Quadratic Functions
Lesson 3: Linking Different Forms

Australian Curriculum: Mathematics (Year 10)

ACMNA239: Explore the connection between algebraic and graphical representation in relation to simple quadratics … using digital technology as appropriate.

- Sketching graphs of parabolas.
- Applying transformations, reflections and stretches to parabolas.

Lesson abstract

Students compare different symbolic forms for the same quadratic function using GeoGebra, and then by hand sketching. Further practice using text book type examples is suggested for fluency.

Mathematical purpose (for students)

Algebra, graphs and tables work together in maths.

Mathematical purpose (for teachers)

Transforming an algebraic expression can reveal different critical features of a quadratic function.

Lesson Length 50 minutes approximately

Vocabulary to Reinforce

- quadratic function, parabola
- parameter, variable, constant
- translate, reflect, dilate, shift
- turning point
- function rule
- axes, x-axis, y-axis
- x-intercept, y-intercept
- symmetric, axis of symmetry
- maximum, minimum

Lesson Materials

- Access to Geogebra or similar software
- Student Sheet 1 – Linking Different Forms (1 per student)
- The worksheet has small grids but additional graph paper would be helpful for some students

Linking Abstract Representations

This lesson requires students to use alternative transformations of quadratic function rules and tables of strategic values to approach "by-hand" graph sketching.

Instructions for students are on Student Sheet 1 – Linking Different Forms.

- Task 1: students work with technology to revisit the role of various forms of the quadratic rule and the information provided by the various parameter.
- Task 2: Students are encouraged to work technology free to identify critical values and sketch quadratics. Technology can be used for checking or to support students having difficulty. Students should be encouraged to be judicious in their use of technology and to consider the more general question of when technology is helpful and when it is more efficient or more satisfying to work by-hand or in your head.

Conclusion

Draw the lesson together by reviewing the tasks in the student work sheet OR looking together at the new problem below that draws on the students learning from those tasks.

Problem:

A. $p=(m-5)(m+7)$
B. $p= m^2 +2m-35$
C. $p=(m+1)^2-36$

1. What do the functions described by the rules in A, B and C have in common?
2. Explain the steps you would go through to sketch the graph of one of these functions.
3. If we restrict the domain to $m>-7$ these functions approximately modelled the profits (p) from my start-up company in terms of the number of months (m) it has been running (where zero is today and the negative are past months). When was I making the greatest loss? When do I expect to start making a profit?

Further Activities

At this stage students should be well equipped to tackle the standard text book quadratics questions for further practice or challenge.

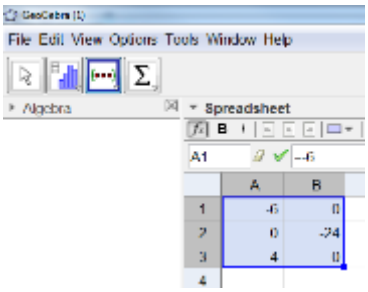
Linking Different Forms

Name: _____

Explore the graphs of quadratics using tables of values, points, parabolas, function rules in 3 different forms, critical values, axis of symmetry. Think about when technology makes things easier and clearer and when it seems like a hurdle.

TASK 1: Technology in focus

Open Geogebra with Algebra, Spreadsheet and Graphics windows showing. The following instructions (left to right) will help you create a quadratic function and label its features.

Start by working with a simple quadratic function rule e.g. y=(x-4)(x+6)	
Work first in the spreadsheet to make a table of values.	Based on the rule above enter -6, 0 and 4 in the A column (Why choose these numbers?)
Remember *x* and *y* are variables. Column A contains some examples of values of *x*, the rule produces the related value of *y*	Enter the rule into B1 =(A1-4)(A1+6) Drag the rule down column B to B3
Link the table of values to points in the graphics window You will need to **Zoom out** and move the graphics canvas to see all three points.	Highlight your table of values and select List of Points from the drop down menu  Leave them as **dependent objects** and select **Create**
Move to the graph window Write down the expanded version of your rule that appears in the Algebra window Write down the turning point form of your rule Link to your table of values by entering -1 in your A column and dragging the rule down column B.	**Input** your quadratic rule to see the parabola Right click on the expanded rule and select turning point form Place a point at the Turning Point using the Input line P=(-1,-25)

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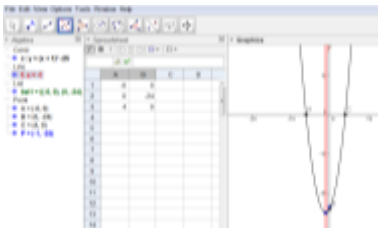
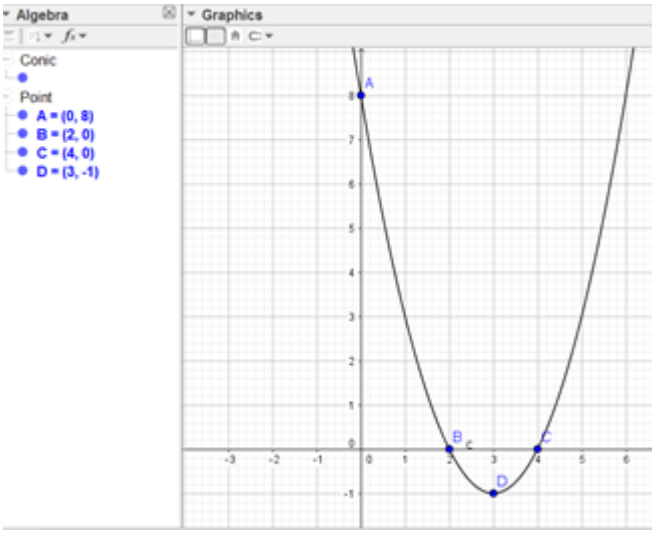
Mark the axis of symmetry in red	In the Graphics window select perpendicular bisector form the drop down menu then select the *x*-intercepts (points A and C) 
Invent a story to give context to this quadratic function and its critical values. Perhaps a science fiction planet where gravity works in reverse! Perhaps a construction, bridge or sculpture…	

TASK 2 Technology in the background

Work with pencil & paper - Use technology to check your sketch or if you are stuck.

Here are 3 equivalent versions of a particular quadratic function rule. From this information we know 4 points that lie on the graph of this function. From $y=x^2-4x-12$ we know (,) From $y=(x+2)(x-6)$ we know (,) and (,) From $y=(x-2)^2-16$ we know (,) Rule and label the axes and mark a suitable scale for sketching this graph. Plot the four points and sketch the graph. Use Geogebra to check your work.	

Using the information from the graph and the table of values provided by the points find the function rule for the parabola shown opposite. Explain your thinking so that someone else would know how to do this.	
Sketch the graph of $y=-(x+1)^2+3$ Explain your thinking so that someone else would know how to do this.	
Sketch the graph of $-\frac{1}{2}(x-2)^2-9$ Explain your thinking so that someone else would know how to do this.	