re(Solve) MATHS BY INQUIRY

Trigonometric Trajectories

Y10

Summary of learning goals

- Students' understanding of trigonometric and Pythagorean relationships are reinforced through two engaging real-world contexts: researching and building a thrilling (yet safe) zip-line ride for a doll, and outlining a proposal for building glider poles in their local area.

Australian Curriculum: Mathematics (Year 10)

ACMMG245: Solve right-angled triangle problems, including those involving direction and angles of elevation and depression.

Summary of lessons

Who is this sequence for?

- This sequence is for students who are familiar with Pythagoras' theorem and angles of depression and elevation. There are opportunities for students to independently design and conduct practical experiments by creating models, drawing diagrams, and conducting and refining practical experiments.

Lesson 1: World's Greatest Zip-lines

Students apply known trigonometric and Pythagorean relationships to investigate the dimensions of adventure zip-lines around the world. They plan, draw, model and construct a zip-line for a Barbie doll.

Lesson 2: Glider Poles

Students learn about the importance of building glider poles by the sides of highways to allow gliding mammals to cross wide roads. They then design a pair of glider poles that are appropriate for the animals and roads in their region, using their knowledge of trigonometry, particularly angles of elevation and depression.

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Reflection on this sequence

Rationale

This sequence contains two intriguing real-world applications of trigonometry, providing opportunities for students to use mathematics to investigate real-life contexts and solve real-life problems.

reSolve mathematics is purposeful

- Students build fluency with calculations in engaging real-world contexts.
- Students explore issues of real-world relevance and concern: preserving the Australian ecosystem and investigating the safety issues in popular adventure activities.

reSolve tasks are inclusive and challenging

- There are multiple possible approaches to building a safe, yet thrilling, zip-line in Lesson 1: detailed diagrams and calculations in the planning stage or repeated experimentation and reiteration.
- Both lessons allow students to set their own level of difficulty based on the location and conditions they choose.

reSolve classrooms have a knowledge-building culture

- Students collaborate to model, design, experiment and redesign.

Acknowledgements

Lesson 2 draws on the work of Ross Goldingay and, in particular, Goldingay R, Taylor B and Parkyn J, 2018, 'Use of tall wooden poles by four species of gliding mammal provides further proof of concept for habitat restoration', *Australian Mammalogy* 41: 255–61.

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LESSON 1

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Trigonometric Trajectories

World's Greatest Zip-lines

Y10

About this lesson

Students apply known trigonometric and Pythagorean relationships to investigate the dimensions of adventure zip-lines around the world. They plan, diagram, model and construct a zip-line for a Barbie doll.

Australian Curriculum: Mathematics (Year 10)

ACMMG245: Solve right-angled triangle problems, including those involving direction and angles of elevation and depression.

Mathematical purpose

- To use data about popular zip-lines around the world to design and build a miniature zip-line.

Learning intention

- To create a thrilling, yet safe, zip-line for a Barbie (or similar) doll.

Time

Two lessons of approximately 1 hour each.

Vocabulary

- angle of depression
- gradient
- slope

Resources

- Equipment for building zip-lines:
  - string/fishing line or similar
  - dolls/action figures
  - large bulldog clips
  - measuring tape
- Student Sheet 1 – Zip-line Data (one per student)

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What makes a good zip-line?

Discuss students' experiences with thrilling theme park rides/extreme adventure activities (e.g. bungee jumping, rollercoaster rides, zip-lines, etc.). Ask: *What makes a really good scary ride? What are some important design features?*

Show the following three videos from zip-lines around the world:

- 'The world's longest zip-line' – Jebel Jais Flight in Ras al Khaimah, UAE (2018)
- 'The world's fastest zip-line' – Zip World Velocity 2 in Penrhyn Quarry, Bethesda, Wales (2013)
- 'The world's steepest zip-line' – Flying Dutchman in St Maarten, the Caribbean (2018)

Discuss how the rides are similar and different.

The claims quoted above ('longest', 'fastest', 'steepest zip-line') are from the websites of each ride. Prompt students to evaluate the claims mathematically. *What data would be needed in order for you to confirm these claims?*

Discuss the key characteristics of 'boring', 'deadly', and 'just right' zip-lines.

Possible student responses:

- Deadly zip-line: *Goes straight into a mountain, is vertical, is a freefall, is too high up, too many people on it at one time, is very steep, doesn't slow down at the end…*
- Boring zip-line: *Just a horizontal line, is not steep, goes really slowly, is too short, you stop in the middle…*
- Just right zip-line: *Is steep but not too steep, has interesting scenery, is not too high or too low, lets the rider wiggle around…*

Teacher notes:

- Students might be interested in researching dangerous accidents on zip-lines around the world. Most injuries or deaths on zip-lines are due to:
  - fast-moving riders kicking against trees or other features in an attempt to slow themselves down, often because the speed of descent feels unsafe; or
  - riders losing momentum and slowing to a complete stop before reaching the end of the ride, and then being struck by the next rider.

Collecting zip-line data

Resources: Give each student a copy of Student Sheet 1 – Zip-line Data.

This sheet contains a table with data about real zip-line rides around the world, drawn from claims made on their websites. Ask students to complete the table as best they can by drawing diagrams and applying Pythagoras' theorem and other trigonometric formulas. In some cases, they will find they do not have enough data or that the supplied data prompt further investigation. Discuss:

- *How might the data help you to compare the different zip-lines?*
- *Do the supplied data seem reasonable? If not, how do you think the company arrived at this claim?*
- *Is it possible to claim that one zip-line is better than another? What factors would you need to consider?*

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Creating a zip-line

Place students in groups of three to four and challenge groups to design and create an outdoor zip-line for a doll or action figure.

Ask students: *What are important issues to consider when building a zip-line?*

Use the following prompts to guide discussion:

- length of ride
- maximum/minimum height above ground level
- horizontal and vertical distances
- angle of depression
- good locations for zip-line (e.g. surrounding scenery)
- ways to slow the rider at the end of the ride
- How taut/slack should the zip-line be?
- Equipment: What should the zip-line be made out of? How is the rider attached?
- Are there other safety concerns?
- Should we build a model or prototype to test ideas before building the real thing?
- How can we compare our zip-line to the zip-lines given in the table in the Student Sheet? (Discuss scale.)

The recommended process for students to follow is:

1. Brainstorm elements of your zip-line and draw a diagram.
2. Create a classroom prototype of the zip-line (taking scale into consideration) and consider what adjustments might be necessary (particularly regarding speed and safety).
3. Create a new diagram of your zip-line, taking your findings into account.
4. Build your outdoor zip-line.
5. Launch the doll and make a video recording of the action.

T Teacher notes:

- This is an intentionally open-ended design brief.
- Students are encouraged to make preliminary sketches and calculations, construct their prototype (indoors), and test its functionality before building the final zip-line. They need to ensure that their rider arrives safely at the lower end, and that there is some sense of thrill to the ride. Students should be given simple materials, but teachers should also allow for any modifications to the materials that students might suggest.

Reflection

In their groups, students present their design process and final launch video to the rest of the class. The class discusses each ride, focusing on: *Did the rider make it to the end of the zip-line? Did the rider land safely or did they die? Did the rider experience a thrilling ride?*

Finally, discuss as a class: *Which ride would have given the rider the most thrills?*

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Zip-line Data

Name: _____

		Cable length	Vertical drop	Horizontal distance	Angle of depression
World's longest zip-line	Jebel Jais Flight UAE	2832.88 m	1622 m		
	Copper Canyon ZipRider Mexico	8350 ft			10°
	Icy Strait Point ZipRider Alaska			5334 ft	25%
	Zip 2000 South Africa		918 ft	1980.33 m	
World's steepest zip-line	Letalnica bratov Gorišek Slovenia	566 m		529 m	
	The Flying Dutchman St Maarten, Caribbean	2538 ft			42%
	Zipflyer Nepal	1800 m	2000 ft		56°
	XTREME ZipRider Utah		500 ft		34%
My own design					

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LESSON 2

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Trigonometric Trajectories

Glider Poles

Y10

About this lesson

Students learn about the importance of building glider poles by the sides of highways to allow gliding mammals to cross wide roads. They then design a pair of glider poles that are appropriate for the animals and roads in their region, using their knowledge of trigonometry, particularly angles of elevation and depression.

Australian Curriculum: Mathematics (Year 10)

ACMMG245: Solve right-angled triangle problems, including those involving direction and angles of elevation and depression.

Mathematical purpose

- To investigate a practical real-world application of trigonometry.

Learning intention

- To learn how trigonometry is used to protect Australian wildlife.

Time

Two lessons of approximately 1 hour each.

Resources

- reSolve PowerPoint *2a Glider Poles*

Vocabulary

- angle of depression
- ratio

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Introduction

Ask students to read this article ('Proof glider poles are the solution to Australia's highway crisis' from news.com.au). Students might also be interested in this article ('A glider's best friend'), in which readers have submitted guesses about the purpose of these 'strange-looking poles'.

Explain that students will be designing a set of glider poles that could help gliding mammals cross a local highway. Discuss important factors that students will need to take into consideration, including:

- which animals live in the area and their glide ratio
- the width of nearby highways
- the type of traffic in the area.

Planning glider poles

Resources: Show slides 2–4 of reSolve PowerPoint *2a Glider Poles*.

Have students record which gliders can be found in their area and the glide ratio of those gliders. Ask students to calculate the angle of depression for each glider.

Teacher notes:

Glider	Average glide ratio	Angle of depression	Glider	Average glide ratio	Angle of depression
squirrel glider	1.84	28.5	feathertail glider	1.6	32.0
yellow-bellied glider	2.0	26.6	mahogany glider	1.91	27.6
sugar glider	1.82	28.8	reSolve glider	1.2	39.8
greater glider	1.5	33.7			

Have students visit Google Earth and identify a suitable highway near the school where their glider poles could be placed, then measure the width of the road.

Give students the following conditions (also given on slide 5):

1. The poles cannot be placed right next to the road edge, as there must be a margin for safety. Note: in the original 2013 study by Taylor and Goldingay, poles were placed 2 m from the road edge.
2. In most of Australia, vehicles have a maximum height limit of 4.6 metres, so the gliders should not come lower than 4.6 metres above the road while gliding.
3. Poles need to be buried 2 m into the ground to be stable.
4. At the top of each pole there is a 1 m long horizontal glide beam pointing to the pole opposite (see image on slide 5).

Issue students the challenge outlined on slide 6. Discuss what a glide ratio for a pole-to-pole glide might look like and why it is necessary. Would you expect this to be a *larger* or *smaller* value than the glide ratio of your given glider? [A smaller value.] If the pole-to-pole glide ratio is larger than the glide ratio of your glider, that means that, on average, your glider is going to crash into the road instead of reaching the opposite pole.

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Reflection

Students present to the class.

Further activity

Research whether there are any glider poles in your local area.

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