Pyramids in a Box

Lesson 1: Discover Phase

Australian Curriculum: Mathematics (Year 6)

ACMMG140: Construct simple prisms and pyramids.

- Constructing prisms and pyramids from nets, and skeletal models.

Lesson abstract

Students are introduced to the challenge of designing a good box to hold two different sized pyramids. The teacher reviews and extends prior knowledge of pyramids and prisms. Students manipulate, three dimensional objects and geometric construction materials to draw nets, making the connection between the shapes and positions of the faces on the object and in the net. They explore the different nets that can make one three-dimensional object.

Mathematical purpose (for students)

Multiple nets can be drawn for pyramids and prisms by manipulating the connections and positions of the faces.

Mathematical purpose (for teachers)

Polyhedra can be described using geometric terms. Connections between three dimensional objects and the shape and position of their faces are made in order to draw multiple nets for each object.

At the end of the Discover phase, students will be able to:

- Describe pyramids and prisms using geometric terminology.
- Distinguish key features of pyramids and prisms, acknowledging both as special polyhedra.
- Recognise that a two-dimensional net can be folded to form a three-dimensional object.
- Observe that multiple nets may represent the same object.

Lesson Length 45-60 minutes

Vocabulary Encountered

- height, length, width
- edge, face, vertex, apex, base
- net
- prism
- polyhedron (pl. polyhedra or polyhedrons)
- congruent, parallel

Lesson Materials

- Several prisms and pyramids with a variety of bases (e.g. small tea box, Toblerone box, Curry Shots box, ornamental crystal, toothpaste box, solid building blocks).
- At least one prism or pyramid (everyday object or geometric solid) per pair.
- Class set of geometric construction materials (e.g. Geoshapes)
- Student work books
- iPad or camera – 1 per group (optional)
- Light card or paper on which to trace nets.

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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Thinking about Pyramids and Prisms

Inquiry Question:
What is the best box to hold two different sized items that are packaged as pyramids?

Exploring the inquiry question

1. To promote enthusiasm and engagement in this Inquiry, inform students that they have two gifts that are in the shape of pyramids and they wish to send them in one gift box to their friend who lives overseas. The focus will not be on the materials, but on the size and shape of the box containing the two different sized pyramids (e.g., bubble wrap for packaging is not a necessity).
2. Display the Inquiry question, *"What is the best box to hold two different sized items that are packaged as pyramids?"* Ask students to record the question in their workbook and add the title DISCOVER.
3. Inform them today in the Discover phase they will be exploring the mathematics required to answer the question. Brainstorm with students the mathematical opportunities in the inquiry question. *What mathematics do you think we might use to answer this question?*
4. Show a pyramid and prism. Have students share with a partner what they already know about pyramids and prisms (box shapes), before **sharing with the class**. This provides an opportunity to ascertain students' prior knowledge of the features and terminology of 2D shapes and 3D objects. Emphasise the value in using mathematical terminology while sharing. During the class sharing ensure students have a clear understanding about the differences between 2D shapes and 3D objects and that the following terminology is reviewed: base, height, length, width, edge, face, vertex, apex, triangle, square, rectangle. Revoice student responses with the mathematical terminology, as needed.
5. As this inquiry focuses primarily on pyramids and prisms and their construction from nets, it is important that students have a clear understanding that:
   - Pyramids and prisms are both polyhedra (closed three dimensional objects formed by four or more flat faces joined along edges).
   - Pyramids are named according to the shape of the base. The remaining faces are triangular and meet at an apex (which is one of the vertices). Examples are square pyramid, and hexagonal pyramid.
   - Prisms have two parallel, congruent faces (often placed to be the 'top' and the 'bottom') These faces are connected by parallelograms (usually rectangles). These are often called the 'sides'. Prisms are named according to the shape of the base (e.g. rectangular prism (most boxes); triangular prism such as a classic tent design – use one of the triangles as the base to see the prism most clearly).

   Discuss these definitions with students and review or introduce the terms polyhedron (plural: polyhedra or polyhedrons), congruent, parallel and parallelogram. The face of a prism that is regarded as the base can change according to orientation.

Drawing Nets

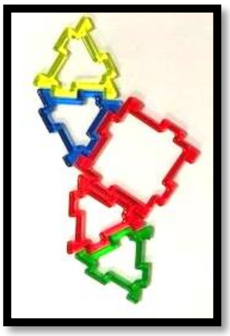
6. To help students make the connection between the two-dimensional faces and the three-dimensional object, provide each pair with a suitably-sized everyday object or a geometric solid in the shape of a pyramid or prism. Discuss the features of the objects and explain their geometric names.
7. Ask students to create different possible nets for the object by tracing around each face. Model how to trace faces of an object so that they are connected along edges to form the net of the object in just one piece. **Alternatively** provide pairs with geometric construction materials such as polyhedron frameworks, polyhedron magnetic strips or Geoshapes to construct a variety of nets for prisms and pyramids. Have them record their work by drawing the variety of nets in their workbook or photographing them into a digital notebook.
8. Allow a few minutes for students to try to create some different nets of the object.

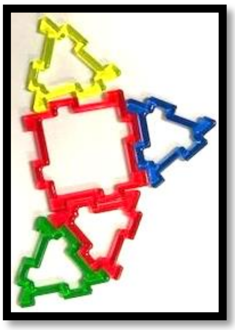
9. Share students' tracings or photographs of nets made, comparing similarities and differences. Select different nets to discover if they represent the same three-dimensional object (see pyramid and cube examples below). Ask students if the different nets will make the same three-dimensional object and what they could infer from this.

    a. It may be necessary to provide explicit instruction on how the shape and the relative position of each face determine the net of the object. Allow a student who realises their net is incomplete, or has extra faces, the opportunity to correct it. If a student does not realise their net is faulty, prompt them to match the number and type of faces in the net to those on the object.

    b. Highlight which of the edges of the solids are evident in the nets (joining adjacent faces).

    c. Some of the cardboard solids (e.g. toothpaste box) can be unfolded to see the nets that were used to make them.

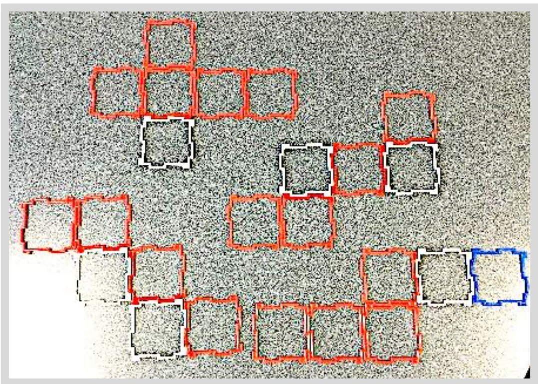




Both nets are for square based pyramids.

How are they different?

What similarities do you notice?



Will all these nets make the same three-dimensional object (ignore colours)?

How could we check?

Can you suggest another net for this three-dimensional object without actually making it? How do you know this net will work?

What do you need to know about a three-dimensional object to enable you to visualise its net?

Conclusion

10. Ensure that students understand that a net is a two-dimensional representation that can be folded to form a polyhedron. Have students review this by sharing with a partner what they define as a net. Share definitions before each student writes down their own definitions. For example,

- A net is a two-dimensional representation that can be folded to form a three-dimensional object.
- A net is a two-dimensional diagram which shows how the edges and faces of a polyhedron are joined.)

Mathematical curiosities

11. Some students may be interested to know that:

- Some complicated nets can be folded in different ways to make different polyhedrons. For example, a net of an octahedron can be folded one way to make an octahedron and another way to make a 'boat'.
- Not all polyhedrons have nets that can be made from one piece of paper – for some complicated polyhedrons, the faces will always overlap each other when flattened.