re(Solve) MATHS BY INQUIRY Odds and Evens

Y4

Summary of learning goals

- This sequence focuses on developing students' understanding of the properties of odd and even numbers. Students explore the results of adding and subtracting odd and even numbers, and use these results to make generalisations.
- The primary generalisation that forms through the sequence is that adding an odd number of odds will always produce an odd total. Students apply this generalisation to solve problems and to explore patterns.

Australian Curriculum: Mathematics (Year 4)

ACMNA071: Investigate and use the properties of odd and even numbers.

ACMNA083: Find unknown quantities in number sentences involving addition and subtraction, and identify equivalent number sentences involving addition and subtraction.

Summary of lessons

Who is this sequence for?

- Students should have prior understanding of what makes a number even or odd. They need to view even numbers as numbers that are divisible by 2 with no remainder, and view odd numbers as those with a remainder of 1 when divided by 2.
- Students employ their mental computation skills.

Lesson 1: Number Maze

Students are presented with a number maze. As they move through the maze, students need to total the numbers on the path they follow. The challenge is to complete the maze with an odd total. By exploring possible solutions to the maze, students form generalisations about the results of adding odd and even numbers. They see that an odd number of odds in the sum is required to get an odd total.

Lesson 2: 10 to 1

This task challenges students to use addition, subtraction and the numbers 10 down to 1 to make a total of 27 and a total of 12. They will see that it is possible to create only an odd total with these numbers. Students look at the largest and smallest positive totals for the problem. They also look at the ways in which they can modify the task to create even totals.

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Reflection on this sequence

Rationale

An important aspect of algebra is that of generalising number and arithmetic structures. This sequence focuses on forming generalisations when performing additive calculations with odd and even numbers. Students explore addition and subtraction with differing amounts of odd and even numbers to determine if their sum will be odd or even. Students' understanding and the ability to form a generalisation is reliant on understanding odd numbers as those numbers that when divided by 2 have a remainder of 1; and even numbers as those that can be evenly divided by 2.

reSolve mathematics is purposeful

- Students are encouraged to experiment, form hypotheses, and then test and prove their theories.
- This sequence draws on prior knowledge but introduces new contexts for study. It is a rigorous examination of a simple set of concepts with practical applications to number sense.

reSolve tasks are inclusive and challenging

- Both tasks in this sequence begin with students being given an unstructured problem and then allowed time to interact with the problem on their own terms and to experiment with its possibilities.
- As the sequence focuses on finding and testing hypotheses, a range of approaches to suit students' skill levels are provided.
- The lessons focus on students sharing and working together to recognise patterns in their collective data, and are structured so that all students can make a valuable contribution.

reSolve classrooms have a knowledge-building culture

- This sequence relies on the class to collect and analyse data as a group. Independently, students will struggle to collect enough data to draw generalisable conclusions.
- The focus here is independent exploration and then collective discussion and analysis during which each student provides their own examples.

LESSON 1

re(Solve) MATHS BY INQUIRY

Odds and Evens

Number Maze

Y4

About this lesson

The task presents students with a number maze. As they move through the maze they need to total the numbers on the path they follow. The challenge is to complete the maze with an odd total. By exploring possible solutions to the maze, the students form generalisations about the results of adding odd and even numbers. They see that an odd number of odds in the sum is required to get an odd total.

Australian Curriculum: Mathematics (Year 4)

ACMNA071: Investigate and use the properties of odd and even numbers.

ACMNA083: Find unknown quantities in number sentences involving addition and subtraction, and identify equivalent number sentences involving addition and subtraction.

Mathematical purpose

- The students develop an understanding of adding different combinations of odd and even numbers. Using the properties of these numbers they explain why:
  - ◊ odd + odd = even
  - ◊ even + even = even
  - ◊ odd + even = odd
- They will see that an odd number of odds is required for the total to be odd.

Learning intention

- To explore the properties of numbers to create odd and even totals using addition.

Time

A lesson of approximately 1 hour.

Vocabulary

- 'Odd' and 'even' numbers, as defined in the Teacher background information.

Resources

- Student Sheet 1 – Number Maze (one per student)

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Teacher background information

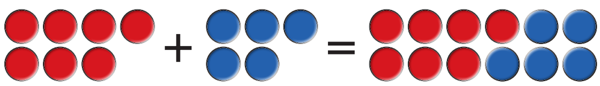
An odd number of odd numbers is always required in addition to give an odd sum. This is due to the structure of odd and even numbers.

When asked to define odd and even numbers, students often explain that odds end in 1, 3, 5, 7 and 9, whereas evens end in 0, 2, 4, 6 and 8. This is a way to quickly identify whether a number is odd or even, but it does not define 'oddness' and 'evenness'.

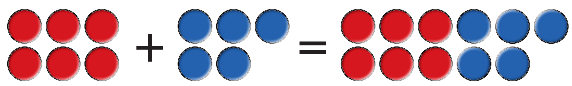
Students need to understand even numbers as those divisible by 2 with no remainders, and odd numbers as those divisible by 2 with 1 remainder.

Consider 7 + 5 and 6 + 5 with the numbers represented in two rows:

$$7 + 5 = 12$$

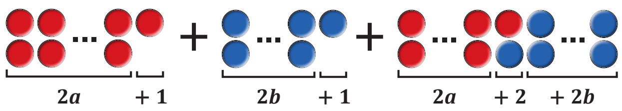


$$6 + 5 = 11$$

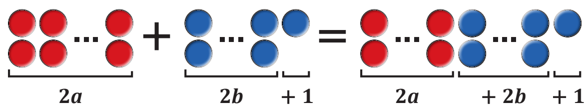


This helps explain the general case:

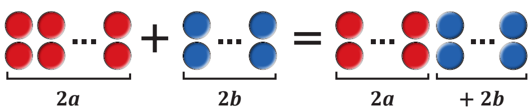
odd + odd = even



even + odd = odd



even + even = even



Exploring the maze

Resources: Present students with Student Sheet 1 – Number Maze.

Pose the challenge:

- *You need to move from 5 at the start through to 14 at the end.*
- *You can use horizontal, vertical and diagonal movements.*
- *Add together the numbers of each box you pass through.*
- *The aim is to finish with an odd total.*

5	32	4	29	26
13	6	18	27	3
14	15	10	38	12
22	42	9	16	2
8	19	12	40	14

Allow students to explore the maze. As they work, discuss some of the computational strategies that are being used.

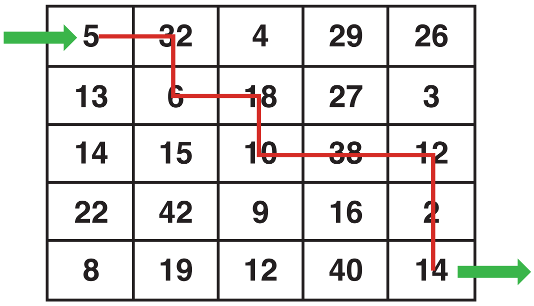
Enabling prompt:

- If students are caught up in the sums, they can use a calculator. The focus of this task is not completing the arithmetic.

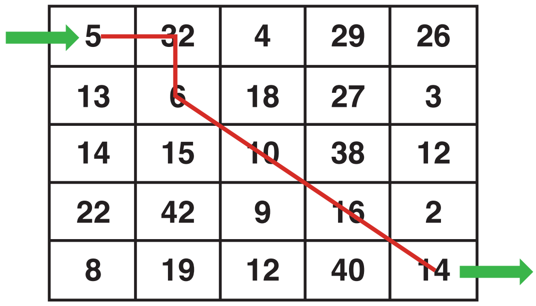
Once students have found one pathway with an odd total, challenge them to find others. There are multiple ways through the maze that generate an odd total. Using different coloured pencils makes it easy to keep track of all the discovered paths.

Possible student responses:

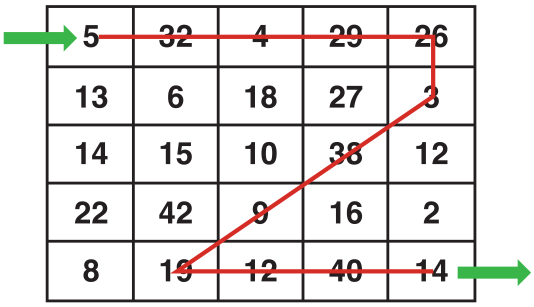
Four possible pathways that will add to an odd total are illustrated.
These are not the only solutions to the problem.



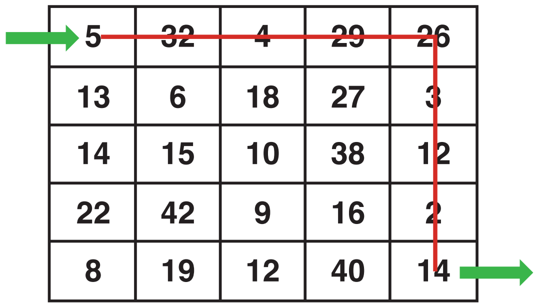
Total is 137.



Total is 83.



Total is 231.



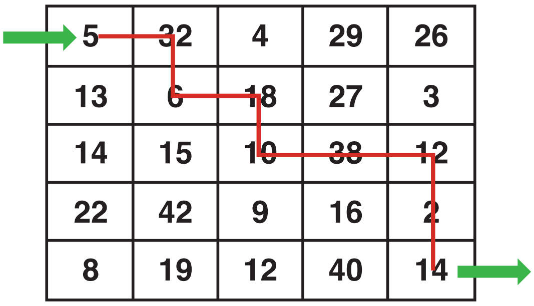
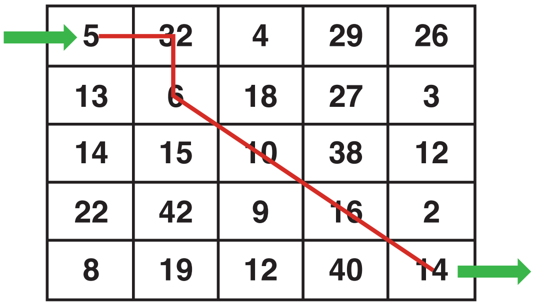
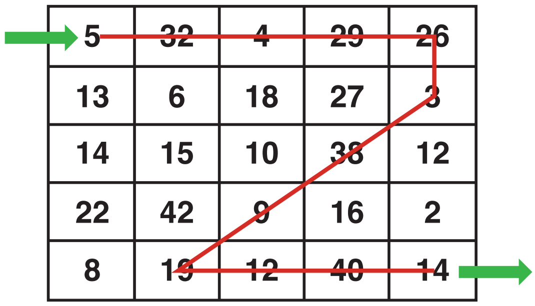
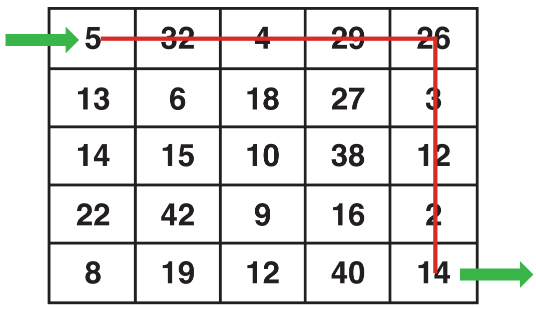
Total is 127.

After the students have found a few pathways, **pose the question:** *Is it possible to know that your pathway will have an odd total without actually adding all the numbers? How do you know?*

This challenge guides the remainder of the investigation.

Gathering data

Have students collect data on the possible pathways by recording how many odd and even numbers are in each path. The four possibilities shown previously are included in the table below.
Draw a similar table on the whiteboard to record some of the possible pathways students have found.

	Number of odd numbers	Number of even numbers	Total
	1	8	137
	1	5	83
	5	6	213
	3	6	127

Ask the students to look at the table and see if they can make any observations.

T **Teacher note:**

- Looking at the example table, it is clear that an odd total requires an odd number of odds. There can be an odd or even number of even numbers. Make sure to collect enough data as a class that the students can make this observation.

Pose the challenge: *Can you get an odd total with an even number of odds?*

Allow the students to explore this question. As they do, they can contribute further data to the table for new pathways discovered.

Pose the questions: *Why does an odd total require an odd number of odds in the sum?*
Is this always the case?

Allow the students to explore.

Investigation

Prompts to direct the investigation and challenge students' thinking and reasoning

- *What if you started with a smaller sum of numbers?*
  - ◊ Starting with a smaller problem is a useful strategy at this stage. Using just two numbers it can be seen that:
    - ▪ even + even = even
    - ▪ odd + odd = even
    - ▪ even + odd = odd
  - ◊ Looking at the sum of three numbers:
    - ▪ even + even + even = even
    - ▪ even + even + odd = odd
    - ▪ even + odd + odd = even
    - ▪ odd + odd + odd = odd
- *What happens when you subtract an odd number from an even number or subtract an even number from an odd number? What about subtracting two evens or two odds?*
  - ◊ Subtracting two evens or two odds always results in an even.
  - ◊ Subtracting an even from an odd or an odd from an even always produces an odd.
  - ◊ See Teacher background information for algebraic representation.
- *Will this work with any numbers? What about two- or three-digit numbers?*
  - ◊ Students should be encouraged to explore a variety of numbers to move toward making the generalisation.
- *Why is an odd number of odds needed in the sum?*
  - ◊ One way this can be shown is as an equation:
    - ▪ odd + odd + even + even = even
    - ▪ (odd + odd) + (even + even) = even
    - ▪ even + even = even
  - ◊ whereas:
    - ▪ (odd + odd) + (odd + even) = odd
    - ▪ even + odd = odd
- *Is it possible to go through every number in the maze and finish with an odd total?*
  - ◊ There is an even number of odds in the maze, which will result in an even total. Ask the students in what way they could change the maze to create an odd total. They should see that changing one even number to an odd number will give an odd total.

Reflection

Select some students to present their working to the class.

Ask the students to look for a path that will produce an odd total and one that will produce an even total. Ask them to justify how they know whether the total will be odd or even.

The focus of the reflection is to move students from the need to calculate the total to generalising that:

- even + even = even
- odd + odd = even
- even + odd = odd

...and from this, generalising that:

- An odd number of odds will produce an odd total.
- An even number of odds will produce an even total.
- The number of evens is not relevant when deciding if the total is odd or even.

Further activities

Activity 1

Provide the students with some addition problems. Rather than calculating the total, ask the students to quickly decide if the total would be an odd or even number.

Activity 2

Have the students design their own number maze in a 3×3 or 4×4 grid. Make sure they use both odd and even numbers.

- *How many pathways can you find that give an odd total?*
- *How many pathways can you find that give an even total?*
- *Does it matter where the odd and even numbers are placed?*
- *Does it matter how many odd and even numbers are in the grid?*

Where to next?

Lesson 2: 10 to 1 explores whether the conclusions drawn in this lesson are applicable to both addition and subtraction.

Number Maze

Name: ______________________________

5	32	4	29	26
13	6	18	27	3
14	15	10	38	12
22	42	9	16	2
8	19	12	40	14

5	32	4	29	26
13	6	18	27	3
14	15	10	38	12
22	42	9	16	2
8	19	12	40	14

5	32	4	29	26
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8	19	12	40	14

5	32	4	29	26
13	6	18	27	3
14	15	10	38	12
22	42	9	16	2
8	19	12	40	14

LESSON 2

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Odds and Evens

10 to 1

Y4

About this lesson

This task challenges students to use addition, subtraction and the numbers 10 down to 1 to make a total of 27 and a total of 12. They will see that it is possible to create only an odd total with these numbers. Students look at the largest and smallest positive totals for the problem. They also look at the ways in which they can modify the task to create even totals.

Australian Curriculum: Mathematics (Year 4)

ACMNA071: Investigate and use the properties of odd and even numbers.

ACMNA083: Find unknown quantities in number sentences involving addition and subtraction, and identify equivalent number sentences involving addition and subtraction.

Mathematical purpose

- This task applies the learning from the previous lesson on odd and even numbers. Students apply the generalisation that an odd number of odds will give an odd total, and see that this generalisation is true when working with both addition and subtraction.

Learning intention

- To explore the properties of numbers to create odd and even totals, using addition and subtraction.

Time

A lesson of approximately 1 hour.

Vocabulary

- 'Odd' and 'even' numbers, as defined in the Teacher background information.

Resources

- 10 square tiles; nine of the tiles need '+ (number)' written on one side and '– (number)' on the other, with one tile for each number from 1 through to 9. The tenth tile needs 10 written on both sides (without the +/– symbols).
- Alternatively, copies of the 10 to 1 Template for each student can be used. (Students should be able to easily change the numbers in the box. Two easy ways to do this are laminating the strips and using a whiteboard marker or using counters with a '+' on one side and a '–' on the other, flipping to change symbols as needed.)

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Teacher background information

The previous task in this sequence showed that an odd number of odds will always produce an odd total. Between 10 and 1 there are five odd numbers. Therefore, it will be possible to create only odd totals using + and – between the numbers 10 to 1.

The largest number that can be made is 55, using only addition signs. Encourage students to consider efficiency when calculating this total. There are four pairs of numbers that add to 10, with 10 and 5 left over:

$$10 + (9 + 1) + (8 + 2) + (7 + 3) + (6 + 4) + 5$$

Alternatively, there are five pairs of numbers that add to 11:

$$(10 + 1) + (9 + 2) + (8 + 3) + (7 + 4) + (6 + 5)$$

The smallest (positive) number that can be made is 1:

$$10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 - 2 - 1$$

All odd numbers between 1 and 55 can be made.

Introduction

Resources: Distribute sets of the flip tiles to students (or copies of the 10 to 1 Template if being used).

Pose these challenges to the students:

- Flip between addition and subtraction so that the result is 27.
- Flip between addition and subtraction so that the result is 15.
- Flip between addition and subtraction so that the result is 12.

Exploring variations

Allow the students time to investigate whether it is possible to make the numbers 27 and 12. They should begin with 27 and then move on to 15 and 12. Ask students to think carefully about the strategies they are using and to record any solutions that they find.

Enabling prompt:

- *Flip between addition and subtraction on the tiles. What different results can you make?*
- Students can record the different numbers that they make. They will start to see a pattern emerge that will lead them to the same thinking as that required in the main task.
- **Further enabling prompt:** *Using only the numbers 5 to 1, what totals can you make?*

Teacher notes:

- It is possible to make 27 and 15, but it is impossible to make 12. The reasoning for this is the focus of the task – see Teacher background information.

When most students have made 27, share some solutions and the strategies that they have used. Repeat for 15.

Give the students time to experiment with making 12. It is important that they attempt to do so, even though it is not possible. The students should reach the point where they are questioning whether it is possible, perhaps even starting to get frustrated. At this point, pose the question:

Which numbers can or cannot be made using addition or subtraction with the numbers 10 to 1? Why?

- No even totals can be produced (see Teacher background information).

Extending prompt:

- *Can you make every odd number between 1 and 55?*
  ◊ This is possible – working systematically from 1 or from 55 will help make each odd number.

Discussion

Select students to share the strategies they used to solve the problem. If members of the class have not found all solutions for every odd number between 1 and 55, this could become a class challenge.

Pose the question: *Can you modify the task to make both odd and even numbers?*

Students should see that it is possible to have only all odd totals or all even totals when using addition and subtraction. This is because an odd number of odds produces an odd total, and an even number of odds produces an even total. There are no other possibilities.

Further activities

Activity 1

Review the Number Maze from Lesson 1. The first time the maze was used, students used only addition. Moving through this time, they can use addition and subtraction. Students can choose where they add or subtract; however, the total should always be kept positive. Does the rule discovered in this lesson for creating an odd total change? Students should find that nothing changes, as addition and subtraction are inverse operations.

Activity 2

Investigate what the pattern of odds and evens looks like in Pascal's triangle. You could provide students with a completed (i.e. number-filled) Pascal's triangle and have them colour in every odd number, or even provide a blank template for Pascal's triangle and explain the way in which the numbers are generated.

Activity 3

Explore odd and even patterns in other number patterns. The Fibonacci sequence has the pattern odd, odd, even. Why is this so?

10 to 1 Template

Name: ________________________________

10 ☐ 9 ☐ 8 ☐ 7 ☐ 6 ☐ 5 ☐ 4 ☐ 3 ☐ 2 ☐ 1

10 ☐ 9 ☐ 8 ☐ 7 ☐ 6 ☐ 5 ☐ 4 ☐ 3 ☐ 2 ☐ 1

10 ☐ 9 ☐ 8 ☐ 7 ☐ 6 ☐ 5 ☐ 4 ☐ 3 ☐ 2 ☐ 1

10 ☐ 9 ☐ 8 ☐ 7 ☐ 6 ☐ 5 ☐ 4 ☐ 3 ☐ 2 ☐ 1