INTRODUCTION

re(Solve) MATHS BY INQUIRY

Working with Algebra

Y8

Summary of learning goals

- Students explore open tasks that have various solutions or strategies, which focus on developing core algebraic skills while enriching their experiences with using algebra. Students benefit from tackling tasks they do not already know how to do. Through comparing and contrasting ideas and experiences, students develop networks of concepts for themselves.

Australian Curriculum: Mathematics (Year 8)

ACMNA190: Extend and apply the distributive law to the expansion of algebraic expressions.

- Applying the distributive law to the expansion of algebraic expressions using strategies such as the area model.

ACMNA191: Factorise algebraic expressions by identifying numerical factors.

- Recognising the relationship between factorising and expanding.
- Identifying the greatest common divisor (highest common factor) of numeric and algebraic expressions and using a range of strategies to factorise algebraic expressions.

ACMNA192: Simplify algebraic expressions involving the four operations.

- Understanding that the laws used with numbers can also be used with algebra.

Summary of lessons

Who is this sequence for?

- These lessons consist of stand-alone activities that provide consolidation of algebra skills and optional extension. At a minimum, students should understand the meaning of simple algebraic expressions such as $5a + 1$, $5ab$ and $2(5a + 1)$; be able to combine terms to create new expressions; and be able to substitute values. They need to understand that some 'equations' are true for just a few values of the unknown(s), and others (the identities) are true for all values. Students who have more algebraic skills will be able to create more advanced solutions.

Lesson 1: Like Terms

This resource contains a collection of tasks focusing on like terms. In the task How Can You Make It?, students create a given expression using a range of provided terms and then share their strategies. In the task Algebra Card Set, students place mathematical operation arrows between expressions to show the relationship between those expressions. In the task Composite Areas, students determine the area of various composite shapes made of squares and quadrants, in terms of given areas.

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Lesson 2: Substitution

This resource contains a collection of tasks focusing on substitution. In the task Expressing Relationships, students determine a range of values that make a two-variable equation true. In the task What Can It Be?, students work with an incomplete identity and explore options that will make the identity true. In the task Temperature Conversion, students find values that satisfy the relationship between temperatures expressed in degrees Fahrenheit and temperatures expressed in degrees Celsius.

Reflection on this sequence

Rationale

This resource consists of a series of open-ended tasks, discussion starters and mathematically interesting challenges. The tasks are not intended to be used sequentially; it is recommended that they be interspersed with regular algebra teaching and learning. Most tasks can be easily adapted to use algebraic expressions of varying complexity, and so the lessons provide teachers with some templates to create tasks for practising other skills.

reSolve mathematics is purposeful

- Fluency is addressed through the variety of tasks that are encountered so that practice is not mechanical. In many tasks, students can set their own level of challenge for practice of algebraic skills.
- Reasoning is developed as students justify their solution strategies.

reSolve tasks are challenging yet accessible

- All tasks are provided with enabling and extending prompts. Most have a simple way to start and allow students choice in the complexity of the algebra they employ.

reSolve classrooms have a knowledge-building culture

- The tasks are conducted either in small groups or consolidated through active participation in small and large groups. Various strategies for eliciting group discussion are included throughout this resource.

Acknowledgements

Many of the tasks within this sequence were developed as a result of inspiration from the following:

- Mathematics Task Centre (specifically Task: *Algebra Through Geometry*)
- Mason J, Pimm D, Graham A *et al.*, 1985, *Routes to/Roots of Algebra*, Centre for Mathematics Education: The Open University Press. (Australian adaptation by B. Henry, 1987)

LESSON 1

re(Solve) MATHS BY INQUIRY

Working with Algebra

Like Terms

Y8

About this lesson

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ACMNA192: Simplify algebraic expressions involving the four operations.

- Understanding that the laws used with numbers can also be used with algebra.

Mathematical purpose

- These tasks can develop fluency with manipulation of like terms. Tasks should be attempted by students with minimal introduction to encourage deep reasoning and communication. Students will discover that multiple expressions and equations can describe the same relationship. Teachers can create tasks using algebraic expressions with any desired complexity, and students can vary the level of complexity in their solutions.

Learning intention

- To practise operating with like terms in algebraic expressions.

Time

20 to 30 minutes for each task.

Vocabulary

- coefficient
- expression
- index
- power
- square
- term
- trial and error
- variable

Resources

- Student Sheet 1 – How Can You Make It? (one per five students)
- Student Sheet 2 – Algebra Card Set (one per group)
- Student Sheet 3 – Algebra Card Set Board (one per group)
- Student Sheet 4 – Composite Areas (one per student)

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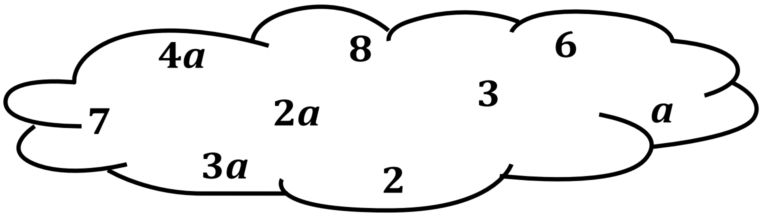
Task: How Can You Make It?

Choose terms from the cloud and write some expressions that equal:

$$5a + 9$$



Getting started:

- Initiate brief classroom discussions by asking:
  - ◊ What is a variable?
  - ◊ How is $5a$ different from $5 + a$?
  - ◊ Why does $a + 8 + a + 1 = 2a + 9$?
- Provide each student with a task card from Student Sheet 1 – How Can You Make It? (or draw it on the board) and pose the problem.

Enabling prompts:

- How can you make $5a$ with the options you have?
  - ◊ Can you do it another way?
- How can you make 9 with the options you have?
  - ◊ Can you do it another way?

Extending prompts:

- How many different expressions for $5a + 9$ can you write?
  - ◊ Use more terms.
  - ◊ Use other operations (i.e. subtraction, multiplication, division, indices, brackets).
- How many different expressions that equal $5a + 9$ can you write using each of the terms exactly once?
- Using all of the terms, create an expression that equates to zero.

Summarising:

- Encourage students to share solutions and strategies for making $5a + 9$, in groups and with the class.

Possible student responses:

- Below is a small sample of possible solutions, including some options using every term.

$$4a + a + 6 + 3 \qquad 2a + 3a + 7 + 2 \qquad 4a + 2a - a + 8 + 3 - 2$$

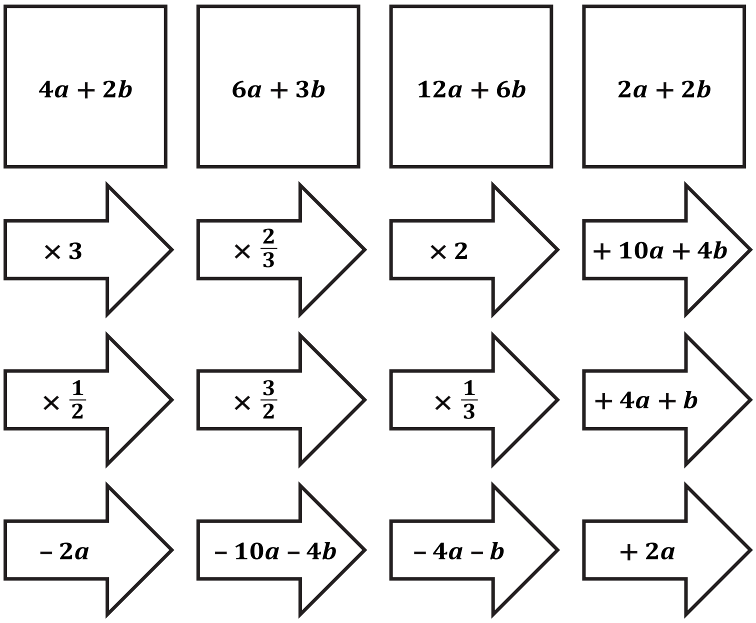
$$2a + 3a + 2 \times 6 - 3 \qquad 2 \times 2a + a + 7 + 8 - 6 \qquad (8 - 3) \times a + \frac{6 \times 3a}{2a}$$

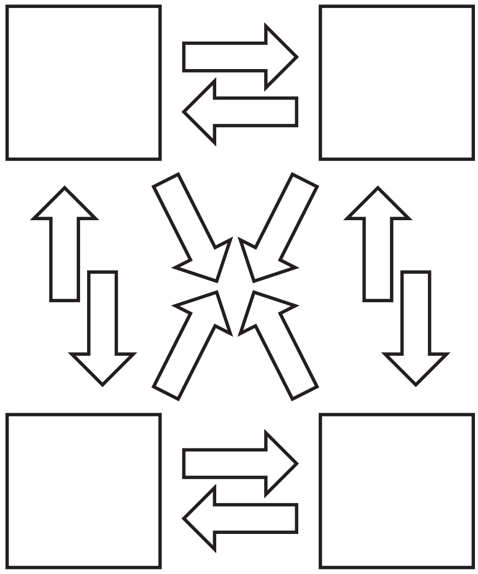
$$\frac{(4a \times a)^2}{(2a)^3} + 3a + 8 + 7 - 6 \qquad \frac{4a \times 3a}{2a} - a + \left(\frac{6}{2 \times 3}\right)^7 + 8$$

$$a + 4a - 2a - 3a + 7 + 8 - 3 - 2 \times 6 = 0 \qquad \left(\frac{4a - 3a}{a} + 2 \times 3 - 6 + 7 - 8\right)^{2a} = 0$$

Task: Algebra Card Set

Students sort the expression squares and operational arrows below (at left) into an arrangement like that shown at right.





Materials required:

- Student Sheet 2 – Algebra Card Set: one per group, cut into individual arrows and squares.
- Student Sheet 3 – Algebra Card Set Board: one per group.

Note: The four additional blank arrows on the student sheet can be used as an extension activity for students to create their own operations.

Enabling prompts:

- What do you notice about the squares? What do you notice about the arrows?
- Make a start by putting two expressions into any two corners of the board.
- What do you do to one expression to change it to another?

Extending prompts:

- Create your own version:
  - ◊ using the given expression squares, but your own operation arrows.
  - ◊ using the given operation arrows, but your own expression squares.
  - ◊ using your own expression squares and your own operation arrows.

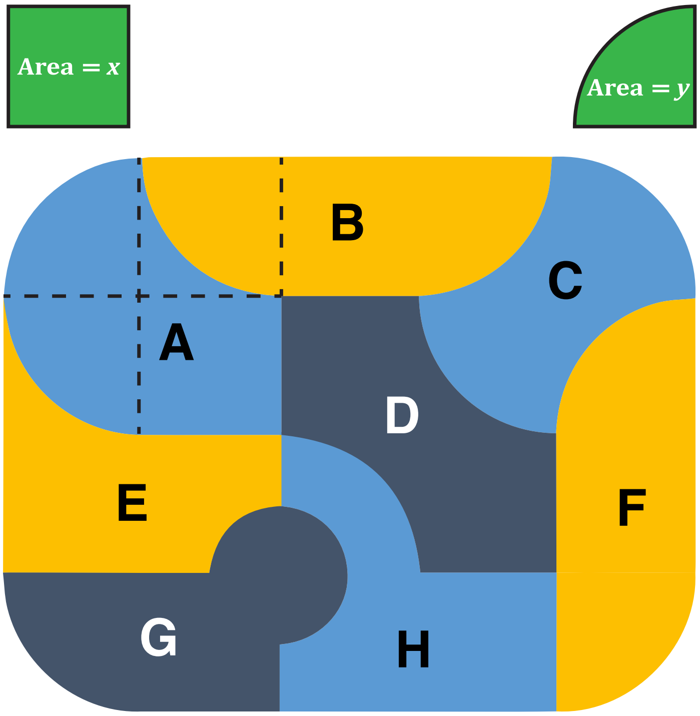
Summarising:

- Encourage students to share different strategies for sorting the expressions and arrows, in groups and with the class.
- The task can be used as a catalyst for similar tasks of increased complexity and to facilitate classroom discussions about the patterns they noticed.

Possible student responses:

- A possible solution is available in Teacher Sheet 1 – Algebra Card Set. Student solutions may have the expression cards in different places, but the operation arrows between two given expressions will be the same as those in the example solution.

Task: Composite Areas



Given a square with area x, and a quadrant with area y, determine the area of the individual shapes (A–H) within the larger composite shape below.

Materials required:

- Student Sheet 4 – Composite Areas: one per student.

Enabling prompts:

- What are composite shapes?
- Start with shape B.
- Cut out (and use) the green square with area x and the green quadrant with area y.
- Make some shapes with areas $2x$, $x + y$ and $x - y$.
- Use a square and quarter square to show that when the side length is halved, the area is divided by 4. Students will need to understand that this is a general rule that also applies to circles. They will need this to calculate the area of the small quadrant for shapes E, G and H.

Extending prompts:

- Explain why the area of the entire shape must be $16x + 4y$. Check that the sum of the individual areas is $16x + 4y$.
- When $x = 4$, what is y? What would be the area of each individual shape and of the entire shape?
- Maintain the total area and design your own tangram shapes, using the square with area x and the quadrant with area y.

Summarising:

- In groups and with the class, encourage students to share different strategies for working out each area. Start with shapes B and F, which require only addition of areas.

Solutions:

- Solutions are provided in Teacher Sheet 2 – Composite Areas.

How Can You Make It?

Choose terms from the cloud and write some expressions that equal:

$5a + 9$

$4a$ 8 6 3 7 $2a$ a $3a$ 2

Choose terms from the cloud and write some expressions that equal:

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$4a$ 8 6 3 7 $2a$ a $3a$ 2

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Choose terms from the cloud and write some expressions that equal:

$5a + 9$

$4a$ 8 6 3 7 $2a$ a $3a$ 2

Algebra Card Set Task

$\times 3$

$\times \frac{1}{2}$

$-2a$

$\times \frac{2}{3}$

$\times \frac{3}{2}$

$-10a-4b$

$\times 2$

$\times \frac{1}{3}$

$-4a-b$

$+10a+4b$

$+4a+b$

$+2a$

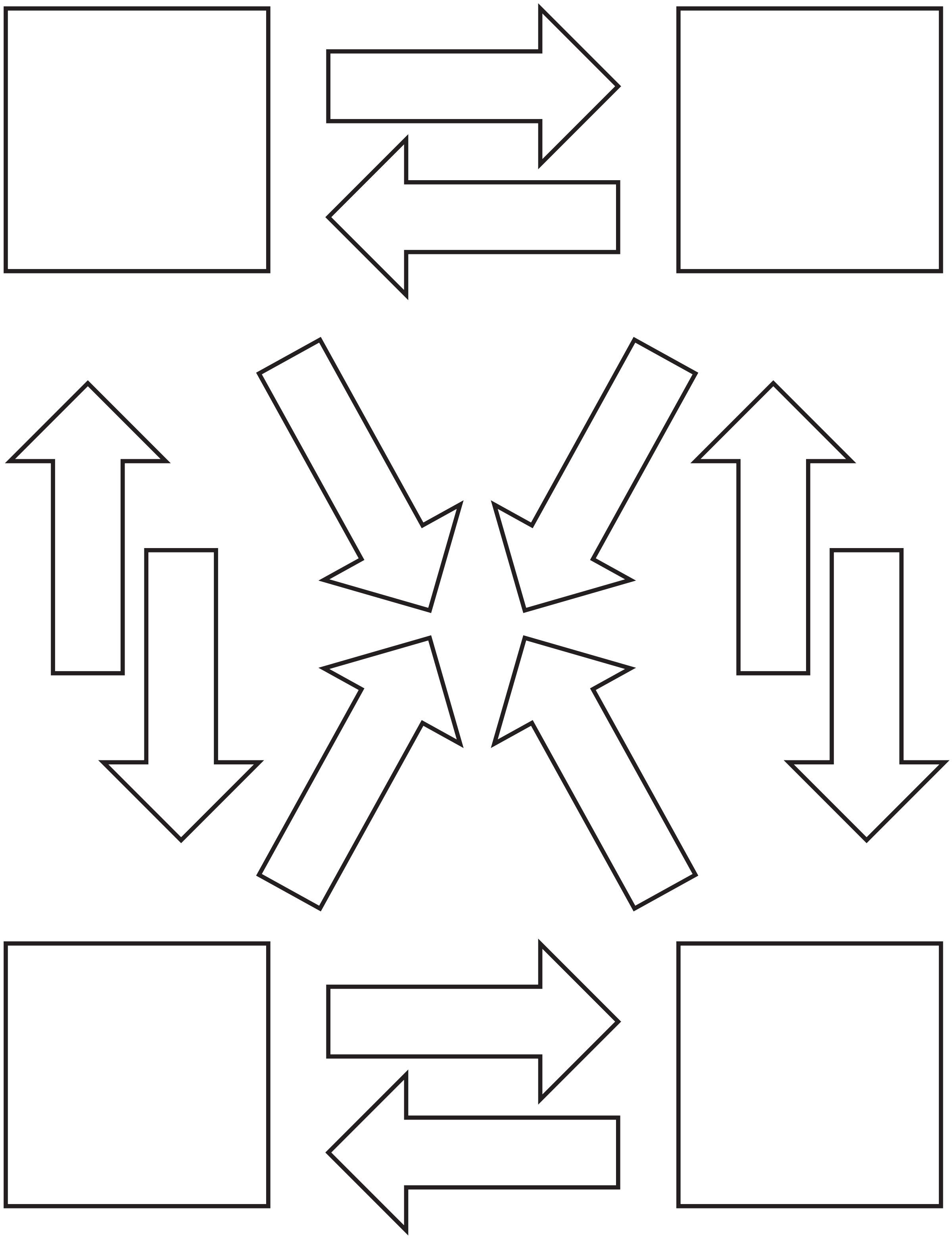
$4a+2b$

$12a+6b$

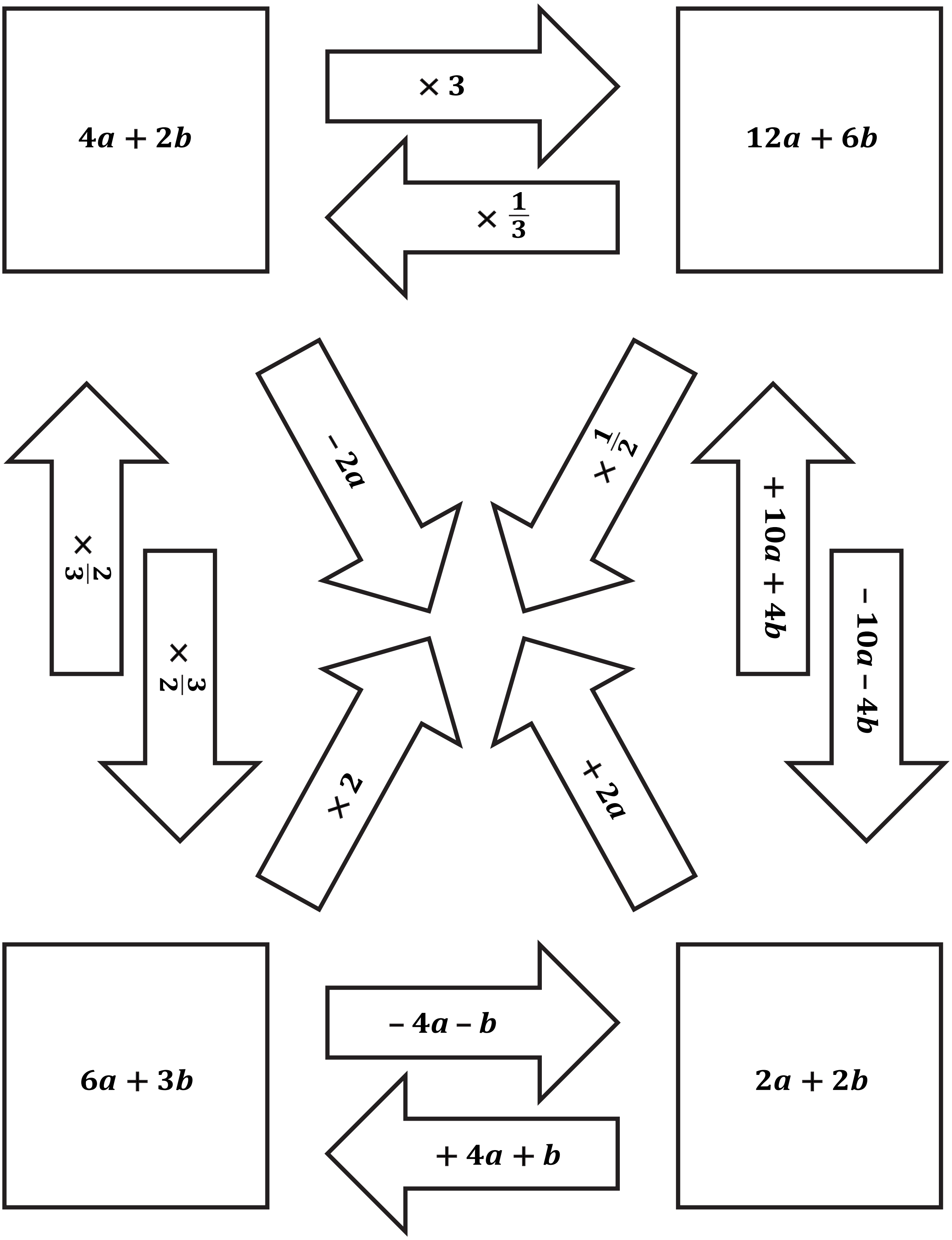
$6a+3b$

$2a+2b$

Algebra Card Set Board

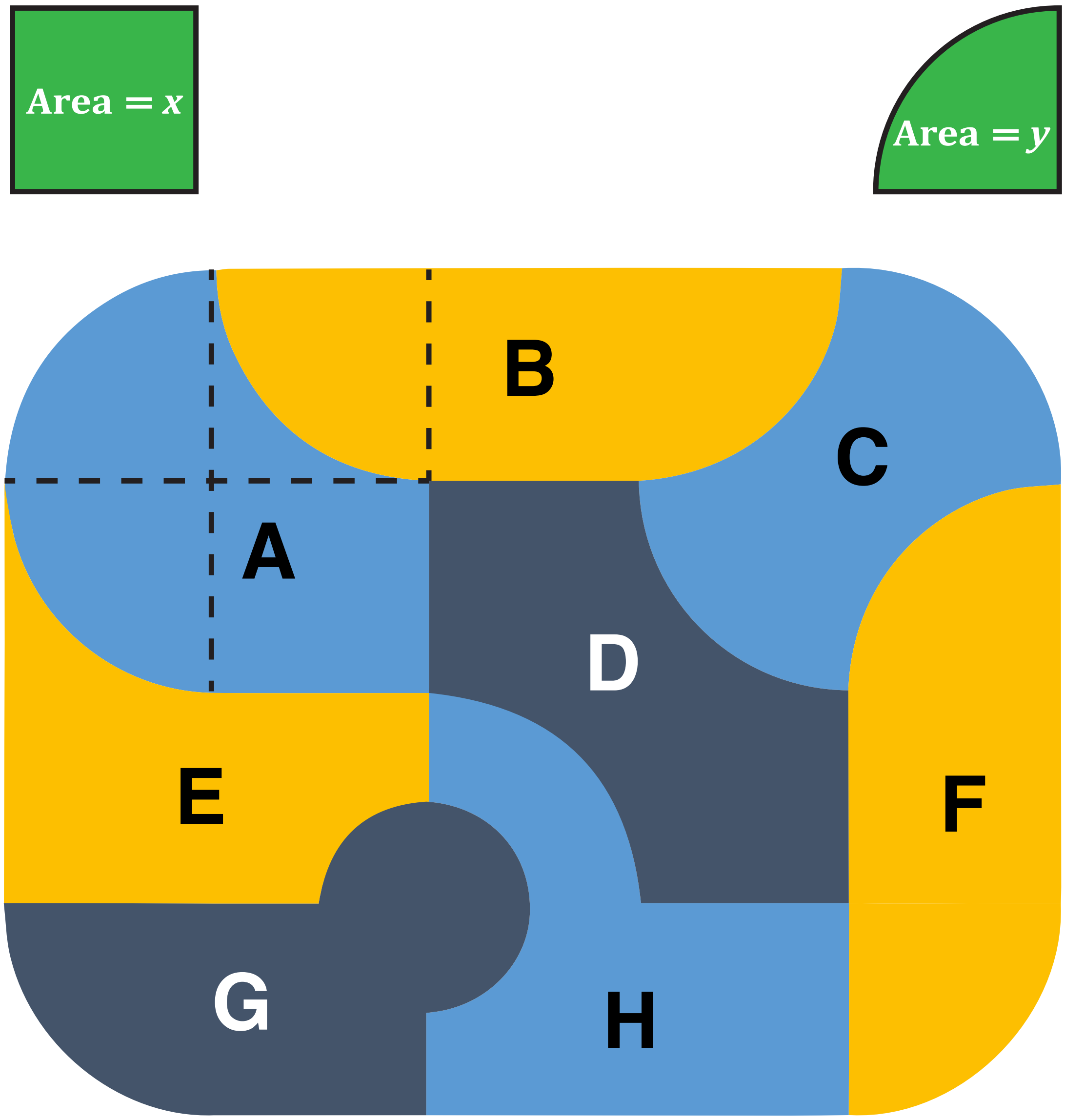


Teacher Sheet 1 – Algebra Card Set



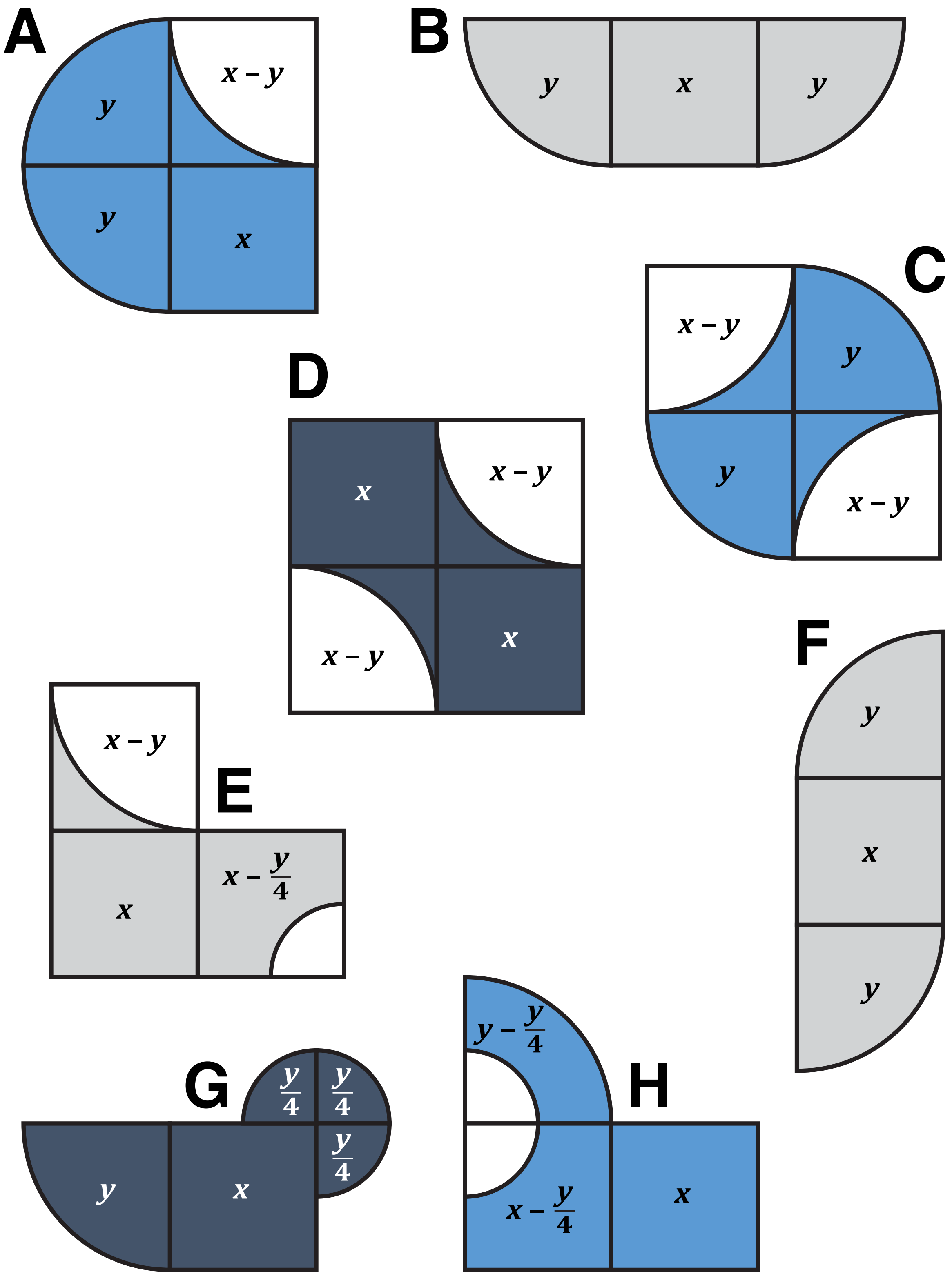
Composite Areas

Given a square with area x, and a quadrant with area y, determine the area of the individual shapes (A–H) within the larger composite shape below.



Teacher Sheet 2 – Composite Areas

A	B	C	D	E	F	G	H
$2x+y$	$x+2y$	$2x$	$4x-2y$	$3x-\frac{5y}{4}$	$x+2y$	$x+\frac{7y}{4}$	$2x+\frac{y}{2}$



LESSON 2

re(Solve) MATHS BY INQUIRY

Working with Algebra

Substitution

Y8

About this lesson

This resource contains a collection of tasks focusing on substitution. In the task Expressing Relationships, students determine a range of values that make a two-variable equation true. In the task What Can It Be?, students explore options that will make an incomplete identity true. In the task Temperature Conversion, students find values satisfying the relationship between temperatures expressed in degrees Fahrenheit and temperatures expressed in degrees Celsius.

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ACMNA192: Simplify algebraic expressions involving the four operations.

- Understanding that the laws used with numbers can also be used with algebra.

Mathematical purpose

- These tasks develop fluency with substitution into algebraic expressions. They enable teachers to identify and address student misunderstandings about the use of algebraic notation. The tasks offer students opportunities to create and communicate a variety of solutions. Teachers can create similar tasks involving algebraic expressions of any desired complexity.

Learning intention

- To practise substitution in algebraic expressions.

Time

20 to 30 minutes for each task.

Vocabulary

- coefficient
- equation
- equivalent
- expression
- identity
- square
- substitute
- term
- trial and error

Resources

- Student Sheet 1 – Expressing Relationships (one per 10 students)
- Student Sheet 2 – What Can It Be? (one per five students)
- Student Sheet 3 – Temperature Conversion (one per five students)

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Task: Expressing Relationships

I know that $\frac{2a}{3} + \frac{3b}{4} = 120$.

What might be the values of a and b?

(Give a range of possible answers.)

Getting started:

- Provide students with the task cards from Student Sheet 1 – Expressing Relationships and pose the task problem.
- Before beginning individual work, conduct a brief class discussion about the task, encouraging students' use of mathematical terminology such as: variable, term, expression, substitute, solve and satisfy.
- Key discussion points might include:
  - ◊ Do the values of a and b need to be whole numbers or positive? (Answer: no)
  - ◊ How many answers are there likely to be?

Enabling prompts:

- Try a simpler problem first: I know that $a + 2b = 60$.
  What might be the values of a and b? (Give a variety of possible answers.)
- Try simple special cases for the given problem; for example:
  When $a = 0$, what is b?
  When $b = 0$, what is a?
- Choose values for a that are divisible by 3, to make the arithmetic easier.

Extending prompts:

- What if $a = b$?
  Solve this problem using logical reasoning, then using guess–check–improve, and then using algebraic equation solving.
- Represent all the solutions graphically.

Summarising:

- Encourage students to share different strategies for finding values of a and b, in groups and with the class.

Solutions:

- There are many solutions. Some solutions and strategies are shown in Teacher Sheet 1 – Expressing Relationships.

Task: What Can It Be?

The underlined box means that something is missing.

What might be the letters or numbers missing in the following?

$$3(a + \square) - \square = \square a + \square$$

Getting started:

- Provide students with the task cards from Student Sheet 2 – What Can It Be? or write it on the board.
- Initiate a brief class discussion about the task, encouraging students' use of mathematical terminology such as: variable, coefficient, expression, equation and identity.
- Clarify that this task is about finding different ways of making the expression on the left equivalent to the expression on the right for all values of a (i.e. creating an identity).

Enabling prompts:

- Try an easier problem; for example:
  Fill the boxes to make $2(a - \square) = 2a - \square$ for all values of a.
- Check your answers numerically by substitution, as well as algebraically by expanding the left-hand side of your identity to check that it is the same as the right-hand side.

Extending prompts:

- Find another way to solve this problem.
- Explain why the missing values cannot be all the same.
- Fill some of the boxes with numerical values that are not positive integers.
- Fill some of the boxes with variables.

Summarising:

- Encourage students to share different strategies and solutions, in groups and with the class. Ask them how they know that they have created an equation that is true for all values of a.

Solutions:

- Some possible student responses are listed in Teacher Sheet 2 – What Can It Be?

Task: Temperature Conversion

There is a rule for converting degrees Fahrenheit (used in the United States) to degrees Celsius (used almost everywhere else):

$$\mathbf{F = \frac{9}{5} + 32}$$

What might be the values for F and for C?

(Give a range of possible answers.)

Getting started:

- Provide students with the task cards from Student Sheet 3 – Temperature Conversion or write it on the board.
- Before starting individual work, initiate a brief class discussion, discussing the two temperature scales with reference to the weather forecast (for example) or a thermometer that has both scales visible.

Enabling prompts:

- If the temperature is 20 °C, what is it in °F?
- What is the freezing point of water, in °C? What is this in °F?
- What is the boiling point of water, in °C? What is this in °F?
- Make a table of values with °C in the first row and °F in the second row.

Extending prompts:

- Represent the relationship graphically.
- At what temperature is the number the same, regardless of whether it is written in degrees Celsius or degrees Fahrenheit?
- What is the Kelvin scale? Write a relationship between °F and K.

Summarising:

- In groups and with the class, encourage students to share different strategies and solutions.

Solutions:

- There is an infinite number of solutions. Some are shown in Teacher Sheet 3 – Temperature Conversion.

Expressing Relationships

I know that $\frac{2a}{3} + \frac{3b}{4} = 120$.
What might be the values of a and b?
(Give a range of possible answers.)

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Teacher Sheet 1 – Expressing Relationships

Setting each term equal:

$$\frac{2a}{3} + \frac{3b}{4} = 120 \qquad 60 + 60 = 120 \qquad \text{Let } \frac{2a}{3} = 60. \qquad \text{So } a = 90.$$

$$\text{Let } \frac{3b}{4} = 60. \qquad \text{So } b = 80.$$

Setting a value for *a*:

$$\frac{2a}{3} + \frac{3b}{4} = 120 \qquad \text{Let } a = 45. \longrightarrow 30 + \frac{3b}{4} = 60 \qquad 3b = 360 \qquad \therefore b = 120$$

Setting a value for *b*:

$$\frac{2a}{3} + \frac{3b}{4} = 120 \qquad \text{Let } b = 40. \longrightarrow \frac{2a}{3} + 30 = 120 \qquad 2a = 270 \qquad \therefore a = 135$$

Common denominator and table of values or graph:

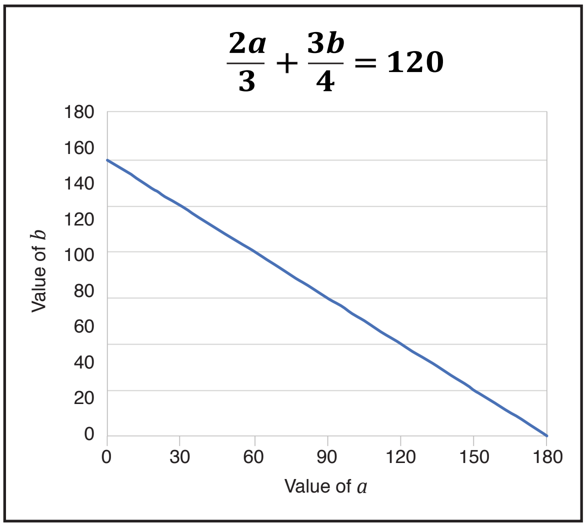
$$\frac{2a}{3} + \frac{3b}{4} = 120$$

$$8a + 9b = 1440$$

$$9b = 1440 - 8a$$

$$b = 160 - \frac{8}{9}a$$

a	0	9	18	27	...	180
b	160	152	144	136	...	0



What Can It Be?

The underlined box means that something is missing.

What might be the letters or numbers missing in the following?

$$3(a + \underline{\square}) - \underline{\square} = \underline{\square}a + \underline{\square}$$

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Teacher Sheet 2 – What Can It Be?

Solution 1

Use 3 as the coefficient of a on the RHS:

$$3(a + \boxed{}) - \boxed{} = \underline{3}a + \boxed{}$$

We now have a range of possible sets of values that could complete the equation, such as:

$3(a + \underline{1}) - \underline{3} = \underline{3}a + \underline{0}$

$3(a + \underline{2}) - \underline{3} = \underline{3}a + \underline{3}$

Solution 2

Use 2 as the coefficient of a on the RHS:

$$3(a + \boxed{}) - \boxed{} = \underline{2}a + \boxed{}$$

To get $3a$ on the RHS, we could use a in the last box.

$$3(a + \boxed{}) - \boxed{} = \underline{2}a + \underline{a}$$

This now means that the LHS must be $3a$, giving a range of possible solutions all of the form:

$3(a + \underline{1}) - \underline{3} = \underline{2}a + \underline{a}$

$3(a + \underline{2}) - \underline{6} = \underline{2}a + \underline{a}$

where the second box must be three times the first box.

Solution 3

Use a as the missing value in the first box.

$$3(a + \underline{a}) - \boxed{} = \underline{3}a + \boxed{}$$

We now have $6a$ on the LHS, which we could balance by adding $3a$ on the RHS. This gives:

$3(a + \underline{a}) - 0 = \underline{3}a + \underline{3a}$

Alternatively, we could use $2a$ on the LHS and a on the RHS:

$3(a + \underline{a}) - \underline{2a} = \underline{3}a + \underline{a}$

There is a range of other possible solutions.

Temperature Conversion

There is a rule for converting degrees Fahrenheit (used in the United States) to degrees Celsius (used almost everywhere else):

$$F = \frac{9}{5} + 32$$

What might be the values for F and for C?

(Give a range of possible answers.)

There is a rule for converting degrees Fahrenheit (used in the United States) to degrees Celsius (used almost everywhere else):

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What might be the values for F and for C?

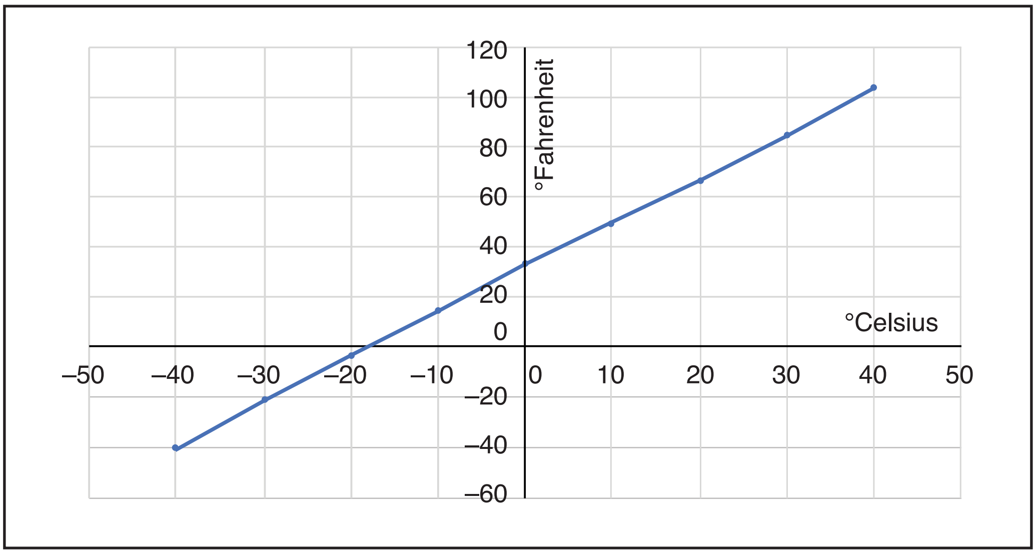
(Give a range of possible answers.)

Teacher Sheet 3 – Temperature Conversion

Choose values of °C that are multiples of 10.

°C	−40	−30	−20	−10	0	10	20	30	40
°F	−40	−22	−4	14	32	50	68	86	104

Choose values of °C that are multiples of 10.



Let F be the temperature with the same number of degrees in Fahrenheit and in Celsius.

Then $F = \frac{9}{5}F + 32$.

$-\frac{4}{5}F = 32$

$\therefore F = -40$

The Kelvin scale uses the same units as the Celsius scale, so one kelvin (1 K) is the same as one degree Celsius (1 °C). 0 K represents the temperature at which all thermal motion ceases. 0 K is known as absolute zero and is equivalent to −273.15 °C.

So, $C = K - 273.15$; hence, $F = \frac{9}{5}K - 459.67$.