re(Solve) MATHS BY INQUIRY

Lesson 3: Falling Balls

Australian Curriculum: Mathematics — Years 5 and 6

ACMSP118: Pose questions and collect categorical or numerical data by observation or survey.

ACMSP119: Construct displays, including column graphs, dot plots and tables, appropriate for data type, with and without the use of digital technologies.

ACMNA131: Make connections between equivalent fractions, decimals and percentages.

- Connecting fractions, decimals and percentages as different representations of the same number, moving fluently between representations and choosing the appropriate one for the problem being solved.

ACMMG135: Connect decimal representations to the metric system

- Recognising the equivalence of measurements such as 1.25 metres and 125 centimetres.

Australian Curriculum: Science — Years 5 & 6

ACSIS090: Construct and use a range of representations, including tables and graphs, to represent and describe observations, patterns or relationships in data using digital technologies as appropriate.

- Constructing tables, graphs and other graphic organisers to show trends in data.
- Identifying patterns in data and developing explanations that fit these patterns.

ACSIS218: Compare data with predictions and use as evidence in developing explanations.

- Sharing ideas as to whether observations match predictions, and discussing possible reasons for predictions being incorrect.

Lesson abstract

In this lesson students discover the height from which a ball needs to be released for it to take 1 second to hit the ground. Students use a variety of measuring devices to find the heights for a half second and a quarter second drop and use their results to predict and then measure the height for a 1 second drop.

Mathematical purpose (for students)

To predict, measure and interpret the motion of a falling ball.

Mathematical purpose (for teachers)

The focus of this lesson is that objects that are falling just under the influence of gravity experience acceleration. Students will see that increasing the falling time by increments of 0.25 seconds requires increasing the distance by amounts that get much bigger each time. Gravity as a force is consolidated. They graph the distance a ball falls in given times, and make a streamer graph which shows that the ball is accelerating. Students learn that experimental data represented graphically can be used as a model to predict the results of further experiments.

Lesson Length 90 minutes approximately

Vocabulary Encountered

- Experimental error

Materials — see Teachers' Guide: Appendix B for full details

- Stopwatches/tablets/g-ball, tennis balls, measuring tape, streamers etc.
- Workbook *ST3_Motion_Y5&6_3a_Workbook.pdf* (1 per student)
- Video for teacher: Video ST3 Motion 3b Falling Balls 290818

We value your feedback after these lessons via https://www.surveymonkey.com/r/JJCGHVX

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Australian Academy of Science

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Background

The challenge in this lesson is “From how high do you need to drop a ball for it to take 1 second to hit the ground?”

There is further background information about the equipment and about gravity in the Modelling Motion Teachers’ Guide (*ST3_Motion_Teachers_Guide.pdf*) and in earlier lessons.
A video of key points for conducting the lesson is available at [Video ST3 Motion 3b Falling Balls 290818]

Students drop a ball from various heights and time how long the fall takes. Free falling objects (i.e. falling without being touched by anything) experience acceleration because of the force of gravity. This acceleration is shown by the rapidly increasing distances the ball can fall in small increases in falling time. In theory a ball will fall a total of about 30 centimetres in 0.25 seconds, about 120 centimetres in 0.5 seconds, about 270 centimetres in 0.75 seconds, and about 490 centimetres in 1 second! This dramatic increase is clear in the graphs the students draw.

The heights involved mean that you need a step ladder to mark the 270 centimetre height for the 0.75 second fall, and an upstairs balcony or stairwell with a drop of at least 5 metres (preferably more) to allow the ball to fall for 1 second.

During the lesson, students can all participate by measuring the times of the falls using a variety of devices, including stopwatches (including on phones and tables), from a video, or using a special ‘g-ball’, depending on availability.

When Galileo investigated this acceleration, he used ramps (like in Lesson 2) to slow down the motion because it was hard for him to measure the time accurately. We have a similar problem: even using a good stopwatch will be inaccurate for the shorter times, as human reaction time is similar to the times being measured!

Accuracy for this experiment can be increased by using a g-ball. This is a ball with onboard electronics which measure the time from release to hitting the ground https://www.arborsci.com/g-ball.html). Alternatively, video the fall and use video editing software to find the elapsed time.

Using the video software

We suggest using *CoachMyVideo*, *BSPlayer or VideoPad NCH* for ipads, tablets or computers.

- If you can play back frame by frame, find the number of frames per second when recording (usually 30). The time for the drop can be calculated by counting the number of frames from the initial drop to the time the ball hits the ground. When recording at 30 frames per second (30 fps), one frame is one thirtieth of a second. Divide the number of frames by 30 to get the time in seconds: 3 frames equal 0.1 seconds, and 8 frames would be approximately 0.27 seconds.
- Experience from trialling suggests that students find it easy to download and use the software, but may need help to understand how to do these calculations.
- If frame by frame playback is not possible, students can identify the frames when the ball starts and finishes, then calculate the difference in displayed times.
- If you have a choice, pick video software that gives greater accuracy (e.g. to a tenth of a second for tablets).

Accuracy and error

Performing the drop and measuring the time taken to fall will raise issues about fair tests and experimental error. When we say a measurement is accurate, we mean that it is very close to the real thing being measured. Accuracy in experiments is about whether we managed to measure what actually happened. ‘Error’ means the difference between what happened and what was measured.

Sometimes it is very difficult to measure accurately. Scientists must design experiments so that it is as easy as possible to get good measurements. Some error in this lesson is due to unavoidable factors such as human reaction time. Students will develop rules to follow to increase the accuracy of their measurements and the consistency of their repeated experiments. These rules may involve taking the average of several time measurements, ignoring measurements that seem ‘way off’ (outliers), and making precise instructions for how to release the ball.

Measuring the Time Taken to Fall

Introduce the challenge for this lesson: "From how high do you need to drop a ball for it to take 1 second to hit the ground?"

In the first part of the lesson, students are introduced to measuring time in tenths and hundredths of a second using different equipment, and they develop procedures to increase accuracy.

Different Ways to Measure the Time

Measuring time with stopwatches and tablets

- Ask a tall student to drop a tennis ball from as high as they can while the rest of the class times how long it takes to hit the ground using stopwatches, tablets or smart phones (not video).
    - Record and compare different results.
    - Discuss sources of differences, e.g. different reaction times, difficulty seeing exactly when the ball is released or hits the ground.
    - Minimise the effect of reaction time at the start of the drop by saying 'Ready, Set, Go!'
    - Repeat until there is some agreement.
- If using a g-Ball, introduce it as a ball that measures the time it has been falling to the nearest hundredth of a second. It is like a specialised stopwatch.
    - Demonstrate how to 'set' the ball and show how the timer stops when the ball hits the floor.
    - *How do we read the time taken to fall from the g-Ball?* [The g-Ball is like a stopwatch. The time between being released and hitting the ground is displayed in tenths and hundredths of seconds.]
    - To avoid breakage, the g-Ball should be released with the timer at the top, and onto carpet.
    - For maximum accuracy, the student releasing the g-Ball should squeeze the button and release it gently, releasing the ball and the button simultaneously.

Using *CoachMyVideo*

- If using frame-by-frame video analysis software, such as *CoachMyVideo* or *BS Player*, introduce it to students and explain how it will be used to measure the time taken to drop the tennis ball to fractions of a second.
    - Students will need time to download the software and may need to practise recording video.
    - *How do we work out the time taken to fall from the video?* (If using video analysis software) establish that all students understand how to convert from the number of frames per second to hundredths of seconds.
- Let different students try drop the g-Ball or tennis ball, ensuring the ball is dropped, not thrown down.
    - Ask students to estimate how long the ball will take to fall for each student's dropping height.
    - Record and compare results from different students and different devices for each drop.
    - If using the tennis ball, here is a good place to decide how to get a consensus from all the different stopwatches, tablets, and videos that are measuring the fall time. Suggestions include taking the average of the measurements after removing any that are 'way off'.

Reducing experimental error

Next, students develop class protocols for better experimentation and more accurate measurement.

- Choose a student to drop the ball from any height they choose. (It could be 'from about the height of this table' or 'from 1 metre' or just dropped without measuring.) Record the time on the whiteboard.
- Ask another student to try and repeat the experiment. They are trying to drop from the same height and get the same time. Record the time. Ask a third student to try as well, and record the time.

- *Was it easy to repeat the drop and get the same time? How could we make it easier?* Suggestions include:
  - Mark the height. We could put a long strip of paper down the wall, so we can draw on it.
  - Give detailed instructions, e.g. 'start with the bottom of the ball lined up with the top of the table' or 'hold the ball by putting your hand over the top, then release it by lifting your fingers'.
  - Practise releasing the ball so that everyone knows how and is doing it the same way.
- *Why is it important to be able to repeat an experiment?* [If your experiment gives important or surprising results, you will need to be able to show it to other people to prove what happened. Also, if the experiment can be done over and over, you can be sure that what happened wasn't just an accident.]
- Choose a student to drop the ball again, this time using the suggestions above to improve the repeatability.
- Ask a few other students to repeat the drop, and compare the times. *Are they closer than before? Are they close enough? Is there anything else we could improve?* [There will always be some error, but it is a good idea to be thinking about ways of reducing it.]

Dropping the ball from a fixed height

Height for a 0.5 and 0.25 Second Fall

- Explain that before we find the height needed for a 1 second fall we are going to start with something smaller.
- Choose someone to mark a height on the wall from which they think the ball will take 0.5 seconds to reach the ground. Measure this height and record the fall time. (Use sticky page markers to mark the height, or attach a long strip of paper to the wall for students to write on.)
- Now have everyone write their own estimate [Workbook Step 1].
- *What was your estimate? Why did you choose this?* [For example, if the first estimate gave a fall time of 0.42 seconds, it makes sense to choose a bigger height.]
- *Which estimate should we try next?*
  - Let a chosen student experiment to find the height for 0.5 seconds by trial and error.
- Mark and label that height on wall. Students record it in their workbooks [Workbook Step 1].
- Choose another student and repeat the whole procedure for a fall of 0.25 seconds [Workbook Step 2].
- Ask students if twice the time means twice the distance. [It is a lot more than twice the distance. At this point in the lesson there is no need to discuss why this is the case. For more details, see Teachers' Guide.]

Height for a 1 Second Fall

- Ask for suggestions for the height required for a fall to take 1 second.
  - *Will it be a little higher, or a lot higher? Why?* [The height for 0.5 seconds was more than twice the height for 0.25 seconds, so we could expect that the height for 1 second will be more than twice the height for 0.5 seconds.]
- Ask students to record their estimates [Workbook Step 3].
- Choose a student to test their estimate of the height to fall 1 second. The step ladder will be needed.
  - Students will discover that there is not enough height in the room.
  - Suggest that they find the height to fall 0.75 seconds instead. (This can still be done in the classroom.)
  - Let the student experiment to find the height for 0.75 seconds by trial and error.
- Ask students to record their results for times of 0.5, 0.25 and 0.75 seconds in the table [Workbook Step 4].

- Go to a place where there is enough height to drop the ball from at least 5 m [stairwell or balcony — see examples below].
    - o Set up the 5m or 8m tape measure.
    - o Let the students experiment to find the height for 1 second by trial and error.
    - o Ask students to complete the table [Workbook Step 4].

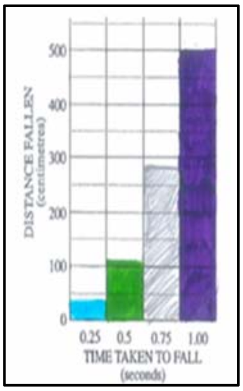
TIME TO FALL (SECONDS)	DISTANCE FALLEN (CENTIMETRES)
0.25 secs	35 cm
0.5 secs	110 cm
1 secs	500 cm
0.75 secs	280 CM

One student's table

Different places teachers found to use for the 1 second drop

Modelling the Motion of the Falling Ball

- Ask students to draw the bar graph of times and heights [Workbook Step 5] and write what they think is happening to the speed of the ball [Workbook Step 6].
    - o Perfect data would have the distances in the ratio 1:4:9:16, but the experimental data may only show a rapid increase.



One student's graph

Class Discussion

- Drawing graphs to scale can be used to gauge students' proficiency in graphing. Students' reasoning and communication skills are involved in interpreting the results and creating (possibly informal or intuitive) models for the motion — such as 'for each extra 0.25 seconds the extra distance fallen keeps getting bigger and bigger'.
- Ask some students to share their responses to Workbook Step 6. Focus on responses that suggest the ball is accelerating (getting faster and faster). Ask students to explain how the data or graph shows that the ball is accelerating.
    - o Students may reason that if the ball was moving at a constant speed, then it would go twice as far in twice the time, i.e. If the ball was going at the same speed then the distance for 0.5 seconds would be twice that for 0.25 seconds, but it is much bigger than this.
    - o Students might comment that the columns in the graph are getting bigger and that this means the speed is getting bigger. However, in this graph the columns do not show the speed. It is important at this point to clarify the difference between the column graph they have drawn and the streamer graphs from Lessons 1 and 2 (see next point).

It slowly gets faster and faster then it hits the ground but the reason it gets faster is because of the gravity pulling it.

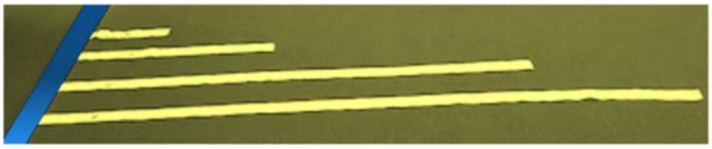
One student's explanation

- *Is this graph similar to one you have made in previous lessons? How is it similar? How is it different?*
    - o Similarities to the Green graph in Lesson 2 include:
        - ▪ Shows distance and time.

        - The columns get longer.
    - Differences include:
        - The time goes up by 0.25s each column. In the Green graph each column was a whole second.
        - In this experiment the ball went (500cm) in one second. In Lesson 2 the ball went a much shorter distance in the first second (the length of the first strip on a Green graph) which made it easier to measure.
        - (Key point) The 'distance' and the 'time' mean different things in the two graphs. In this graph each column is the *total distance travelled* in 0.25s, 0.5s, 0.75s and 1s. In the Green graph, each strip was the *distance travelled in just that second*.
- *How could we make this data into a streamer graph like the Green graph from Lesson 2? (i.e. How could we show the speed?)*
    - *We can imagine that we have a metronome that ticks every 0.25 seconds, and very quick Markers to mark where the ball is at every tick. One of the markers would have to be on the step ladder! What do you think the streamer would look like after we marked it?*
        - Key point: Students will need to realise that the first 0.25 seconds of a 1 second fall will look just the same as a completed 0.25 second fall.
    - We could think of the last column of the graph we drew as a streamer, and the tops of the other columns are where we would mark the streamer to cut it. (Imagine the streamer has been put upside down.)

Making a streamer graph

- Lay out a streamer on the floor and cut it to the length of the 1 second fall.
- *Thinking about our 1 second fall, how far do you think the ball fell in the first 0.25 seconds?*
    - Establish that it is reasonable to assume this would be the same distance as the column for 0.25 seconds.
    - Mark this length on the streamer & label it as the first strip.
- *Still thinking about our 1 second fall, how far do you think the ball fell in the NEXT 0.25 seconds?*
    - This is the difference between the 0.5s column and the 0.25s column.
    - Instead of calculating the difference, it is easier to just measure out the 0.5s distance from the start of the streamer and put a mark there.
- Repeat for 0.75 seconds.
- Cut the streamer into the 4 pieces and create a streamer graph on the floor.
    - The length of the strips should be increasing by close to a constant amount – of course this will depend on the accuracy of the data. Perfect data would show the strips in a ratio of 1:3:5:7.



A streamer graph on the floor

- Alternatively, students can find the distance travelled in each 0.25 second interval using the table from Workbook Step 4. They will need to put the times in increasing order and add an extra column to show the differences between the distances.

Class Discussion

- Discuss with students how each of these columns can be thought of as the speed of the ball for that time interval, just like the Green graphs, but the units will be a bit unusual: 'centimetres per quarter second'.
- Students should be able to recognise that the increasing strip lengths in this graph directly show that the speed is increasing (so the ball is accelerating), or 'going faster and faster'.
- *Where is the ball travelling fastest?* [At the end of the fall, just before it hits the floor]

- *If we drop two balls, one from (500cm) and one from (110cm), which one will be going faster when it hits the ground? Why?* [The ball dropped from higher, because it has had more time to get faster and faster.]
    - What causes the ball to speed up? [Do not suggest gravity but let all suggestions be aired and let students explain their ideas. Rather than just using the magic word 'gravity', encourage students to talk in terms of the ball being pulled towards the ground, i.e. a force.]
    - Discussion may reveal that some students believe, correctly, that gravity is the force involved; other students will not yet have a well-developed notion of force. Be aware that students may use the word 'gravity' as an explanation without understanding that it is a force, as they may have been rewarded in the past for this 'magic' word.
- Ask students to record their explanations [Workbook Step 7].

Prediction and Reflection

Estimating the Height for a Fall of 2 Seconds

- Ask students to think about (and discuss with a partner) the height from which they think the ball will take 2 seconds to reach the ground.
- Ask students to record their estimate and write their explanations [Workbook Step 8].

Class Discussion

- Discuss students' estimates and explanations. What model did they use to arrive at their estimates? That is, what pattern did they see in the heights and times so far, and then use to predict a new height? For example:
    - The columns on the graph get bigger and bigger.
    - The last column on the graphs shows about 2m higher than the one before, so maybe about 7m — i.e. add 2m. (This does not take into account that 2 seconds would not be the next column.)
    - Each column gets bigger by more each time, so I added a bigger amount for each column. So 1.25s might be 8m, 1.5s might be 12m, 1.75s might be 17m, and 2s might be 23m.
    - (Correct but probably unlikely at Years 5 & 6) Doubling the time seems to multiply the distance by four). So, the height for a 2 second fall will be about 20m.

Student Reflection

- Ask students to write what they have learned about using graphs to understand and make predictions about the motion of a falling ball by doing this activity [Workbook Step 9].
    - You may wish to discuss this with the class before they write anything and/or ask a few selected students to read their answers afterwards.

Just for interest: the 12 second drop

The Milford Track, in the South Island of New Zealand, goes over Mackinnon Pass. The pass is 1154 m above sea level and it is the highest point on the track, although the nearby mountains tower above it. At the top of the pass, the track passes a cliff called the "12 second drop". It is said that a stone dropped from the top would take 12 seconds to hit the bottom. Images of the track and the 12 second drop can be found on the web.

How high is the cliff above the valley floor?