right section of the table.

Lesson Introduction

The challenge in this lesson is to investigate the path of a ball when it is launched across a sloping table.

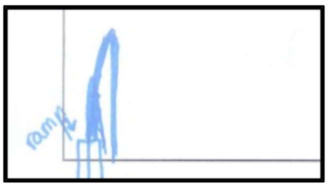
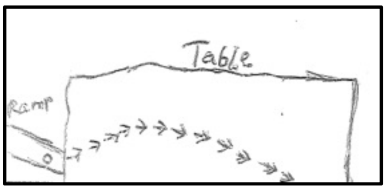
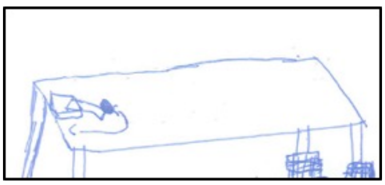
Predicting the Motion of the Ball

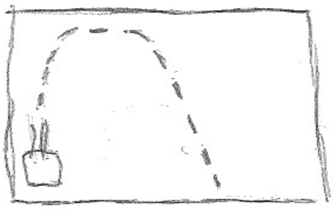
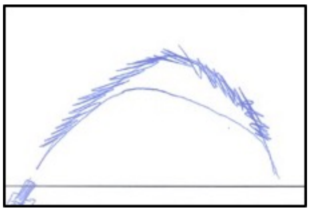
- Ask students to draw what they think will happen when the ball is launched diagonally across the table [Workbook Step 1].
- Students should now write in their workbooks the reason why they think the ball will move as they have predicted [Workbook Step 2].

  You may wish to discuss this with the class before they write anything and/or ask a few selected students to read their answers during the class discussion below.

Class Discussion

- Gather students' predictions for the likely path of the ball by asking some students to draw their responses to Workbook Step 1 on the whiteboard and describe their predictions to the rest of the class. Ask students to also explain why they think the ball will behave as they have predicted.
- There is no need to decide which predictions are better at this point. Note any interesting ideas that are brought up, such as:
  - o The ball starts off in the same direction as the launching track.
  - o A curved path, like when a ball is thrown.
  - o Gravity pulling 'down the table'.
  - o The ball continuing to move to the right 'across the table' (because it started off moving a bit to the right).
  - o Changes in speed as the ball travels.
  - o A few students may believe that the ball will travel in a straight line in the same direction as the launching track and run off the opposite edge of the table. (If the ball is going fast enough when it starts, it could run off the top and its path could look almost straight. Ask the student if they think this will always happen, or if there is anything that could stop it happening.)
  - o Some other typical answers are shown here, with Example 1 showing a relatively common idea that the ball will travel in a curve to the top of the path and then travel straight down the table.

Predictions with explanations		
Example 1		because it is on the angle so gravity would pull it strait down
Example 2		Cos in a arch shape or starts going up and then gravuty will pull it down but still be moving forward.
Example 3		I predict that the ball would roll up about 10 - 15cm then roll on a angle because the launcher is on a angle then that means the ball would roll on a angle

Example 4		I think that the marble will go half way up the table and then go down on a angle. I think the ball will go on an angle because the launcher is on a slight angle. I'm pritty shore that the ball will go down hill because the table is on a slop. I think that ~~a~~ the marble will go fast at the start slow in the middle and then really fast at the end.
Example 5		Because it was already going right it is going to keep going right

Observing the ball

- Ask a student to roll the ball down the launching ramp while the class observes the motion of the ball across the table.

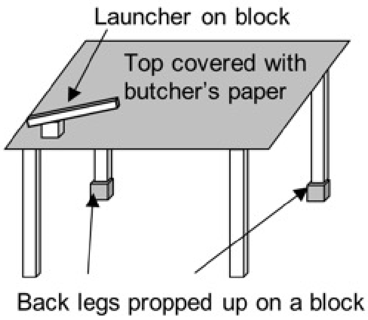
  This may need to be repeated several times to check what happens [Workbook Step 4].

- Ask students whether or not the motion matched their predictions [Workbook Step 5].
  - *Why doesn't the ball go in a straight diagonal line? i.e. why does the ball curve up the table and then down the table?* [Gravity is pulling the ball in the ‘down the table’ direction, so the ball can’t keep going in the direction it was launched at.]
  - *What would happen if the table was level (not propped up)? Why?* [The ball would go in a straight diagonal line in the direction it was launched until it fell off the edge of the table. The weight of the ball would be supported by the table, so gravity would not be affecting the path. Note: if it is suggested that a ball rolling on a flat table would slow down and eventually stop, ask for a reason, i.e. friction from a rough surface causing the ball to stop like in the activity Rolling on a Flat Track in Lesson 4.]
  - *What do you think was happening to the speed of the ball throughout its travel?* [Students may suggest that the ball starts off fast, gets slower in the middle/at the top, then faster again. It is not necessary to give a definite answer here. See Background for more details.]

Setting up Tables in Groups

Explain to students that they will now be carrying out their own experiments in their small groups.

- Ask students to collect all the required equipment [Workbook Step 6].
- Help students set up their tables, checking that the butcher's paper overlaps in the correct direction and that the launching track is placed in the bottom left hand corner [Workbook Step 7].

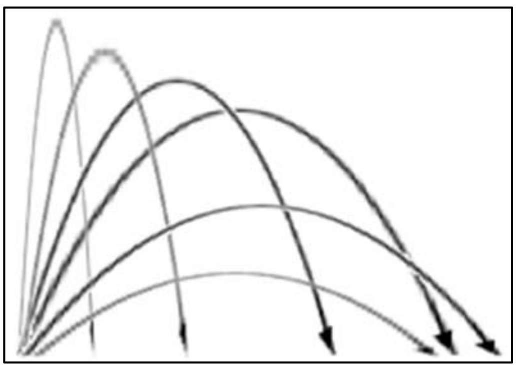


- Ask students to think about a good direction in which to launch the ball so that the path of the ball covers as much room as possible on the table. Tell them that they will be experimenting to find a good direction, and a good release height on the launching track.
    - o The block supporting the launching track (and the back part of the launching track) should always remain in the same position at the bottom left-hand corner of the table, so essentially students are adjusting the angle of launch.
    - o Clarify that they are trying to get arcs that cover a lot of the table.
    - o Each group experiments with the best place to put the front of the launching track (i.e. the direction of launch). Most groups will decide on a direction close to the diagonal.
    - o Students then mark the launching track position (by drawing its outline on the butcher's paper).
    - o Next, students adjust the release height of the ball on the launching track (which affects the intial speed, and how far the ball gets). They are trying to get an arc that finishes somewhere near the bottom right corner.
    - o Students then mark the release height by placing a mark on the launching track.
    - o Explain to students that we are marking the release height so that it can be kept the same throughout the next activities. We will be changing the position of the front of the launching track to see how changing the angle/direction changes the path. In Science it is a good idea to only change one thing at a time, so that we can clearly see its effect.

Tracing the Path of the Ball

- Ask each group of students to choose a Launcher, while the other members of the group become the Markers and line up around the table ready to mark the path of the ball using a pencil and small dots [Workbook Step 9].
- Ask students to create their first coloured trace on the paper by launching the slightly wet ball, marking the lightly drawn pencil dots, and then joining up the dots with a coloured marker, making sure not to rip the paper [Workbook Step 10].
- Members of the group should then swap roles and repeat Steps 9 and 10, using a different position for the front of the launching track, but the same height as marked in Step 8, and a different coloured marker [Workbook Step 11].

  This should be repeated 4 or 5 times, with each group drawing 4 or 5 paths, resulting in paths something like those shown here.



Hitting the Target

This activity familiarises students with the variety of paths that the ball can take. It allows students to observe many trials and to get a feel for the behaviour of the ball. Trying to hit the top left corner should result in motion directly up and down the table, helping students make connections to Lesson 4: Rolling Uphill.

- Tell students that they are going to keep launching the ball but this time they will be trying to hit a target placed on the table. Give a copy of the target to each group and ask students to place their target on the table as shown [Workbook Step 12].
    - They do not need to trace the path any more, so the ball does not need to be wet.
    - They can change the direction of the launching track but should keep the release height the same.
    - The position of the back of the launching track should be kept the same as for the previous activity, so that students can use the traced paths as a guide.

Placing the target on the table

- Once someone has hit the target, the target should be moved to the top left corner (i.e. directly above the launching track). It is important that students try launching directly up the table. If the ball runs off the top, suggest that they try a lower release height (so that they see the ball stop and then go back down).
- Ask students to try other positions on the table, and to work out which positions are possible to hit and which are impossible [Workbook Step 14].
- *Do you think we could we hit these 'impossible' positions if we changed something else about the launching? What would you change and how would it help?* [A different release height gives a different initial speed, which will make the arc bigger overall, so it can reach different areas of the table.]

Class Discussion

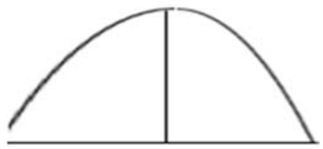
The ideas here are developed further in the next class discussion.

- *Did you notice anything the same about all the paths you have seen?*
    - All/most of the paths were curved.
    - The ball reached a 'high point' each time and then travelled back down.
    - The ball never moved to the left. It kept going to the right.
    - The ball never went straight down from the launching track: it always starts off in the same direction as the launching track.
- *Did you notice any differences between the paths you have seen?*
    - Some paths were stretched further across the table and didn't go as far up the table.
    - Sometimes the ball went all the way to the other side of the table, and sometimes it fell off the bottom of the table.
- *What happened when you launched the ball straight 'up the table' to hit the top left corner?*
    - It went straight up and then off the table. (This will happen if the ball starts off too fast.)
    - It moved in a straight line up and down the table like in the Rolling Uphill activity in Lesson 4.
    - *Why do you think this happened?* [Gravity pulling down the table causing deceleration so the ball gets slower and slower until it stops moving up, and then starts moving down getting faster and faster (accelerating), like the ball in Lesson 4: Rolling Uphill.]
- *Why do you think the ball keeps going to the right?* [In the 'across the table' direction there is only friction to slow it down. The table is quite smooth so there is not a lot of friction, so the ball will keep going for a while. (If the table was very long, the ball would eventually stop, but it wouldn't ever go back to the left.)]

What Does the Path of the Ball Look Like?

The focus of this part of the activity is on the shape of the ball's path and the factors affecting that shape.

- Select one traced path per group for the students to cut out. Select a variety of paths to facilitate discussion of their similarities and differences. Include one path that is similar to the example shown below, to serve as an average or typical example.
- Ask students to mark the highest point of the ball's path and also mark or crease their butcher's paper along the bottom horizontal edge of the table. (These marks are necessary as after the paper has been cut it is very difficult to determine these features.)
    - If students have used two sheets of butcher's paper, ask them to use masking tape to carefully tape them together.
    - Students should now cut the butcher's paper along the curved path-line of the ball & along the mark or crease at the bottom.
    - The cut-out path should then be folded along the perpendicular line from the highest point to the mark or crease at the bottom of the table.
    - Students should then unfold their cut-out paths and stick them on the board [Workbook Step 15].
    - A typical cut-out path is shown here. The left-hand of the path starts slightly above the mark or crease at the bottom of the table and the shape should be close to, but not exactly, symmetrical.



An example of a cut-out path

Class Discussion

The discussion should first draw students' attention to the shape of the paths, then to the forces affecting the paths and the likely acceleration and speed of the ball.

- Focus on the most 'typical' path on the board. Ask students to describe what the path looks like. [For example: an arc, an arch, symmetrical, a parabola.] Ask a student to check how symmetrical the path is (by folding it in half).
- Ask students whether they have seen similar shapes before & if so, where. [For example: buildings, bridges, water fountains, the path of a ball on television.]
- *We talked earlier about the similarities in the paths. Can anyone point out a similarity in the paths AND explain what they think causes it?*
    - Every path has a high point where the ball turns around and starts going down again. I think this is because gravity is always pulling the ball down the table.
    - All the paths are symmetrical (or almost symmetrical). I think this is because gravity is always pulling down with the same size force, so the 'up the table' part looks like the mirror image of the 'down the table' path, a bit like the graphs in Lesson 4: Rolling Uphill and in Lesson 2: Rolling Downhill. (Note: Friction is slowing the ball down slightly over the whole path. This means that the ball doesn't travel as far across the table in the second 'half', making the paths slightly asymmetrical.)
    - The ball always starts off in the direction of the launching track. I think this is because when it is on the track it can only go in that direction. When it leaves the track it can move in any direction on the table.
    - The angle the ball finishes at when it fall off the table is very close to the angle it started at. I think this is for the same reason that the path is close to symmetrical.
    - The ball never goes to the left. I think this is because gravity only pulls down the table, so it doesn't affect how the ball moves across the table. In the across the table direction, friction is pushing back on the ball, but it is only small so it is only a small deceleration. Also, if friction made the ball slow down and stop, it still wouldn't go back to the left because friction happens when the ball is moving (or trying to move).

- *Can you point out a difference between the paths AND explain what you think causes it?*
    - Some paths are stretched further across the table and don't go as far up the table. I think this is because the launching track was at a low angle (i.e. pointed more across the table than up the table), so more of the ball's speed was directed across the table. Gravity still made the ball turn around and go down the table, but the ball got further along the table before the turn happened.
    - Some paths go far up the table but don't go very far across the table. I think this is because the launching track was at a high angle (i.e. pointed more up the table than across the table), so more of the ball's speed was directed up the table. The ball got further up the table before gravity slowed it down enough for it to turn around and go back down the table.
    - *Did anyone try launching the ball directly 'across the table'?* (i.e. horizontally) *What happened/what do you think would happen?* [The ball would start off moving horizontally, and then curve downwards until it fell off the table. In this case, the 'high point' of the path is at the start, just where it left the launching track.
- *How do you think the speed of the ball changes on each path? We haven't measured the speed or the time, so just think about what you have observed, and about what happened in previous lessons.*
    - *When do you think the ball might be moving fastest?* [Approximately at the start and the finish of the path]
    - *When do you think the ball might be moving slowest?* [At the highest point]

Student Reflection

- Ask students to write what they have learned about how the ball moves when it rolls on a sloping table and why [Workbook Step 16].

  You may wish to discuss this with the class before they write anything and/or ask a few selected students to read their answers afterwards.

  Some sample responses are shown here:

> A ball will decellerate and then accellerate and that a curve in scientific language is called a "parabola"!

> I've learnt that ~~ball~~ the speed of the ball & the angle of the table with the launcher work hand in hand.
>
> I also learnt what a parabola is.

> Today I found out that when you change the angle of the track the ball (marble or orange bouncy ball) will either go higher or lower, wider, thiner ext. I think this is because when the angle of ~~ball~~ track moves it might gane exaliration. And the table being on a slop makes it go down hill

> I found out that angles matter alot if you are rolling a marble or ball to a specific spot. Also if the table was flat the ball or marble would just go straight up then back down onto the launcher. But if put on a slope & a angle the ball / marble would go up then roll back down on a angle in what angle the launcher is in.

Some sample responses

Planning a Project

- Suggestions for projects for students to complete are given in *ST3_Motion_Projects.pdf.*