re(Solve) MATHS BY INQUIRY

MECHANICAL LINKAGES and DEDUCTIVE GEOMETRY

Year 9 Similar Triangles

Lesson 1: Ironing Tables

Australian Curriculum: Mathematics (Year 9)

ACMMG220: Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar.

- Establishing the conditions for similarity of two triangles and comparing this to the conditions for congruence.
- Using the properties of similarity and ratio, and correct mathematical notation and language, to solve problems involving enlargement (for example, scale diagrams).
- Using the enlargement transformation to establish similarity, understanding that similarity and congruence help describe relationships between geometrical shapes and are important elements of reasoning and proof.

Lesson abstract

When an ironing table with legs that pivot is raised or lowered, the top always stays parallel to the floor. In this lesson students investigate the triangles formed by the pivoting legs. They make physical models, observe computer simulations and explain with geometry why the table is always horizontal. Students investigate different leg lengths and pivot positions to ensure similar or congruent triangles are formed. Three designs of ironing table are included, involving different geometry.

Mathematical purpose (for students)

Similar and congruent triangles can be used to explain why the top of a folding ironing table stays parallel to the floor as the table is raised or lowered.

Mathematical purpose (for teachers)

The three designs of ironing table that can be considered in this lesson involve geometry of different complexity. The forming of congruent or similar triangles by the pivoted legs of ironing tables ensures that alternate angles are equal and hence that the top is parallel to the floor. This is the converse of '*when two parallel lines are cut by a transversal, alternate angles are equal*'. The geometric reasoning involves vertically opposite angles, isosceles, congruent and similar triangles. Different conditions for similarity apply in different cases. Students may also divide different leg lengths proportionally, to ensure similar triangles are formed. They can also consider the stability and practicality of different designs.

Lesson Length 50-60 minutes approximately

Vocabulary Encountered

- similar triangles
- proportion
- converse
- pivot
- vertically opposite
- isosceles

Lesson Materials

- Geostrips or similar (2 long red and one long yellow per pair of students, or print these strips onto card)
- Per pair of students: 1 paper fastener, scissors, 2 rulers
- Slide show: *ST1_Yr9_1d_IroningImages.pptx*
- GeoGebra file: *ST1_Yr9_1a_Ironing1.ggb* OR https://ggbm.at/x7dMsDbh
- GeoGebra file: *ST1_Yr9_1b_Ironing2.ggb* OR https://ggbm.at/J3fBs66E
- GeoGebra file: *ST1_Yr9_1c_Ironing3.ggb* OR https://ggbm.at/tcrNZaM5
- Student Sheet 1 – Ironing Tables (1 per student)

We value your feedback after these lessons via https://www.surveymonkey.com/r/2JH6Z82

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Introducing Ironing Table Designs

An important feature of an ironing table (or other types of folding table) is that it can be adjusted to comfortable heights for different users, it remains stable, and it will fold neatly for storage. When it is raised or lowered, it is important that the table top stays horizontal. The simple linkage of the pivoting legs achieves this.

This lesson examines three slightly different designs. Table 1 has the simplest geometry, followed by table 2 and then table 3. The lesson can be adapted to the needs of students by selecting the appropriate table designs.

Ironing table 1

The simplest design has legs are of equal length. The pivot point is placed so that the upper sections of both legs are equal in length and also the lower sections, so the legs form congruent isosceles triangles. Alternate angles are therefore equal and hence in every position, the table top is parallel to the floor. This uses the converse of '*when two parallel lines are cut by a transversal, alternate angles are equal*'.

Ironing table 2

This design also has legs are of equal length, but the pivot point is placed above the midpoint, forming two isosceles triangles not of the same size. These two triangles are similar, and alternate angles are therefore equal and hence in every position, the table top is parallel to the floor.

Ironing table 3

This design was patented in 1871 by James Mallory of Indiana, USA. The pivot point is positioned so that the triangles formed by the legs are similar scalene triangles. The leg design means that items being ironed are not caught on the legs as they hang over the edge of the table.

In the patent description, Mallory wrote: '*This invention consists in the production of a table especially adapted to the ironing of clothes, the object being to provide an ironing-table the top of which will be level when in position for use, the height of which may be regulated by suitable means, and the supports of which may be folded up when not in use for convenience where room is an object, and in carrying.*'

An image of the original patent diagram is included in the slide show *ST1_Yr9_1d_IroningImages.pptx*

Getting started

Show students an ironing table, or a small folding table, or a picture (e.g. from the slideshow).

Discuss its important features drawing on students' experiences, including:

- Size and shape of the table for convenient ironing.
- The need for stability.
- Adjustability and the consequent need to stay horizontal.
- The desirability of neat folding.

Now show students the first two ironing table designs (tables 1 and 2) in the slide show *ST1_Yr9_1d_IroningImages.pptx* and ask them to identify some geometric features of the legs.

Expected Student Responses

- The legs form isosceles triangles with the top and the floor.
- At the pivot point, there are vertically opposite angles, so they are equal.
- Students may also notice at this stage
  - Pivoted in the centre of the legs: two congruent isosceles triangles.
  - Pivoted at the same position on each leg: a pair of similar isosceles triangles.

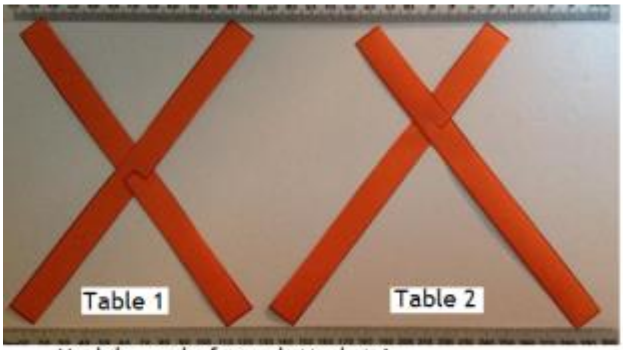
Ironing Tables 1 and 2

Distribute Student Sheet 1 – Ironing Tables

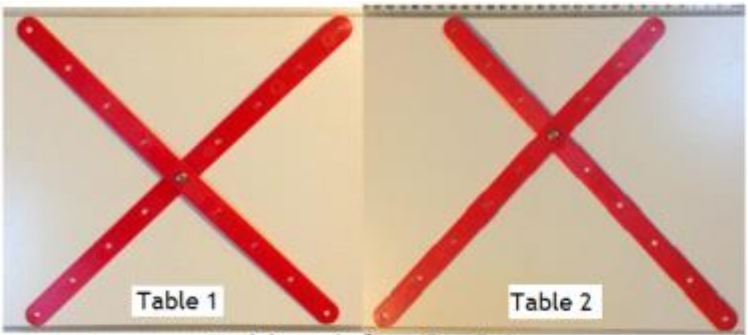
Making physical models

Students can make the models with two identical strips cut from the light card or with Geo Strips as shown below, joined only at the pivot point.

The floor is made from a ruler, and the legs slide along it. The table top can be another ruler or strip, touching the tops of the legs. The table top cannot be attached to the legs in this model.



Models made from slotted strips.



Models made from Geo Strips

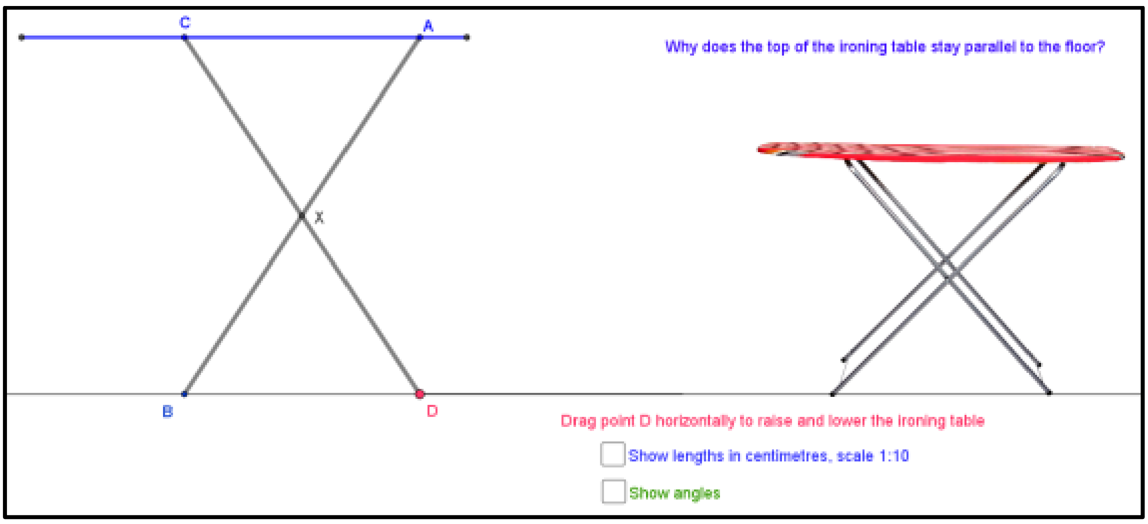
As they work, students should observe how the models move, and note geometric features such as:

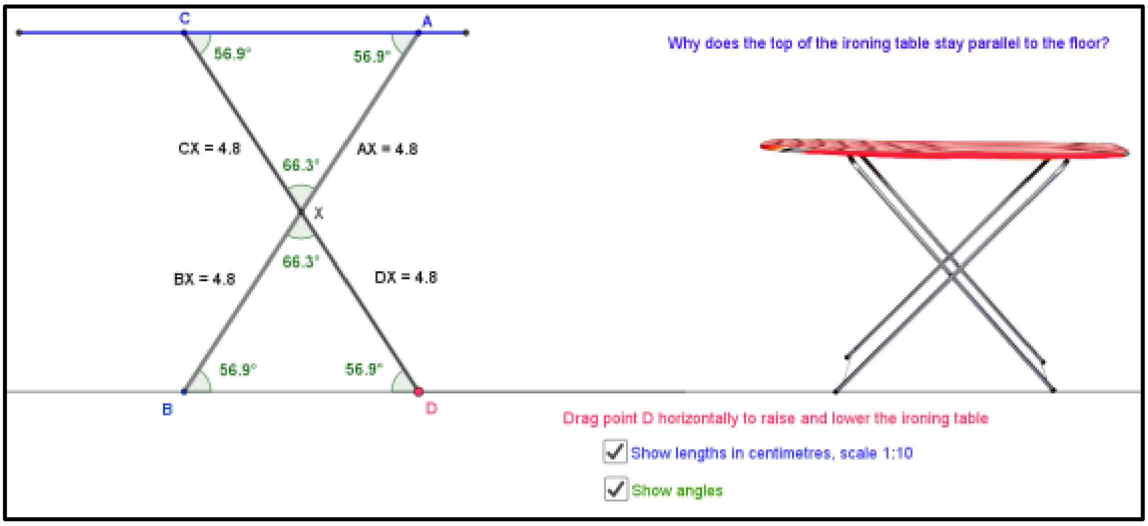
- Some angles are equal.
- The isosceles triangles are congruent if the pivot is in the centre of each leg.
- The isosceles triangles are similar if the pivot is not in the centre but in the same position on each leg.

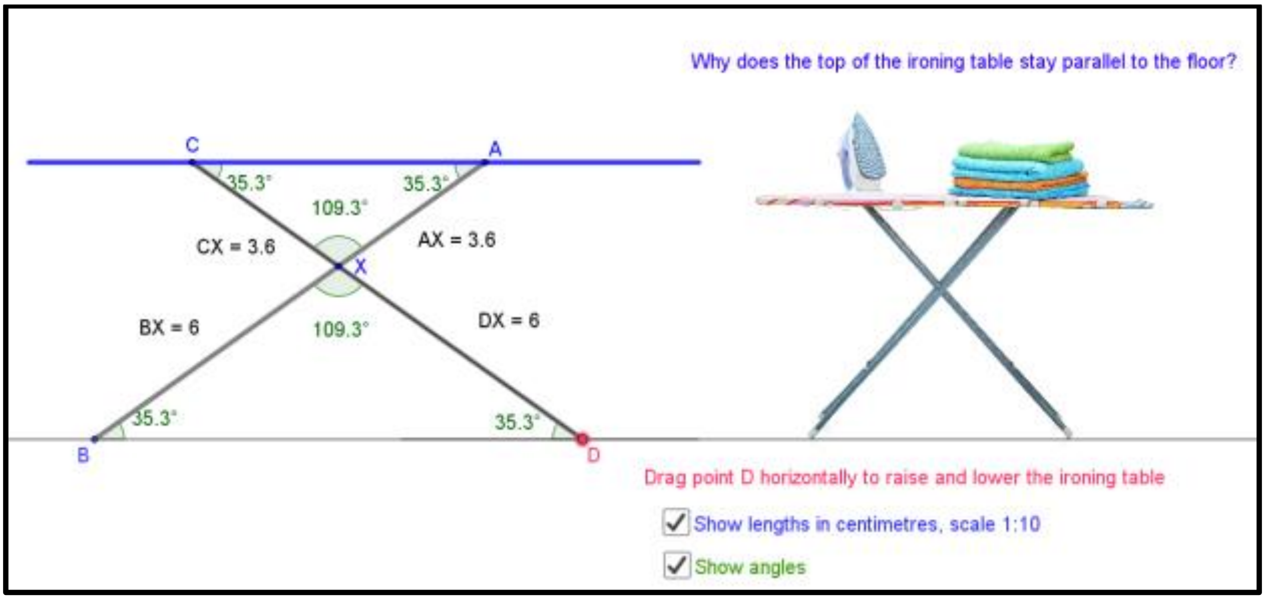
Using computer simulations

Students open the dynamic geometry files:

- To observe the motion of the linkage more closely,
- To see measurements of angles and lengths in different positions,
- And eventually to answer the main question: why does the top of the ironing table stay parallel to the floor?







Expected Student Response for Ironing Table 1

- PROOF 1: using congruent triangles

  The two triangles are congruent: two sides and the included angle (vertically opposite angles) of one triangle are equal to two sides and included angle of the other triangle.

  $\angle ACX = \angle CAX = \angle BDX = \angle DBX$

  Alternate angles between *AC* and *BD* cut by transversal *CD* are equal.

  CA is parallel to *BD*.
- PROOF 2: using isosceles triangles

  The two triangles are isosceles, so the two angles at the floor are equal to each other and the two angles at the table are equal to each other.

  The two angles at X are vertically opposite angles, and hence equal.

  The angle sum of a triangle is 180 degrees, so:

  $\angle ACX = \angle CAX = \angle BDX = \angle DBX$

  Alternate angles between *AC* and *BD* cut by transversal *CD* are equal, so *CA* is parallel to *BD*.

Expected Student Response for Ironing Table 2

- PROOF 1: same as Proof 2 for Table 1.
- PROOF 2: using similar triangles

  The two triangles are similar: two sides of one triangle are in proportion to two sides of the other and the included angle is equal (vertically opposite angles).

  So alternate angles are equal.

  So *CA* is parallel to *BD* (alternate angles between *AC* and *BD* cut by transversal *CD* are equal).

Ironing Table 3 – Design Challenge

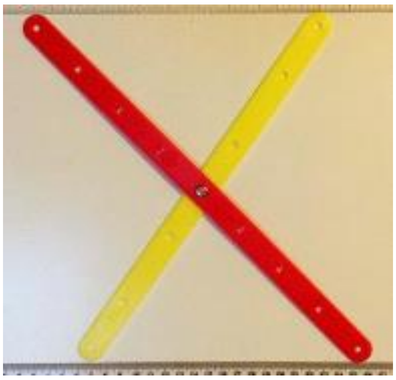
- Pose the question "Do the legs of an ironing board have to be the same length?"
- Allow students some time to investigate, possibly using the geometric ideas above, but more likely beginning by making models to test ideas.
- The key is that two similar triangles must be formed – do not tell students this too early.

Making physical models

Students use strips of unequal length. There are many solutions.

1. Unequal legs with centre pivot.

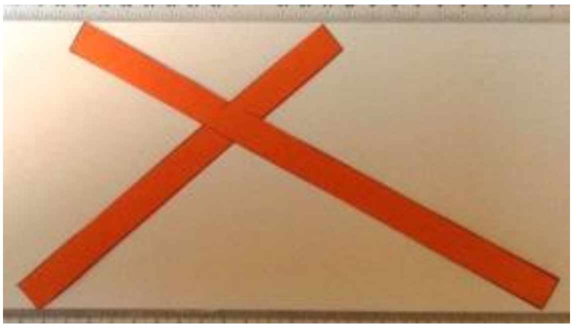
- Congruent triangles are formed.

2. Unequal legs with non-central pivot

- The pivot point must divide the legs in the same proportion to create similar triangles.
- The card strips are of length 18 cm and 24 cm to make it easy for students to find a pivot position which divides each leg in the same proportion.
- A laser printer should give the correct size, but some inkjet printers give larger margins so that the strip length will not be accurate.
- Note: This cannot be modelled with Geo Strips because the holes in the different length strips are not often in suitable positions for dividing two strips in the same proportion other than at the centre.

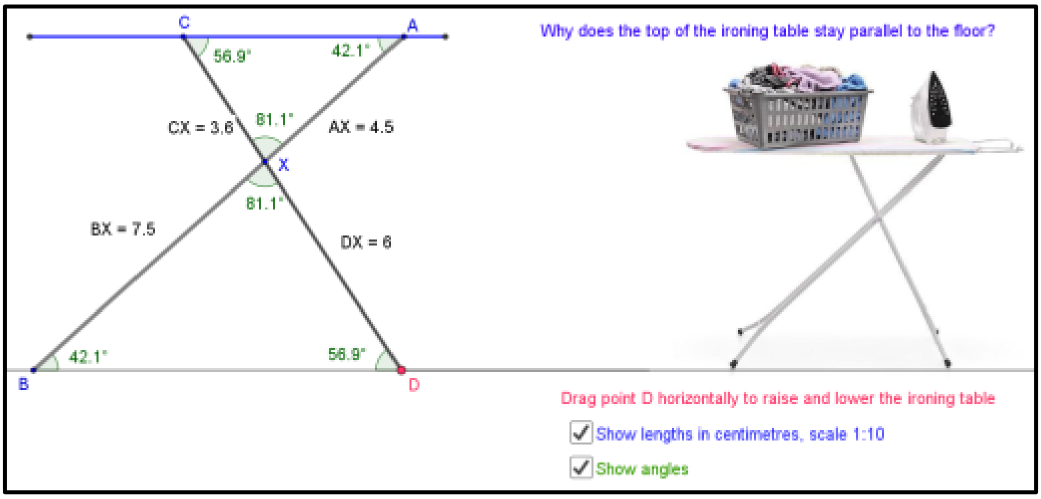
Expected Student Response



- The pivot has to be where two similar triangles are formed.
- The pivot could be one third of the way along each leg: 6 cm from top of 18 cm leg and 8 cm from top of 24 cm leg.
- There are many other legitimate positions, for example, 7.2 cm from top of 18 cm leg and 9.6 cm from top of 24 cm leg.
- The stability of the design also needs to be taken into consideration.

Using a computer simulation

- The computer simulation *ST1_Yr9_1c_Ironing3.ggb* shows the case where the legs are pivoted so that two similar triangles are formed.
- The pivot point must divide each leg in the same proportion.



Expected Student Response

Consider triangles AXC and BXD.

$\angle AXC = \angle BXD$ (vertically opposite angles)

$$\frac{AX}{CX} = \frac{4.5}{3.6} = 1.25$$

$$\frac{BX}{DX} = \frac{7.5}{6.0} = 1.25$$

So the two triangles are similar (two sides of one in proportion to two sides of other and included angles equal)

So the alternate angles $\angle CAX$ and $\angle DBX$ are equal.

So CA is parallel to BD.

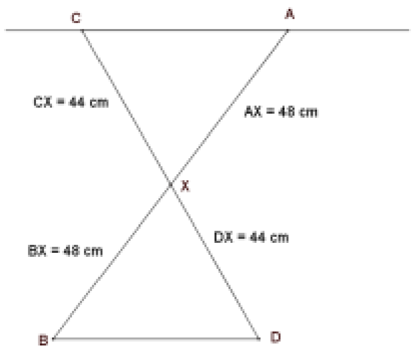
Where should the pivot be?

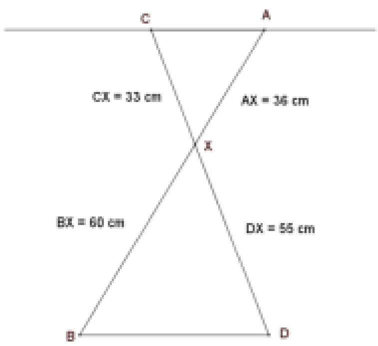
The two legs of an ironing table are 96 cm and 88 cm respectively. Suggest an appropriate position for the pivot so that the top will stay parallel to the floor as the ironing table is raised or lowered. Are there other possible positions for the pivot?

Ans.

Students should recognise that they can create two congruent triangles if the pivot is in the centre of each leg.

They may find it harder to work out positions for the pivot so that similar triangles are formed. They must choose positions so that each leg is divided in the same proportion. By noticing that 8 is a factor of both leg lengths, they can, for example, divide each leg in the proportion three eighths to five eighths as shown below.





Conclusion

The two legs of an ironing table are pivoted in a position so that two congruent triangles or two similar triangles are formed. This means that alternate angles between the top and floor cut by one of the legs are equal and so the top stays parallel to the floor.

Ironing Tables

Name: ____________

When an ironing table is raised or lowered, the top always stays parallel to the floor.

Making Physical Models

- Use 2 strips with the same length to model each of the two ironing tables shown above.
- For the ironing table on the left, the pivot is in the centre of each strip. Cut slits so the two strips can form a pivot.
- For the ironing table on the right, work out a suitable position for the pivot and cut short slits in the two strips.
- Move the legs about the pivot point to represent raising and lowering of the ironing table.
- Use two rulers to represent the table top and floor.
- What do you notice about the triangles formed by the legs, the table top and the floor?

Using Computer Simulations

- Open the GeoGebra files *Ironing Table 1* and *Ironing Table 2*.
- Raise and lower the ironing tables and check the side lengths and angles of the triangles.
- In each case use geometry to explain why the table tops stay parallel to the floor.

Is it possible to make an ironing table with legs of different lengths?

Making a Physical Model

- Use 2 strips with different lengths and a paper fastener to model an ironing table in which the legs have different lengths.
- Try to work out the conditions under which the top would stay parallel to the floor.
- Test your ideas by making slits for the pivot in a pair of legs with different lengths.

Using a Computer Simulation

- Open the GeoGebra file *Ironing Table 3*.
- Raise and lower the ironing table and check the side lengths and angles of the triangles.
- Use geometry to discover whether the table top really stays parallel to the floor.

Where should the pivot be?

- The two legs of an ironing table are 96 cm and 88 cm respectively.
- Is there an appropriate position for the pivot so that the top will stay parallel to the floor?
- Are these appropriate lengths for the legs of an ironing table? What is a sensible range for the lengths of legs of an ironing table.
- Should the pivot for an ironing table be near the floor ends of the legs?   Why or why not?

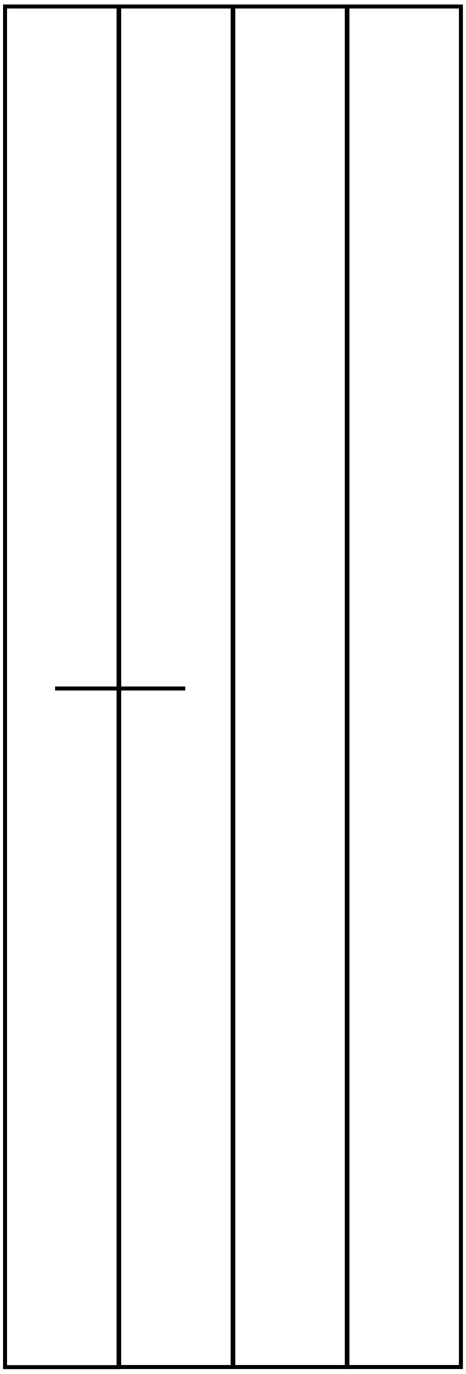
Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

aamt

Ironing table strips

2 sets: same leg length



2 sets: different leg lengths