Pyramids in a Box
Lesson 2: Devise Phase

Australian Curriculum: Mathematics (Year 6)

ACMMG140: Construct simple prisms and pyramids.

- Constructing prisms and pyramids from nets and skeletal models.

ACMMG141: Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles.

Lesson abstract

Students make a plan to answer the Inquiry question. They determine what 'best' means in this context, acknowledging that having a small amount of unused space in the box is a key consideration. Students draw nets and construct their two pyramids to be packaged, before sharing their initial ideas for constructing their best box.

Mathematical purpose (for students)

Dimensions are important when planning and drawing nets.

Mathematical purpose (for teachers)

Appropriate units of measure and accurate measurement are required to record net dimensions. Sharing will highlight ideas that help improve the accuracy of nets and models, including how to get the angles right.

At the end of the Devise phase, groups will be able to:

- Describe the pyramids and box (prism) they have initially considered.
- Share initial planning ideas - nets, dimensions and model(s), seeking clarification from peers.
- Consider strengths and weaknesses of initial plans and pathways to overcome challenges presented.

Lesson Length 60 minutes

Vocabulary Encountered

- dimensions
- model

Lesson Materials

- Student workbooks
- Chart paper for poster (optional)
- Rulers and protractors (1 per student)
- Light card and tape for construction.

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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Criteria for 'Best'

Inquiry Question:
What is the best box to hold two different sized items that are packaged as pyramids?

1. Inform students today's lesson is in the Devise phase and they will be planning initial ideas to assist them as they work towards a solution for the inquiry question. Have them add the title DEVISE underneath the previous lesson's ideas and nets.
2. Revisit the question, "*What is the best box to hold 2 different sized items that are packaged as pyramids*?" Remind students:
    - *They will be constructing a box for birthday gifts to be sent overseas.*
    - *The box needs to hold two different sized objects that are already packaged as pyramids.*
    - *The cost of overseas postage is related to the size and weight.*
    - *Parcels may be exposed to rough handling.*
3. With a partner have students brainstorm what they think the "best box" means and record suggestions in their workbook. '*Best'* is a relative term used to justify their solution later in the Inquiry and is subject to personal preference. Possible attributes might include: lightweight, durable, and an appropriate size that is not too big, easy to make. Share and justify suggestions as a class.
4. During the sharing, remind students that reducing the amount of **left over space** in the box may reduce the postage, so it is a key consideration.
    - *The box needs to be just big enough to hold the pyramids. If there is too much empty space, the pyramids could move around and get damaged.*
    - *Making the box as small as possible may reduce the postage.*
5. Construct a class poster or slideshow slide of '*best'* for this context, to refer to throughout the inquiry.

Plan and Construct Pyramids

6. Have students return to the inquiry question they have recorded in their workbooks and highlight the key mathematical information in the question. For example:

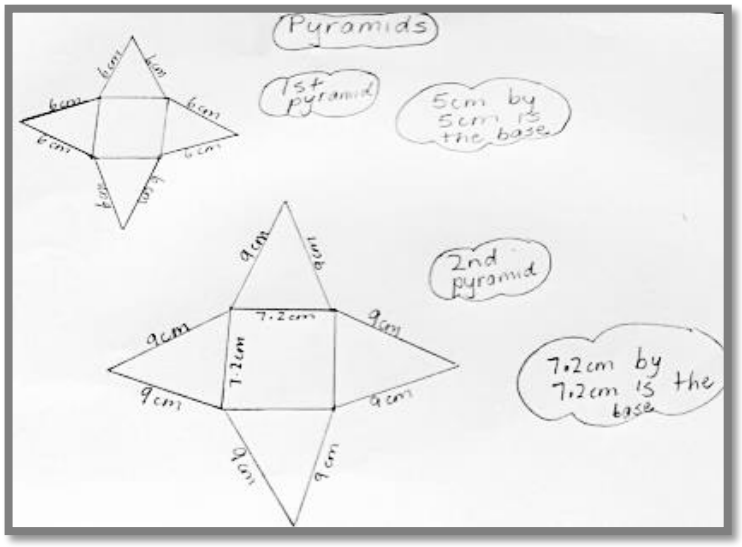
    "*What is the best box to hold 2 different sized items that are packaged as pyramids*?"

7. As a class, discuss how they could proceed to answer the question. Explain that they will need to construct the pyramids first. They can have pyramids of any size and shape that they like, as long as they are different. When they have made their 2 pyramids they can begin to plan their box.
8. Have each group:
    - Determine and record the size and shape of the pyramids they will use (the presents to send overseas).
    - Draw nets for their chosen pyramids in their workbooks and record the dimensions. To help students draw these nets, have them refer to the net representations they made from the geometric shapes and construction materials in the Discover Phase.
    - Groups should refrain from simply copying or tracing around commercially produced nets.

    Remind students that group jobs can be shared but it is the expectation that everybody has drawn and labelled at least one net.
9. Circulate and observe groups as they draw the nets and construct the pyramids to ensure they are recording their net suggestions and measuring accurately. Prompt where necessary *e.g. How are you recording that? If I wanted to make your three-dimensional object, could I do so from what you have written on your plan? What else to you need to add to make it clearer?*

10. After checking their net representations and dimensions with the teacher, have students copy their net onto card and construct their pyramids.
    - To make the net on card, students might cut out one triangle first, and then trace around it four times.
    - Alternatively, they might measure angles with a protractor to construct identical triangles.
    - They will need to make sure that the angles of the base square or other polygon are as required (e.g. measure 90 degrees for a square base).
11. **Allow groups thirty minutes to construct their** pyramids and to begin work on planning their box before moving to the Checkpoint.

Example of Student Sketched Plan



These students chose two square pyramids and observed that the slanting sides will be the same length, so the triangular faces are isosceles. Probably they drew the equal sides of the two isosceles triangles (sides 6 cm, and sides 9 cm), and then measured the third sides to get the dimensions of the bases.

Enabling Prompts (Use when groups encounter setbacks)

- What are you trying to do?
- What have you tried?
- What else could you try?
- What do you need help with to enable you to move forward?
- Have you revisited the nets made in the Discover Phase for an idea?

Extending Prompts

- Challenge groups that need extending to construct more complex polyhedra such as hexagonal pyramids.
- Suggest that students record the height of their constructed pyramids, not just the length of sloping sides.
- Discuss how students could construct their planned pyramids more accurately – not just by trial and error or by tracing around an initial shape.

Checkpoint

12. Groups begin by sharing their ideas, even if incomplete. Remind them that they are expected to listen actively to what other groups have done and to seek clarification if they do not understand. Sharing needs to be done as a whole class activity in the Devise Phase to showcase the ideas of others.
13. As each group shares, have them:
    - Explain what they have done and why they have chosen those three-dimensional objects, nets and dimensions.
    - Answer any requests for clarification.
    - Present any challenges they are facing and require assistance with.
    - Consider how their constructions could be made more accurately.
    - Consider what they will need to do next to answer the question.

Possible Student Work and Questions

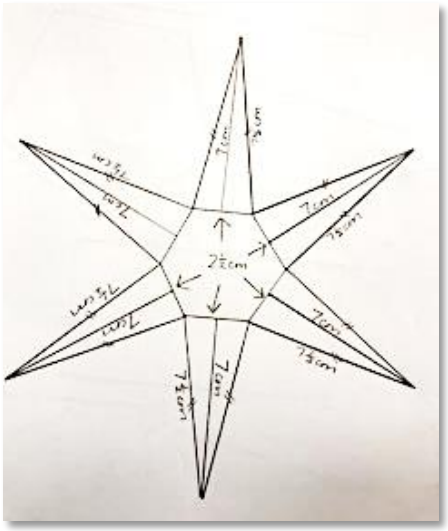
- Ideas which have the potential to improve the accuracy of the mathematics should be highlighted.

Teacher: Why have you placed a line in the centre of the triangular faces?

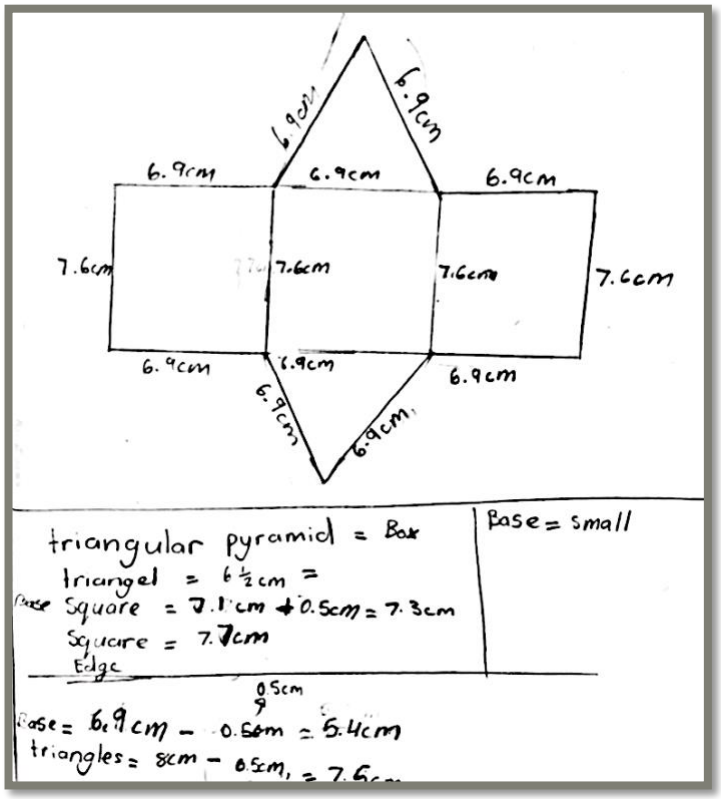
Student: So, my faces are all exactly the same height and will meet right at the apex. They will be equally spaced around it.

Teacher: I really like this labelled diagram with the clear measurements. If I wanted to make the pyramids, I could easily use what you have there.

But how did you make sure the angles were all the same size?



- Modelled requests for clarification:



Response 1

Teacher: You have indicated your box would be a triangular pyramid. What do you know about the faces of a triangular pyramid?

Student: They are all triangles.

Teacher: Look carefully at the faces on your net. Are all your faces triangles?

Student: No, only two are triangles, the rest are rectangles.

Teacher: What do we call an object that has two triangular faces connected with rectangular faces? Try imagining your net folded up.

Student: A triangular prism.

Response 2

Teacher: I'm confused as your calculations do not match the dimensions on your net. Check them to see if there are any mathematical errors or add more detail to make your thinking easier to follow.

Conclusion

14. From the sharing, encourage groups to consider other groups' ideas that they could use. Allow groups a few minutes to consider the strengths and weaknesses of their designs and record ideas that will improve their plan or build on in the Develop phase (e.g. *We could challenge ourselves to try different based pyramids*; *We can see how to make a big box, but tomorrow we need to think how to make it smaller*.)