re(Solve) MATHS BY INQUIRY

Domino Arrays

Y3

Summary of learning goals

- To build students' multiplicative thinking through the use of arrays, whereby all parts of the array are not visible. The sequence encourages students to find the total number of items in an array by multiplication rather than counting by 1s or skip counting.
- The connection between area, arrays and multiplication is also made.

Australian Curriculum: Mathematics (Year 3)

ACMNA056: Recall multiplication facts of two, three, five and ten and related division facts.

ACMNA057: Represent and solve problems involving multiplication, using efficient mental and written strategies and appropriate digital technologies.

Summary of lessons

Who is this sequence for?

- It is assumed that students have used arrays, whereby all parts of the array can be seen to help solve multiplication problems. Students should be starting to build knowledge of their multiplication facts up to 10×10.
- Students should also be familiar with measuring the area of a flat surface by covering with a common unit, without leaving gaps or overlapping. This task reinforces the connection between area, arrays and multiplication.

Lesson 1: Which Uses the Most?

Students explore the number of dominoes required to cover a rectangle. They are presented with four domino designs, which are placed in different orientations. As the students work through the problem, they realise that the same number of dominoes will be used regardless of orientation. Students are then asked to explain why the same number of dominoes is used.

Lesson 2: Six Arrays

Students investigate six arrays that have been covered by dominoes. They are asked to record what they notice about the arrays. They are presented with four observations by other children and match each observation to their relevant arrays.

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Reflection on this sequence

Rationale

Arrays are a common representation of multiplication as coordinated rows and columns. There are many real-world examples of arrays, making arrays an accessible and powerful tool for students. This sequence extends and abstracts the array as a multiplicative structure. This abstraction occurs on two levels. First, students are not shown the full array, which means that they are unable to count the parts of the array in 1s. They are forced to think about the rows and columns simultaneously. This requires them to recognise the coordinated structure of the array, allowing them to multiply the number of rows by the number in each row.

The second way in which the array is abstracted is the use of dominoes. Each domino represents two square units in the array. The dominoes are also placed on the array in different orientations. The connection between the size of the array and the number of dominoes used to cover the array is not easily recognisable.

reSolve mathematics is purposeful

- Students use arrays to reason mathematically about the size of different rectangles. This connects multiplication to another aspect of mathematics: area.

reSolve tasks are inclusive and challenging

- Students are challenged to think more abstractly about arrays.
- Allowing students to use dominoes (either real or cut-outs) provides access for those who require additional support.

reSolve classrooms have a knowledge-building culture

- Students can approach the task in different ways. Through discussion, students build a collective understanding of ways the array can be used for mathematical reasoning.

LESSON 1

re(Solve) MATHS BY INQUIRY Domino Arrays

Which Uses the Most?

Y3

About this lesson

Students explore the number of dominoes required to cover a rectangle. Students are presented with four domino designs, in which the dominoes are placed in different orientations. As the students work through the problem, they realise that the same number of dominoes will be used regardless of orientation. Students are then asked to explain why the same number of dominoes is used in each design.

Australian Curriculum: Mathematics (Year 3)

ACMNA056: Recall multiplication facts of two, three, five and ten and related division facts.

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Mathematical purpose

- To build students' ability to abstract the array. Students are encouraged to find the total in an array by multiplying the number of rows by the number in each row, rather than by counting by 1s or skip counting. Students find that the number of dominoes required to cover a shape is the same no matter which way they are oriented.

Learning intention

- To compare the number of dominoes used to cover a rectangle in different ways.

Time

A lesson of approximately 1 hour.

Resources

- Student Sheet 1 – Domino Arrays (one per student)

Vocabulary

- area
- arrays
- orientation
- square units

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Introduction

Resources: Provide students with a copy of Student Sheet 1 – Domino Arrays.

Explain the context: *A class of students was asked to use dominoes to cover a rectangular piece of paper. These are four different methods that students in the class used. Which method will use the most dominoes?*

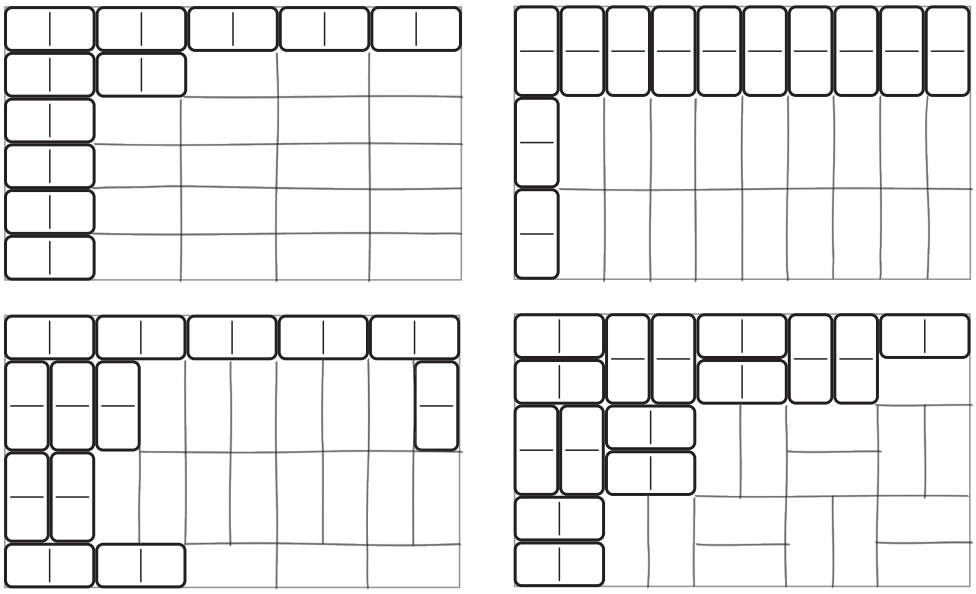
Allow students to use different strategies to determine how many dominoes will be needed in each picture.

Teacher note:

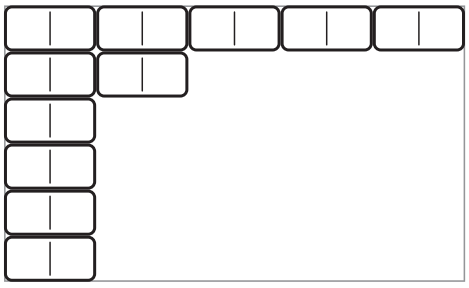
- This task is designed to help students start abstracting the array and links the array to measuring area. It is not immediately obvious to students that all methods will use the same number of dominoes.

Possible student responses:

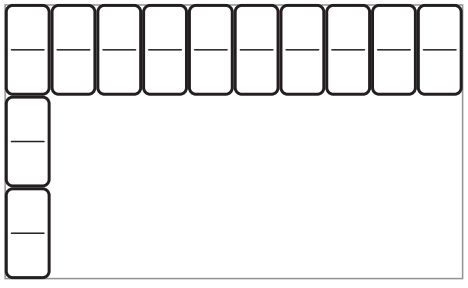
- Students may draw in the missing dominoes and then count how many dominoes are used; for example:



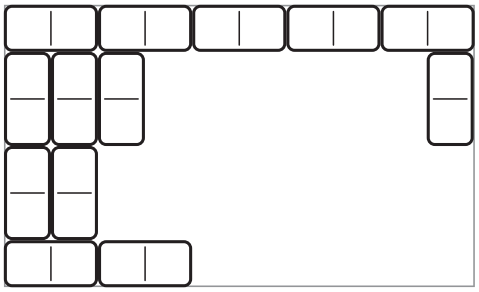
- Encourage students to look for arrays and use skip counting or known multiplication facts to calculate the number of dominoes rather than counting by 1s.



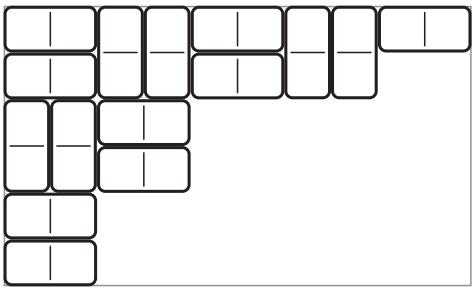
In this array, there are five dominoes across the top and six down the side, creating a 6 × 5 array.



This array shows three rows of 10 dominoes.



This array shows five dominoes across the top and the bottom. In the middle rows, 10 will fit across each row.



Putting another domino under the domino on the right completes a pattern of 10 dominoes. Three rows of each pattern of 10 dominoes can be made.

Ask the students to compare the different arrangements and to see in what ways they are similar and in what ways they are different. Have them develop an explanation for why there is the same number of dominoes in each arrangement.

Teacher note:

- One domino covers two square units of area. Each rectangle is 60 square units. This means that 30 dominoes will be used to cover each rectangle regardless of their orientation.

Extending prompts:

- *What other rectangles (with different dimensions) could you cover using exactly 30 dominoes?*
  - Assuming that one domino covers two square units, 30 dominoes cover any rectangle with an area of 60 square units.
- *Can you use dominoes to cover a rectangle of area 25 square units?*

Reflection

Ask students to share their strategies for:

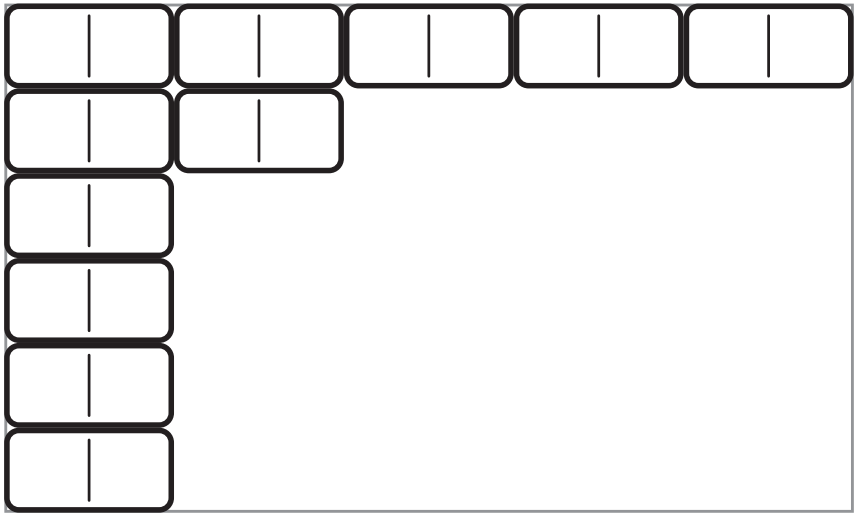
- calculating the number of dominoes needed
- justifying why each rectangle is covered by the same number of dominoes.

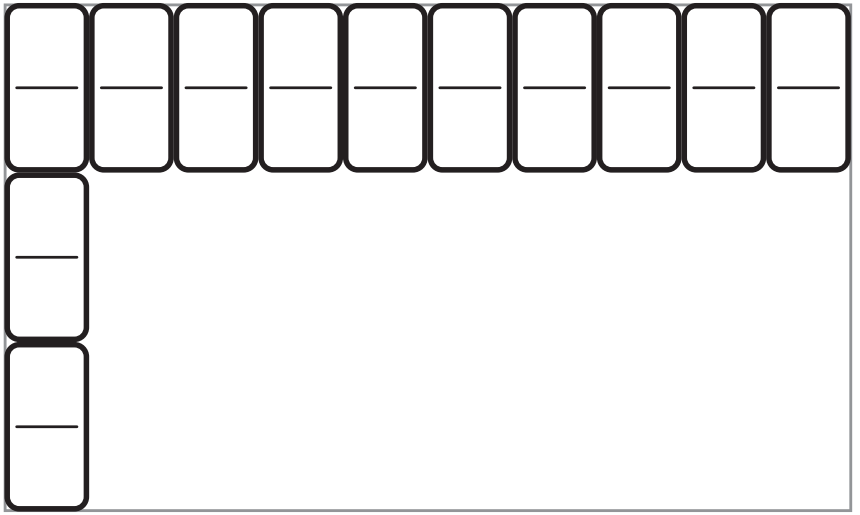
Where to next?

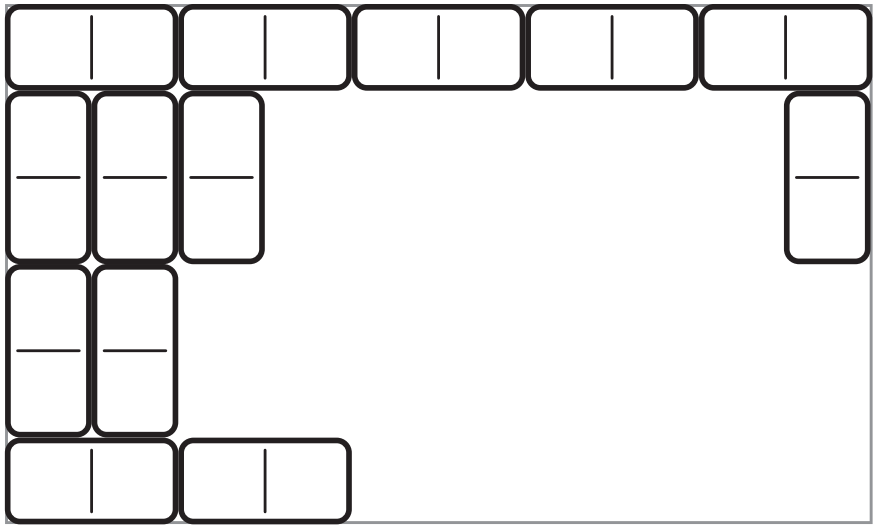
Lesson 2: Six Arrays is the second activity in this sequence. Students are shown six new rectangles covered by dominoes and make observations about the area of each rectangle.

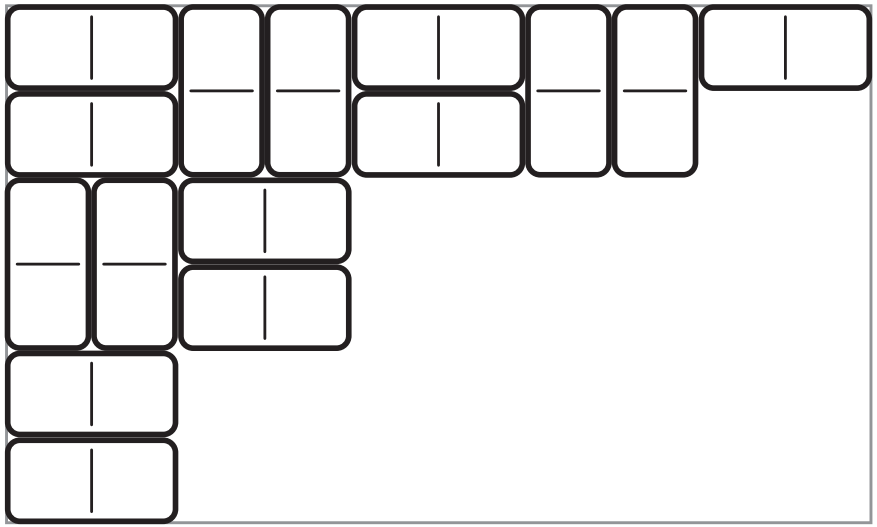
Domino Arrays

Name: _____









LESSON 2

reSolve MATHS BY INQUIRY Domino Arrays

Six Arrays

Y3

About this lesson

Students are presented with six pictures, each showing a rectangle that has been partially covered by dominoes. They are asked to record what they notice about the pictures. They are presented with four observations by other children and match each observation to the relevant picture.

Australian Curriculum: Mathematics (Year 3)

ACMNA056: Recall multiplication facts of two, three, five and ten and related division facts.

ACMNA057: Represent and solve problems involving multiplication using efficient mental and written strategies and appropriate digital technologies.

Mathematical purpose

- To use abstracted arrays to compare areas of different rectangles.

Learning intention

- To compare areas of rectangles, using dominoes.

Time

A lesson of approximately 1 hour.

Resources

- Student Sheet 1 – Six Arrays (one per student)
- reSolve PowerPoint *2a Six Arrays* (for display)

Vocabulary

- area
- arrays
- associativity
- orientation
- square units

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Introduction

Resources: Present students with a copy of Student Sheet 1 – Six Arrays.

Give students time to investigate the six pictures on the sheet and record their observations.

Resources: Show reSolve PowerPoint *2a Six Arrays*.

Read through the observations made by the four children. Ask the students to identify which rectangle each child is talking about.

Teacher note:

- The first observation states, '*One of [the rectangles] cannot be covered by dominoes.*' This means that one of the rectangles cannot be completely covered by dominoes without any of the dominoes overlapping or overhanging.

Questioning

I know which rectangles need the most and the least dominoes

What is the most number of dominoes used? What is the least number used?

- Rectangle 1 uses the most (36 dominoes) and rectangle 3 uses the least (30 dominoes).

I found a square!

How do you know that it is a square?

- When rectangle 4 is covered by dominoes it forms an 8×8 array of unit squares.

Is there only one square? Can you find more than one?

- There is only one square array, although others may appear square.

One of them cannot be covered by dominoes

Why is it that this rectangle cannot be covered by dominoes without any overlapping or overhanging?

- Rectangle 2 cannot be completely covered by dominoes without overlapping or overhanging. The rectangle's area is an odd number. As one domino covers 2 square units, one unit will always be uncovered.

Can you think of some other rectangles that would not be able to be covered completely by dominoes?

- Any rectangle that has an odd length and an odd width. This is because an odd number multiplied by another odd number will form an area with an odd total.

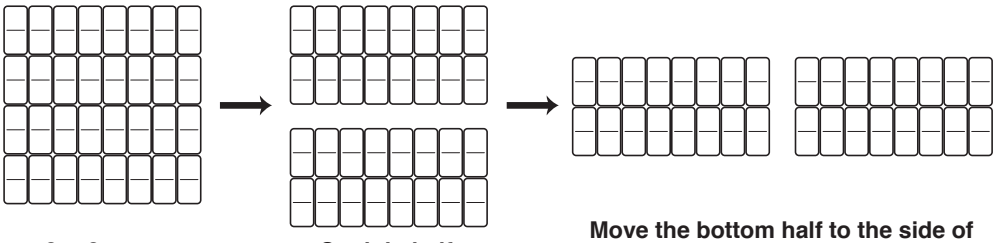
Some have exactly the same area

What are the different ways you can show that these two have the same area?

- Rectangles 4 and 5 have the same area. They both use 32 dominoes and have an area of 64 square units. The completed arrays can be rearranged to visually show they are equal.

Teacher note:

- This is an example of the associative property of multiplication. The associative property states that $a \times (b \times c) = (a \times b) \times c$.



8 × 8 array

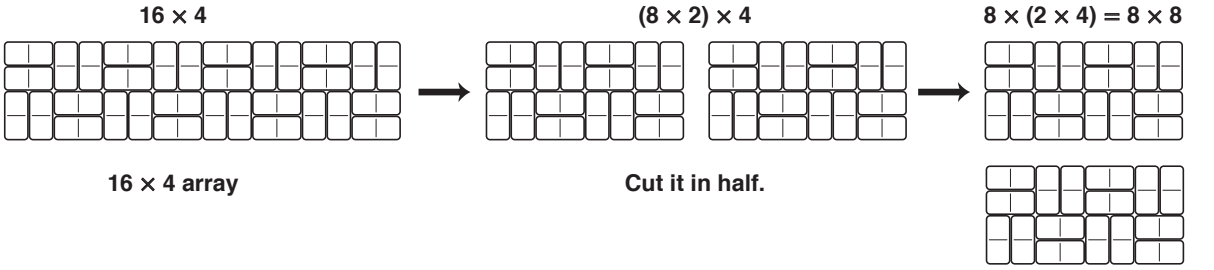
Cut it in half.

Move the bottom half to the side of the top half to form a 16 × 4 array.

8 × 8

(4 × 2) × 8

4 × (2 × 8) = 4 × 16



16 × 4

16 × 4 array

(8 × 2) × 4

Cut it in half.

8 × (2 × 4) = 8 × 8

Move the one half below the other half to form an 8 × 8 array.

Reflection

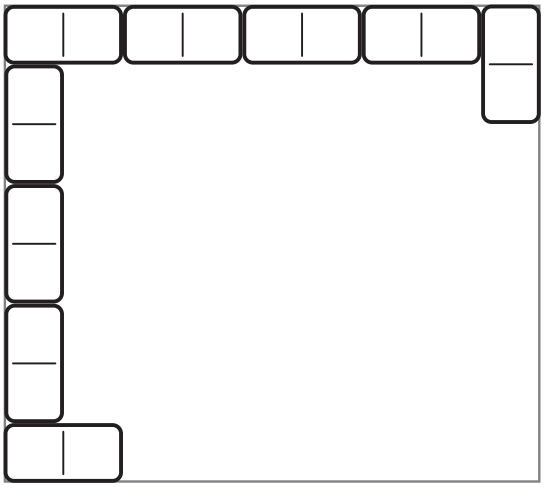
Have students share their observations and strategies.

Six Arrays

Name: _____

Investigate these six domino arrays that are partially covered by dominoes. What can you say about them?

1.



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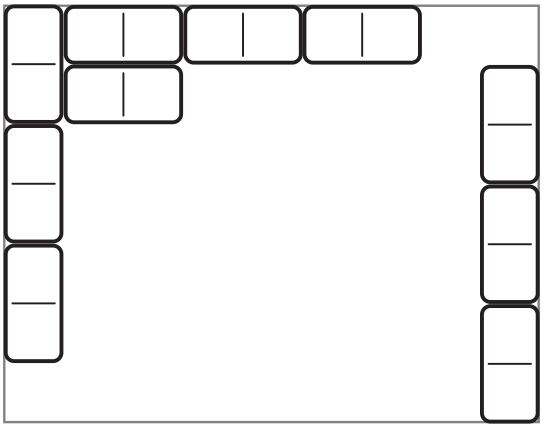

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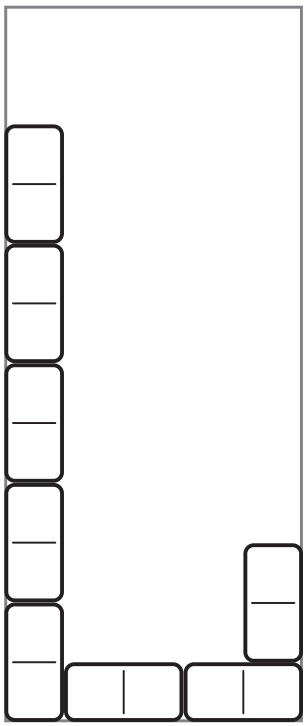

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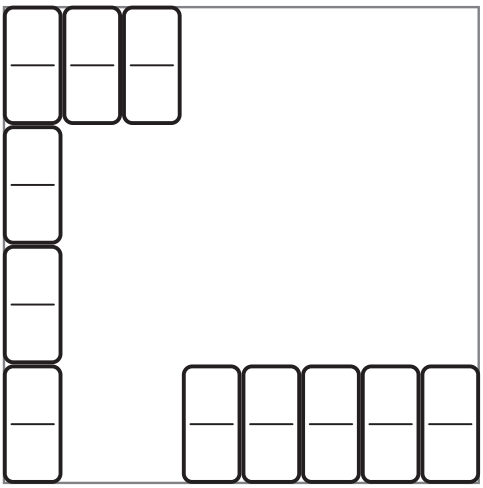

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