Year 7: Angles and Lines
Folding Quadrilaterals

Australian Curriculum: Mathematics (Year 7)

ACMMG165: Classify triangles according to their side and angle properties and describe quadrilaterals

Lesson abstract

Students make models of triangles and quadrilaterals, using strips hinged with paper fasteners. In a series of five challenges, they investigate the lengths of set of strips that can make triangles and quadrilaterals and contrast the rigidity of triangles with the flexibility of quadrilaterals. They then investigate what quadrilaterals fold neatly. An introduction to mechanical linkages can be included.

Mathematical purpose (for students)

A triangle made from hinged bars is a rigid shape, but a quadrilateral is not.

Mathematical purpose (for teachers)

This lesson will familiarise students with the equipment used to make the physical models of mechanical linkages, and highlight the fact that in a mechanical linkage, the side lengths are fixed (nearly always) but the angles change. The challenges can be conducted in an open way, with students making models, collecting data to test conjectures and observing patterns. The answers to the challenges all involve only relationships between side lengths (e.g. sum of lengths of two sides = sum of lengths of other two sides). There is an opportunity to discuss the differences between mathematical findings and the real behavior of physical objects.

Lesson Length 60 minutes approximately

Vocabulary Encountered

- rigid
- mechanical linkage

Lesson Materials

- At least 6 Geo Strips of assorted lengths, four paper fasteners per student. More strips are better.
- Alternatively, students can cut strips from light card or a more rigid material such as corflute. (1 per student, 1 per group of 4 etc)
- Slide show: *ST1_0_Linkages_Intro_Images.pptx*
- Expanding lattice or arm e.g. from a bathroom mirror (optional)
- Other examples of linkages (optional)

We value your feedback after these lessons via https://www.surveymonkey.com/r/2JH6Z82

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Making Shapes with Hinged Strips

What is a mechanical linkage?

(This lesson can begin or end with an introduction to mechanical linkages, but this is not essential to the lesson.)

Explain some of the key points from the "What is a mechanical linkage?" section of the Teachers' Guide: *ST1_0_Teachers Guide*.

The operation of many everyday items—such as folding ironing tables, umbrellas, car jacks and scissor lifts—involves a set of hinged bars which have a fixed length, but are free to move. The lengths of the bars stay the same, but the angles between them can change.

Show some examples and highlight how the bars are of a fixed length but the angles can change:

- real examples are good (e.g. car jack, expanding arm to pull out a mirror, folding umbrella mechanism)
- show the pictures in the slide show *ST1_0_Linkages_Intro_Images.pptx*.

Joining three strips to form a triangle

Students choose three of their Geo Strips and three paper fasteners to make a triangle. Some students will have chosen strips that cannot make a triangle. Ask students to show what they have made (or not made!).

Pose challenge 1 and challenge 2 and allow students time to investigate in pairs.

- Challenge 1: *How can we tell whether or not a set of strips will make a triangle?*
- Challenge 2: *Can you make two different triangles from a given set of strips?*

Students will find that the total lengths of the two shorter strips must be greater than the length of the third strip, otherwise the two shorter strips will not meet. Trying to make a triangle with the strips helps students appreciate the reason for this.

Enabling Prompts

- Be careful to measure the lengths of strips only between the paper fasteners, i.e. between the hinge points.
- Make a list of the lengths of strips that form a triangle, and a list of those that do not.
- Trace around the shapes you make to keep a record of your work and findings.
- Make conjectures and test them. Write your conjectures down clearly.
- Look carefully at what happens when you try to make a triangle. Try to learn from the examples you see, not just collect data.

Discuss students' findings, emphasizing points such as:

- A triangle has a fixed shape. It is rigid. This is why triangles are used where strength is required (for example in building bridges).
- If the side lengths of two triangles are the same, then the triangles will have the same angles. It is not possible to make two different triangles with the same side lengths.

A triangle is a rigid shape.

Some bars will not make a triangle.

A quadrilateral is not rigid.

Joining four strips to form a quadrilateral

Students choose four of their Geo Strips and use four paper fasteners to make a quadrilateral. Some students will have chosen strips that cannot make a quadrilateral. Ask students to show what they have made (or not made!).

Pose challenge 3 and challenge 4 and allow students time to investigate in pairs.

- Challenge 3: *How can we tell whether or not a set of strips will make a quadrilateral?*
- Challenge 4: *Can you make two different quadrilaterals from a given set of strips?*

Students will find that the total lengths of the three shorter strips must be greater than the length of the longest strip, otherwise the quadrilateral will not meet up. Trying to make a quadrilateral with the strips helps students appreciate the reason for this. As soon as the quadrilateral has been constructed, students will note that it can change shape. It is not rigid. This is why quadrilaterals are used where movement is required (for example in a car jack).

Enabling Prompts

- Be careful to measure the lengths of strips only between the paper fasteners.
- Make a list of the lengths of strips that form a quadrilateral, and a list of those that do not.
- Trace around the shapes you make to keep a record of your work and findings.
- Make conjectures and test them. Write your conjectures down clearly.
- Look carefully at what happens when you try to make a quadrilateral. Try to learn from the examples you see – not just collect data.

Extending Prompt

- What happens if you make polygons with more than four sides? (ANS: similar to quadrilaterals).

Discuss students' findings on Challenge 3 and 4, emphasizing points such as:

- A quadrilateral made of hinged strips does not have a fixed shape. It is not rigid. This is why quadrilaterals are not used where strength is required (for example in building bridges). They need bracing.
- A quadrilateral can be made rigid by joining a fifth strip (a brace) as a diagonal to create two triangles.
- There are many different quadrilaterals with the same side lengths. The angles can be different even though the side lengths are the same.
- Many everyday objects make use of the changing shapes of joined bars like the Geo Strip quadrilateral so they can pivot at the corners. This is the basis of mechanical linkages.

Which Quadrilaterals Fold Neatly?

Show students an object with an expanding arm or a lattice -like structure OR show the image of the expanding arm for the bathroom mirror in the slide show *ST1_0_Linkages_Intro_Images.pptx*. Note that it is essential to the design that it folds neatly. This is important in many real situations, for storage and transport.

Ensure that students can identify the quadrilaterals in the arm.

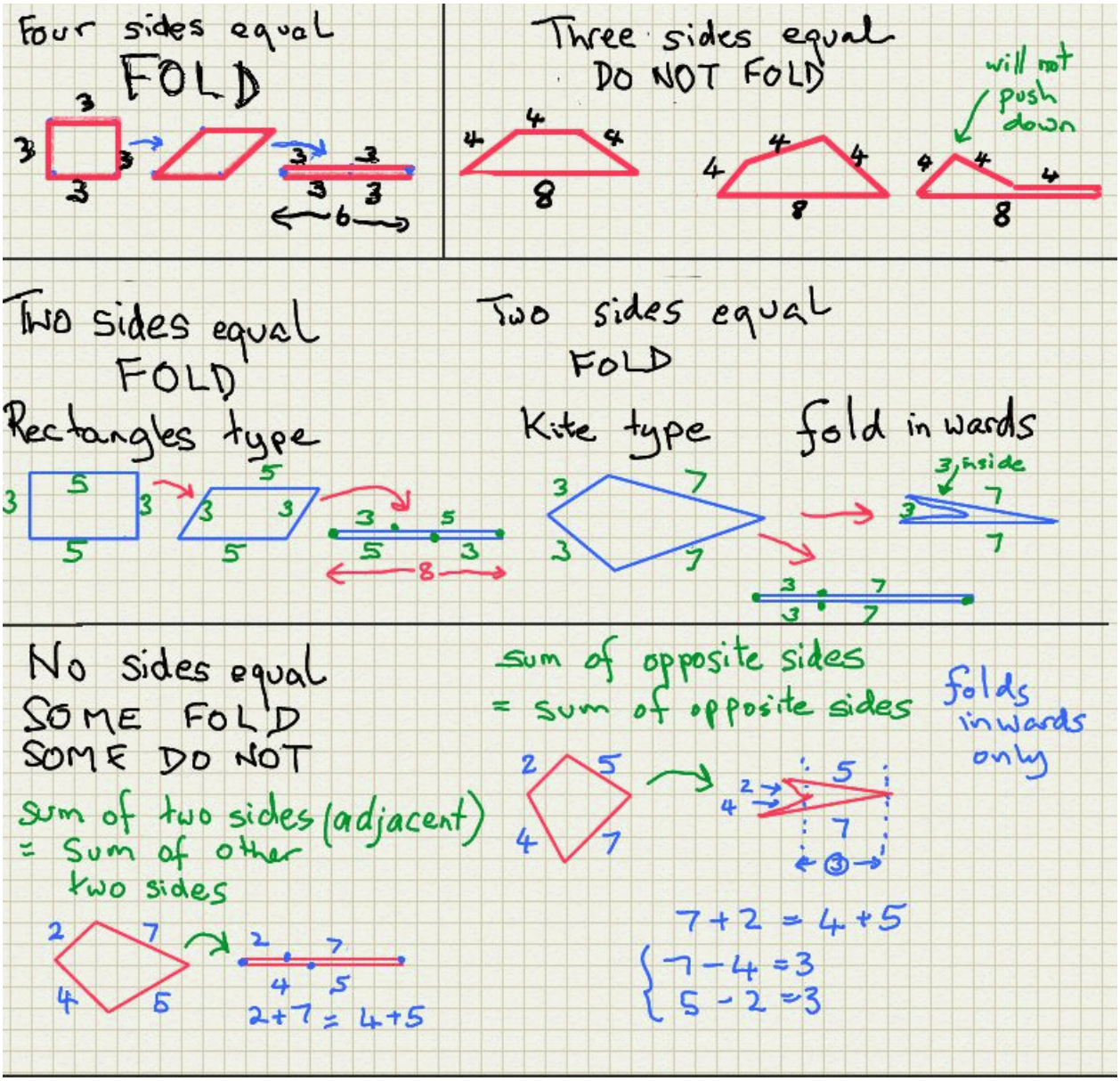
Ask students to look at quadrilaterals that have in front of them to see if they fold neatly or not. Discuss the meaning of neat folding with examples provided by students. Apart from the thickness of the bars (strips), the quadrilateral would be able to be pushed right down to just one line.

Pose challenge 5 and allow students time to investigate in pairs. Enabling prompts are as above.

- Challenge 5: *What quadrilaterals can be folded up really neatly.* (Consider only a single quadrilateral, not a series as in the expanding arm.)

Have groups of students present their findings on Challenge 5. Some groups will present examples of quadrilaterals which do or do not fold down neatly. Others will have found conditions on the lengths of the bars (strips). The main findings are demonstrated below.

- Encourage students to present their conjectures and encourage other students to present confirming or contradictory evidence.
- Students in Year 7 will probably present their findings using numerical examples. Some students can be encouraged to use algebraic letters (e.g. *a, b, c, d*) to make a general statement.
- Discuss whether "folding inwards" is to be counted as 'neat folding'.
- Some students will note that some shapes can fold in two different ways (e.g. parallelograms. kites) but others only fold in one way (e.g. no sides equal).
- Students may note that squares can be deformed into rhombuses, and rectangles can be deformed into parallelograms. This relates to the side properties of these figures. Geometric language can be introduced.



Conclusion

Review, with suggestions from students, the main mathematical findings of the lesson, including

- the conditions for three bars to make a triangle (and why)
- the conditions for four bars to make a quadrilateral (and why)
- the rigidity of triangles and the non-rigidity of quadrilaterals
- how the non-rigidity of quadrilaterals makes them useful in some everyday objects that move
- that many quadrilaterals 'fold neatly' and this has practical uses.

Review, with suggestions from students, the strategies that helped them in their investigations, including

- trying examples and keeping a record of what happened, to look for patterns later
- looking carefully at the examples to see why something is happening
- making conjectures to investigate, and writing them down clearly so they can be tested.