re(Solve) MATHS BY INQUIRY

BRINGING THE REAL WORLD INTO ALGEBRA

Quadratic Functions

Lesson 2: Trajectories

Australian Curriculum: Mathematics (Year 10)

ACMNA239: Explore the connection between algebraic and graphical representation in relation to simple quadratics … using digital technology as appropriate.

- Sketching graphs of parabolas.
- Applying transformations, reflections and stretches to parabolas.

Lesson abstract

Students take a set (burst) of photos of the path of a ball, and fit a parabola to points from the images by systematically altering the parameters of the quadratic function. The usefulness of the turning point form (highest point reached) and of the factorised form (distance travelled) are explored in this context.

Mathematical purpose (for students)

Quadratic functions can model real world events such as the path of a ball. The turning point form and the factorised form can help find useful information about the path.

Mathematical purpose (for teachers)

At the end of this lesson students will be able to:

- Use a quadratic function to model the trajectory of a ball that has been thrown or kicked.
- Describe a quadratic function algebraically including identifying critical points.
- Use technology to explore the impact of changing the parameters in $y=a(x-g)^2+h$ on the graph of this quadratic function.
- Use technology to move between alternative symbolic arrangements of quadratics.
- Articulate the link between the graphic, algebraic, numeric and digital image representations of quadratic functions.

Lesson Length 2x50 minutes approximately

Vocabulary Encountered

- trajectory
- projectile

Lesson Materials

- Access to digital cameras with burst mode (smart phone or tablet is adequate)
- Access to GeoGebra or equivalent software
- Student Sheet 1 -Tracing the Path of a Ball (1 per two students)

We value your feedback after these lessons via <link to be advised>

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Taking Your Own Trajectory Photos

Students will have greater interest and engagement in the mathematical learning if they feel some ownership over the task. We have supplied two series of images of balls being tossed but it would be MUCH better to take the extra time to allow students to take their own burst of images.

This could be done as a class or small groups.

What works?

- Tossing a ball, kicking a footy, aiming to throw a basketball/ netball goal.
- More advanced versions could include juggling and tracing the patterns, the path of an object fired from a sling or trebuchet if you are linking with history!
- Smartphone or tablet cameras are fine.
- The photographer needs hold the "camera" parallel to the expected path of the ball with most of the expected path in view. This will mean holding it horizontal. Note that if the ground is sloping this does not mean parallel to the ground.
- Most important: the "camera" must be held still and NOT FOLLOW THE PATH of the ball.
- With a Smartphone or tablet this is now easy. In photo mode the photographer merely holds the button for a slightly extended time while the ball is thrown and the camera will record a burst of several images. You may have more sophisticated equipment and approaches but this simple strategy does work.
- The students should select 3 or 4 images where the position of the ball is clear and save these or send them to where ever is appropriate at your school. Student account? Public drive? You will need to check where photos are permitted.

Before taking photos

It is a good idea to discuss the following with the class:

- Describe the task of taking photos of the path of a ball.
- Give instructions or draw from students the need to:
  - Keep finger on button so that a series of "stills" will be taken at high speed (not video). Demonstrate (have a student demonstrate) if this seems necessary and project images so class can see.
  - Hold camera still – do not follow the ball
  - Hold camera horizontal
  - Consider the background – discuss good locations where ball will be visible and where height and distance might be estimated by features in background (plain or brick wall for example – but not essential)
- Indicate bounds for where the activity will take place
- Allocate groups
- Indicate time allowed

Have Student Sheet 1 -Tracing the Path of a Ball available when students are back in the classroom as students will be ready at different times.

Using GeoGebra to Trace the Path of a Ball

Students will work through the activities set out in Student Sheet 1 -Tracing the Path of a Ball

Teacher Notes

- Backup: Have a prepared set of photos loaded on an intranet that students can access. Use the ones we have supplied or better still have a set with a teacher shooting a basketball or football goal. Students need to find the path, follow the path – did the shot go in?
- The modelling task can be repeated for a second set of photos: students could swap photo sets and compare results.

Conclusion

Discuss students' experience of the task: difficulties encountered, discoveries made.

Recap the mathematics linking quadratic function rules and their graphs with question such as the following:

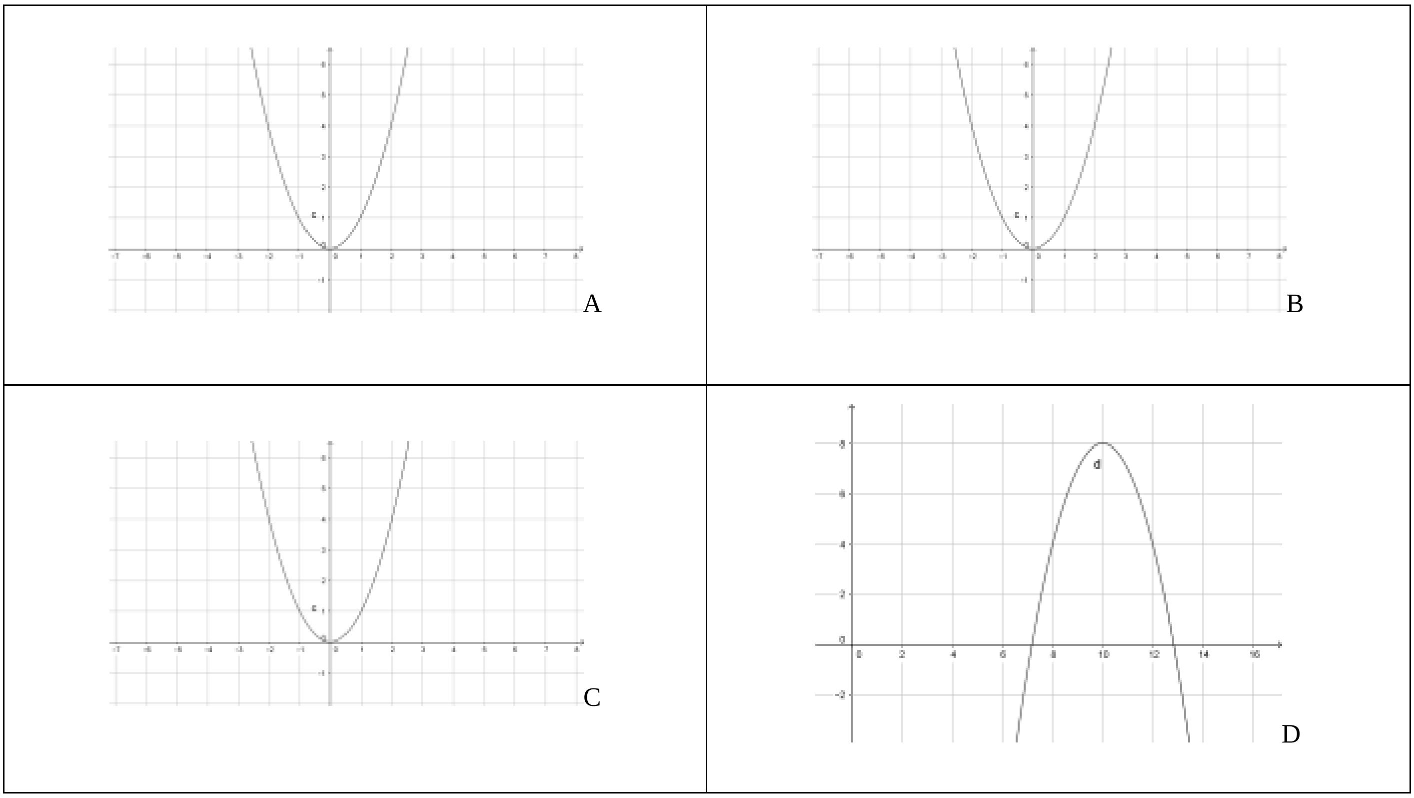
- When the function rule is in the form y=a(x-g)^2+h:
  - What is the effect of changing "a"?
  - What is the effect of changing "g"?
  - What is the effect of changing "h"?
  - How can we find the maximum height of the path?
- What form of the quadratic function rule helps us to find where the ball landed? Why?

You might choose to use check questions like these:

- Choose the correct word or number from the alternatives in **bold:**

The curve for $y= -(x-4)^2-22$ would (**look like or be the reflection in the x-axis of**) the curve for $y=x^2$ except translated (**up or down**) by (**4 or 22**) and (**left or right**) by (**4 or 22**).

- The graphs below show $y=x^2$
  - In window A sketch in $y=x^2+ 3$
  - In window B sketch in $y=(x+ 2)^2$
  - In window C sketch in $y=-x^2+ 5$
  - If the scales in window D are in metres, how high did the ball go and how far did it travel?



Extension

Some students may wish to discuss or investigate the following:

- Why does the ball follow the path of a parabola?
- What factors affect this path?
- What determines the maximum distance travelled by a projectile?

If there is a student who can juggle, say with different coloured balls, then modelling the paths would make an excellent extension.

Tracing the Path of a Ball

Name: ____________

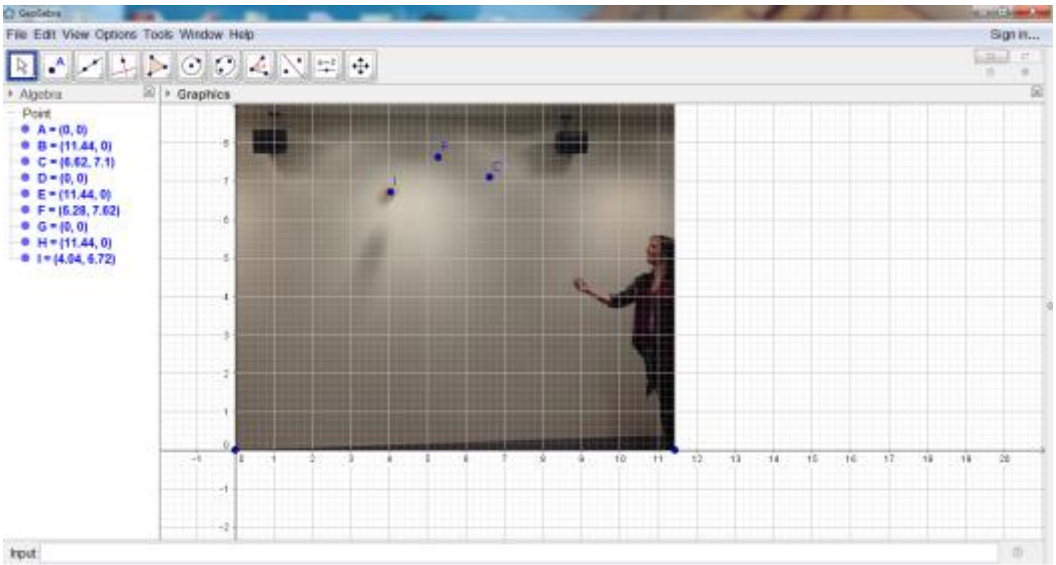
Guide for graphing the curve of a ball through the air

From your photo burst taken while the thrown or kicked ball was in the air select 3 images that clearly show the position of the ball. Save/send these so that you can access them on the computer. Your teacher may have put these on the school public drive.

The notes below will help with remembering how to use Geogebra.

1. As in Lesson 1, in a new Geogebra file insert the first image
2. Place this so that the left corner is at (0,0)
3. When in position, right click on the image. Select Object Properties then Fix Object and Background Image.
4. Place a point at the centre of the ball
5. Insert your second image on top of the first. When in position, right click on the image, select Object Properties then Fix Object and Background Image
6. Place a point at the centre of the ball in its new position (you should be able to see the first point).
7. Insert the third image. When in position, right click on the image, select Object Properties then Fix Object and Background Image.
8. Place a point at the centre of the ball in its new position (you should be able to see the first two points).

   You should now have a Geogebra screen that looks something like this:



Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Find a rule for a quadratic that describes the path of the ball.

- Use trial and error with the rule in turning point form.
- Keep notes – rule entered, rule as it appears in the algebra window, effect on line of graph.

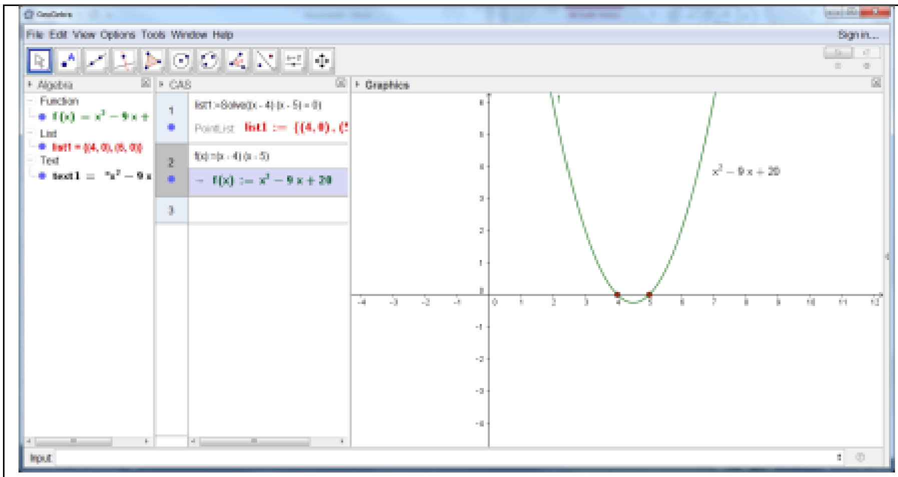
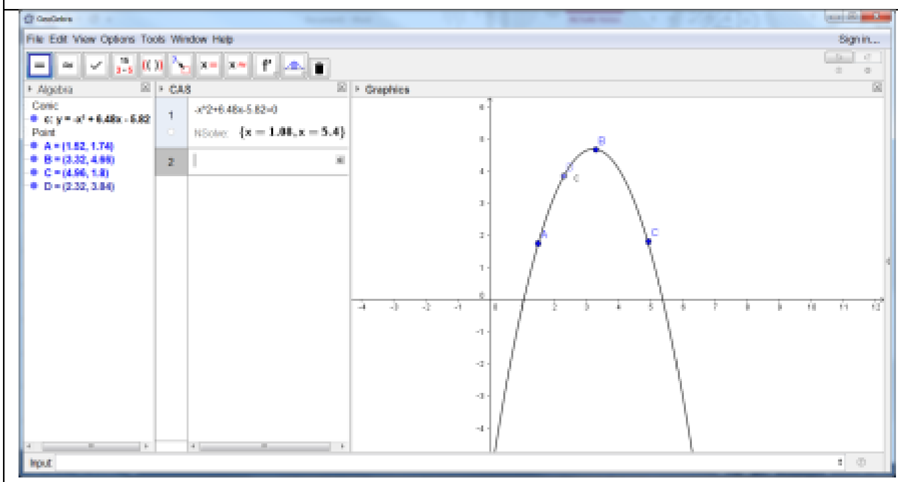
Rule entered	Result in algebra window	Resulting impact on the orientation, shape or position of the graph.

Reminder: These questions will help you to describe the resulting graph

- Is the new curve steeper for flatter than $y=x^2$?
- Is the new curve translated up or down relative to $y=x^2$?
- Is the new curve translate left or right (in the negative or positive direction) relative to $y=x^2$?

Describe the path of the ball.

- How high did it go? How far did it travel?
- The following examples illustrate how you might use CAS in Geogebra to help answer these questions.

	You can use computer algebra (CAS) in Geogebra or other software to help you find the *x*-intercepts for your graph. Choose CAS from the Geogebra **View** menu Use the pointer to check to see what menu feature each of the CAS icon represents. These might be integer values if the expression in your function rule can be simply factorised. Or you may get exact surd values.
	To obtain approximate numerical values that will help you interpret the answer in terms of the position of the ball you will most likely need to Solve Numerically (Nsolve)