BRINGING THE REAL WORLD INTO ALGEBRA

Quadratic Functions

Lesson 1: Fitting Curves

Australian Curriculum: Mathematics (Year 10)

ACMNA239: Explore the connection between algebraic and graphical representation in relation to simple quadratics … using digital technology as appropriate

- Sketching graphs of parabolas
- Applying transformations, reflections and stretches to parabolas.

Proficiencies

- understanding … making the connection between equations of relations and their graphs
- fluency includes factorising and expanding algebraic expressions, using a range of strategies to solve equations
- problem-solving using algebraic and graphical techniques to find solutions to simultaneous equations

Lesson abstract

Students are introduced to GeoGebra (or alternative software), by fitting lines to a digital image. This is extended to curve fitting using the turning point form for quadratic functions. Students make systematic adjustments to the function rule to fit the curve to an image of water from a hose or the path of a ball.

Mathematical purpose (for students)

Parabolas can be seen in the real world.
$y=a(x-g)^2+h$, or turning point form, is helpful for describing the reflection, translation or dilation of the parabola $y=x^2$ needed in order to draw a quadratic curve of a particular size and location.

Mathematical purpose (for teachers)

At the end of this lesson students will be able to:

- Provide examples of where parabolas may be seen in the real world.
- Describe a quadratic function algebraically.
- Use technology to explore the impact of changing the parameters in $y=a(x-g)^2+h$.
- Articulate the link between the graphic and algebraic representations of quadratic functions under transformations to reflect, translate, or dilate.

Lesson Length 2x50minutes approximately

Vocabulary Encountered

- parameter
- variable, constant
- translate/shift
- reflect, dilate

Lesson Materials

- Student Sheet 1 - Images and Graphs (1 per pair)
- Student Sheet 2 - Curve Fitting (1 per pair)
- Photos supplied in *ST2_Quadratic_1a_Images* OR use your own OR have students take their own photos.

We value your feedback after these lessons via https://www.surveymonkey.com/r/RKRDYBW

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Familiarisation with the Software

If students are new to using Geogebra (or your alternative software) or even new to using this software to import images and/or input and edit function rules then they you need to start with Student Sheet 1 - Images and Graphs.

The task:

- Is designed to give students some familiarisation with the software.
- Introduces inserting images and line fitting by working with familiar graphs of linear functions.
- Is designed to be done with students supporting each other by working in pairs.
- It includes help for students who have difficulty with linear functions by offering an initial "black box" approach by inserting and dragging a line into place. You may wish to edit before printing for students.
- Offers a challenge to more able students by introducing the notion of a restricted domain. This is not essential for the work on quadratics. You may wish to edit before printing for students.

Bring the students together for very brief discussion of any common difficulties with the software and, most importantly, drawing attention to the role of parameters in linking the symbolic and graphic representations of functions.

Curve Fitting

The purpose of the activity set out in Student Sheet 2 – Curve Fitting is that students:

- use Geogebra to import an image and fit a parabola by systematic trial and error
- describe the translations of $y=x^2$ indicated by the parameters g and h in $y=a(x-g)^2+h$

Students fit a graph to the curve of water from a hose, then other shapes that may or may not be modelled by a parabolic shape; a tropical palm frond, an egg, a banana. You may want to replace these with other local images.

Getting started

Introduce parabolas in the real world and make the link between the terms "quadratic function" and "parabola":

- Show photo of water from hose.
- Ask questions such as:
  - What shape is the curve?
  - What if someone wants to reproduce that curve?
  - What if I want to make something that shape?
  - What if I want to program a machine to make lots of objects? We need to find an algebraic rule that a machine could use to reproduce that curve.

    OR
  - Why is the curve that shape?
  - What else might follow a curve of this shape? Why?

Set students the task of exploring the link between symbolic quadratic functions rules and the graph of a parabola by working on Student Sheet 2 – Curve Fitting.

Enabling Prompt

- Start by entering $y=x^2$ How could you reflect this parabola in the x-axis?
- How could you move the parabola up/down?

Extending Prompt

- How might the curve change if the water came out under greater pressure?
- What makes the graph of a quadratic different from the graph of other functions? What are its characteristics?
- What points on the graph do you think give us the most useful information for graphing?

Conclusion

Rough script including expected student responses

If possible project the Geogebra image and rework together the basic path of the water from the hose

Keep track on the board of the various suggestions for changing the function rule and draw attention to the need to be systematic and note the impact of changing each parameter.

Start with $y=x^2$

What's wrong? Upside down…too far down

What could we do? Try a transformation. Let's flip it, reflect it in the x-axis

Try $y=-x^2$

too far down – move it up

Translate it… how far up?

6 (what ever they suggest)

OK let's translate that up by 6 so now let's try $y=-x^2+6$

It's too steep / it's too far over etc.

Make appropriate translations or dilation /stretch following students lead

Celebrate a good model

Recap – using the pattern $y=a(x-g)^2+h$

- How did we reflect/flip the graph upside down?
- Which parameter/ letter translated/moved the graph vertically/up and down?
- Which parameter/ letter translated/moved the graph horizontally/sideways?
- Which parameter / letter dilated/stretched the graph? [$0<|a|<1$ flatter, $|a|>1$ steeper]

Check question: Choose the correct word or number from the alternatives in **bold:**

- The curve for $y=(x-3)^2-5$ would look like the curve for $y=x^2$ except translated (**up or down**) by (**3 or 5**) and (**left or right**) by (**3 or 5**).

Further discussion (if time permits):

- What's special about the graph of a quadratic function?
- Which points on the graph do you think are particularly important? Follow up further next lesson.

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Images and Graphs

Name: ______________

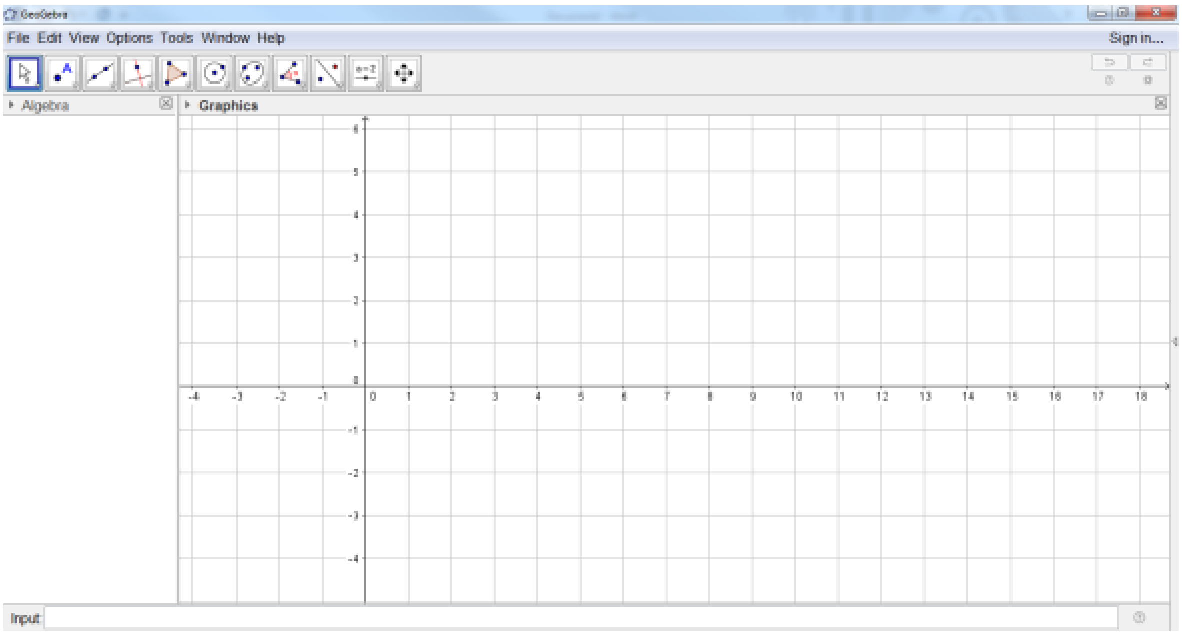
Learning to use Geogebra

This is a photo of part of Ballarat’s traditional railway gates. Your task is to become familiar with Geogebra by fitting lines along the cross of the gates.

Open Geogebra

Expect to see:

- Menu in words across the top [File, Edit etc]
- Menu of icons starting with an arrow [for pointing at things]
- Narrow Algebra window on the left, wide Graphics window on the right
- Input line at the bottom of the screen



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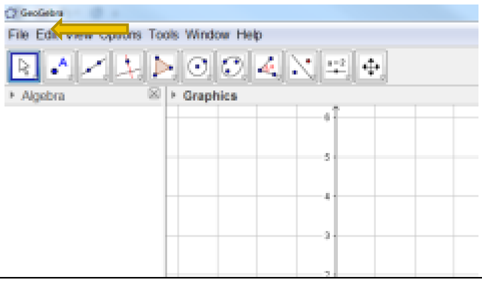
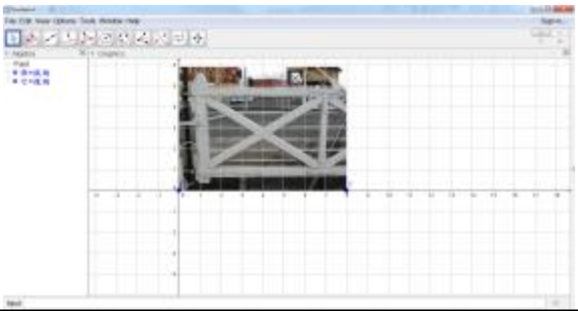
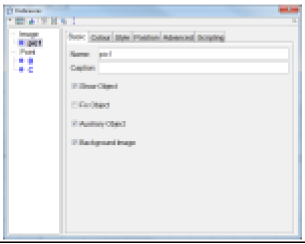
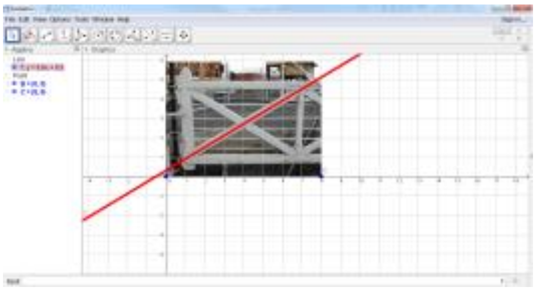
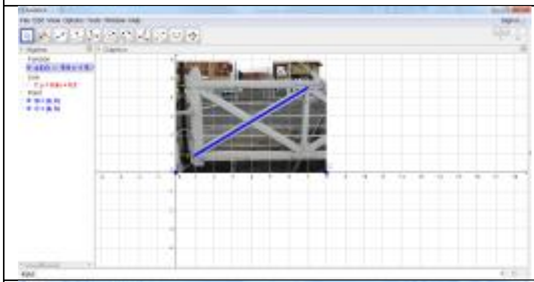
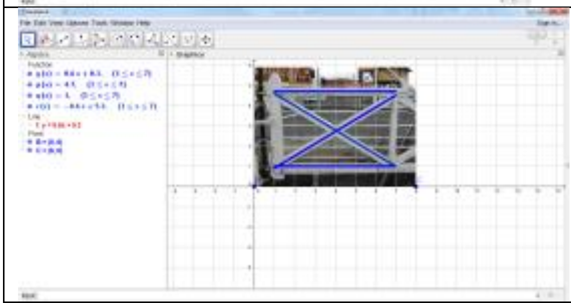
	If the grid is not showing in the graphics window click on **Graphics** (top left of graph window) and select the grid option. Click on EDIT or the second last drop down menu icon and select **Insert Image** Select required photo file from the photos file list
	Use the pointer and the blue points at the base of the photo to drag it to a convenient position (e.g. corner at origin or image symmetrical about y-axis)
	When in position, right click on the image select **Object Properties** then **Fix Object** and **Background Image**
	Enter function rules via the input line. I started with y=x. Next systematically try different values for the parameters “m” and “b” in *y*=m*x* +b You can delete the lines you don’t want or “turn them off” by clicking in the algebra window. Right click on the rule that appears in the algebra window to choose the symbolic form *y*=m*x* +b Once I had the line I thought best I chose to right click on this in the algebra window and edit its colour and style properties. Drag the rule from the algebra list onto the graph canvas to label your line. Now find a rule that will give you the line through the x from top left to bottom right.
STUCK? Having trouble? Forgotten about gradient and intercept of straight lines? Here is some help.	
	Choose Line from the lines menu (3rd icon from left). Place one point on the brace and then a second. The line will pass through these points. A function rule will have appeared in the algebra window. Right click on this and choose the “Equation y=mx+b” format. You can change the rule for this linear function by dragging the line OR you can enter a function rule in the input line at the bottom of the screen. Try this out.

	When you have a line that is a good fit along the slope of the slide drag the rule from the algebra list onto the graph canvas to label your line. Now try entering a rule for a line along the brace in the other direction.
Need a challenge? Keep going to draw lines in colour over the whole structure of this gate.	
	To show only the section of the line required to model the image restrict the domain by naming start and end x values . The syntax is: Function[<Function>, <Start x-Value>, <End x-Value>] In my example in the input line I typed: Function[0.6x+0.3,1,7]
	Note that we cannot draw in vertical lines this way because these are not functions (eg In this example for the vertical line, x=1 there are an infinite number of y values – not just one!) Think to solve this problem by giving function rules for 'almost vertical' lines.
Your teacher may want you to **Print** your image for later discussion	

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Curve Fitting

Name: ______________

Find rule for quadratic that describes the path of the water spouting from the hose by using trial and error to systematically change “a”, “g” and “h” in the rule $y=a(x\text{-}g)^2+h$ where the turning point of the curve will be at (g,h.)

Note: the rule you enter from the input line will appear in expanded ($y=ax^2+bx+c$) form in the algebra window.

Keep a list:

Rule entered	Result in algebra window	Result on orientation, position or shape of the graph

These questions will help you to describe the resulting graph

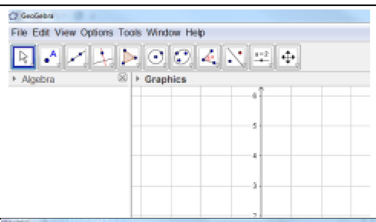
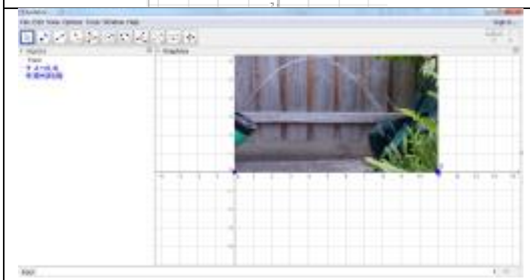
Is the new curve steeper for flatter than $y=x^2$?

Is the new curve translated up or down relative to $y=x^2$?

Is the new curve translated left or right (in the negative or positive direction) relative to $y=x^2$?

Is the new curve upside down? Has it been reflected in the x axis?

The notes below will help with remembering how to use Geogebra:

	Click on the **EDIT** menu and select **Insert image from** Select required photo file from your photo files list that
	Use the pointer and the blue points at the base of the photo to drag it to a convenient position (eg corner at origin) When in position, right click on the image select **Object Properties** then **Fix object** and **Background Image** Click on Graphics and select the grid image. If you need to move the whole canvas click on the last icon on the right and choose **Move graphics view**

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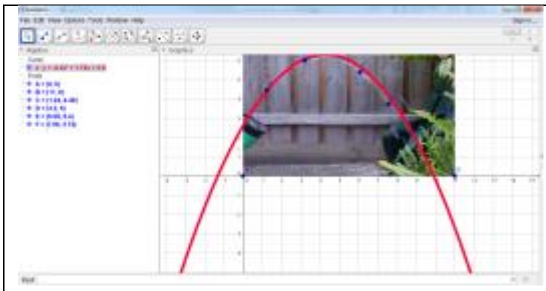	Enter function rules via the input line. I started with $y=x^2$. Then tried $y=-x^2$. Systematically try different values for a, g and h in $y=a(x-g)^2+h$ You can delete the lines you don’t want or “turn them off” by clicking in the algebra window. Once I had the curve I thought best, I chose to right click on this in the algebra window and edit its colour and style properties

Which of the following objects could have their outline modelled by a parabola?

Test your conjectures in GeoGebra.