Pyramids in a Box
Lesson 3: Develop Phase

Australian Curriculum: Mathematics (Year 6)

ACMMG140: Construct simple prisms and pyramids.

- Constructing prisms and pyramids from nets and skeletal models.

ACMMG141: Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles.

Lesson abstract

Students check and refine plans before implementing them to construct their box. They gather mathematical evidence as they work through iterations to reduce the amount of left over space in their box.

Mathematical purpose (for students)

Improvements are necessary to reduce the amount of unused space in the box. In the best package, the pyramids will be packed close together and only a small amount of space will be unused.

Mathematical purpose (for teachers)

As pyramids are positioned more compactly, the opportunity for a smaller sized box becomes apparent. Sharing sessions help ensure mathematical thinking and adjustments are recorded. The mathematics involved in construction in previous lessons is re-used. At the end of the Develop phase, groups will be able to:

- Provide models of different sized pyramids and boxes that have been constructed accurately from nets.
- Share the mathematical thinking and labelled nets they have used to make a better box.

Lesson Length 2 x 60 minutes

Vocabulary Encountered

- iteration
- minimal

Lesson Materials

- Construction materials – paper, lightweight card, tape
- Student workbooks
- Rulers and protractors (1 per student)
- Cameras or tablets (optional)
- Chart paper for poster
- Evidence Triangle poster (optional, see Teachers' Guide, or download from https://www.mathsinquiry.com/resources.html)

We value your feedback after these lessons via https://www.surveymonkey.com/r/CV2TXTT

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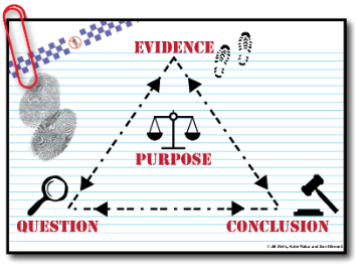
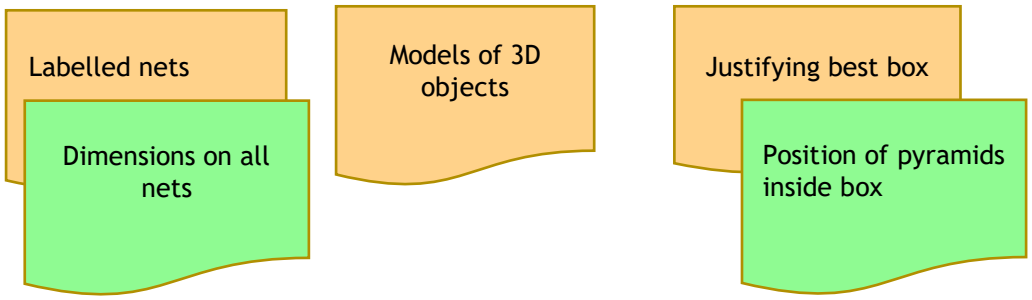
Refine Plans and Complete Constructions

1. Inform students in the Develop Phase they will be refining plans and constructing their remaining three-dimensional objects. Have them add the title DEVELOP underneath the previous lesson's ideas and representations.
2. Focus students on the necessity to provide sufficient mathematical evidence to convince others that their box is the best box, or at least a very good box, to hold their two pyramids.
3. Discuss with students the connections between the inquiry question, the evidence gathered and the conclusion. The Evidence Triangle poster shows this graphically.



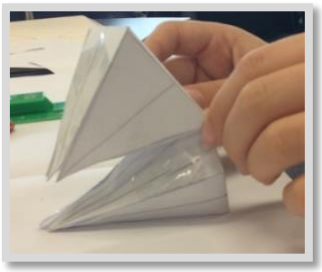
   - *Evidence* is a record of the mathematical thinking and processes used to answer the inquiry *question*.
   - Sufficient *evidence* is required to convince others that the *conclusion* answers the inquiry *question*.
4. Have students Think-Pair-Share (recording each idea on a sticky note) what evidence they would require to convince others that their constructed box is the best. Collect and collate the sticky notes, summarising the ideas on a poster. Encourage groups to refer to the poster as they prepare their evidence.



5. Teacher prompting or additions may be required to ensure the poster includes sufficient forms of mathematical evidence for convincing others. Evidence required should include:
   - Nets of pyramids (gifts) and prism (box) labelled with dimensions
   - Reasons for choosing the dimensions and shape of the box
   - 3D models of pyramids and boxes tried
   - Any adjustments made to nets and reasons why
   - Photographs of possible positions of pyramids and the final positioning used in the box
   - Explanation of the mathematics used

Use feedback to refine and complete plans

6. Have groups revisit their plans made in the Devise Phase and review what they need to do next. This should include drawing the net(s) and constructing their box(es), and manipulating the pyramids to see how they best fit in the box. Ensure students complete all nets (including representations) and the construction of both pyramids and the box.



7. Have groups position the pyramids in the box and draw or photograph them inside for sharing. Once a few groups have completed this, bring the class together for a quick Checkpoint to elicit the need to work through iterations of the box design to achieve the best outcome.

Extending Prompt

- Challenge students who finish early to:
    - Investigate boxes that are not rectangular prisms (e.g. hexagonal prism);
    - Replace one of their pyramids by another that is more difficult to pack near the first;
    - Make an excellent packing box of a different shape (e.g. prism with faces that are parallelograms but not rectangles, truncated pyramid (pyramid with top cut off).

Iterative checkpoints: creating the best box

8. Invite two groups to show their box and the pyramids positioned inside. Remind them the best box will be the box that fits the pyramids with little left-over space. Ask each group to explain whether their box is the best box and what they could do to improve it. Groups are likely to identify they still have too much left over space or insufficient space so their box cannot close.

9. If not identified by students, point out the amount of left over space in their box and ask the group if they can think of a way to reduce it. For example:

   T: Do you think this box is the best box to package the pyramids?

   S: No, there is too much left over space in our triangular prism.

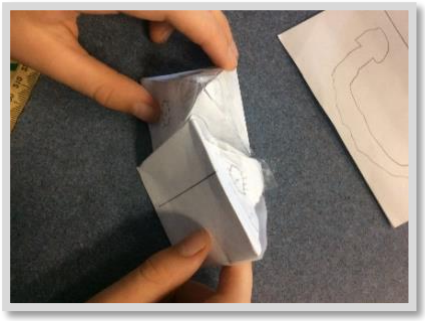
   T: Have you tried positioning the pyramids in another way?

   S: Yes. When we did we found there was 2 centimetres of empty space along the base of the prism. (see RHS image below – note that this is not the only region of empty space in the box)

   T: What could you do to reduce this unused space?

   S. We can make the rectangular faces in the net of our prism each 2 centimetres shorter in length.

   T: Good suggestion. Give it a try.

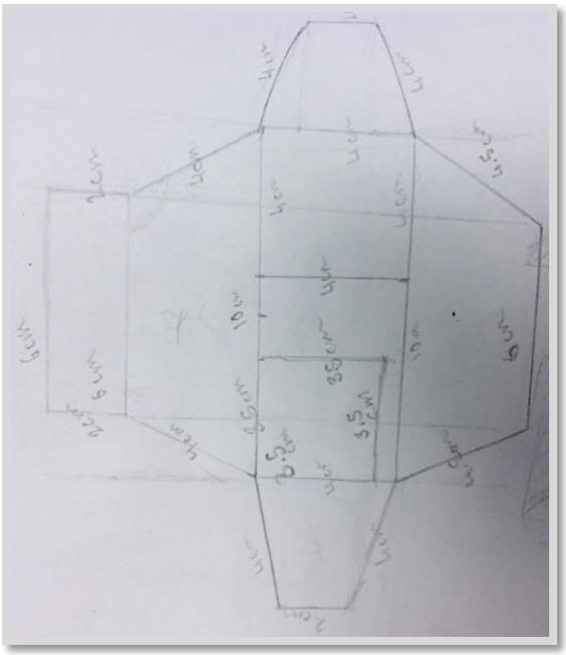

10. After sharing, have groups continue to manipulate their pyramids and work through iterations to refine their box to reduce the amount of left over space. As groups are working on refining their box, visit each group to ensure they are recording mathematical reasoning for box dimensions and not simply relying on trial and error. If students are unsure how to use mathematics to assist them, prompt them with open questions. For example: *Now you have decided how you will position your pyramids, what measurements should you check to help you reduce the amount of left over space?*

11. Once each group has completed at least one iteration of the box, spend some time reviewing progress at another Checkpoint. Choose groups to share their progress and/or challenges faced. This could be groups progressing well, or groups stalled in their progress. This has a two-fold purpose:

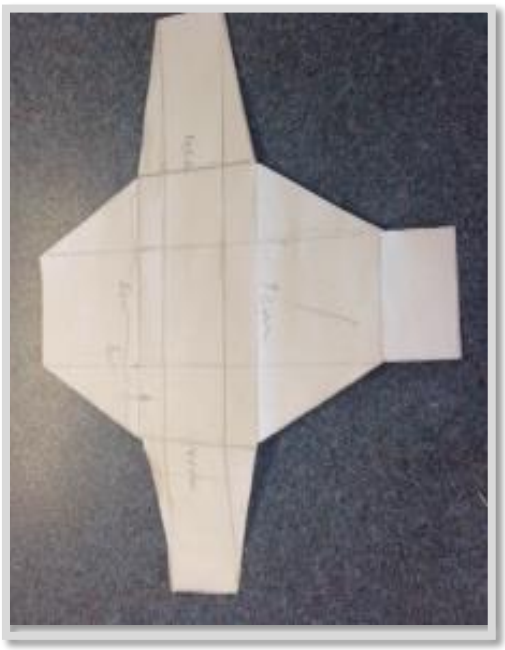
    - Providing other ideas and suggestions for working through challenges.
    - Ensuring sufficient mathematical evidence is being gathered.

Student Group Example 1

First attempt



Final net after iterations



Student Group Example 2

Pictorial record of progress to be used as mathematical evidence when justifying best box.
Example of trapezoidal prism. The trapezium base is the back (or front) of the box in the picture) and the four sides are rectangle. The photos show how to pack the two pyramids.

Conclusion

12. Have groups use ideas or feedback from the sharing to check that they are gathering sufficient to support their conclusion. If they have not done so, they will need to address this before continuing with any further improvements to the box.