RATIONAL TANGLES
Lesson 1: Untangling Tangles

Australian Curriculum: Mathematics (Year 8)

ACMNA183: Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies.

Lesson abstract

Students learn a “dance” with ropes that can be used to represent any rational number. They experiment with the dance to determine its properties.

Mathematical purpose (for students)

We will investigate how this “dance” works and what the different moves do.

Mathematical purpose (for teachers)

Students explore the properties of rational numbers.

Suggested presentation: Two lessons of approximately one hour each.

Vocabulary encountered
- operations
- rational number

Lesson materials
- *1a Untangling Tangles* PowerPoint (for display)
- Ropes or strings approximately 3m long

We value your feedback after these lessons via our website.

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education

Australian Academy of Science

Teacher Background Information

This lesson is based on mathematician John Conway's 'Rational Tangles'. Using two ropes, tangles are created using twists and rotations. A *twist* adds one to the value of the tangle, and a *rotate* changes the value of the tangle to its negative reciprocal. Conway shows that any rational number can be represented by a tangle.

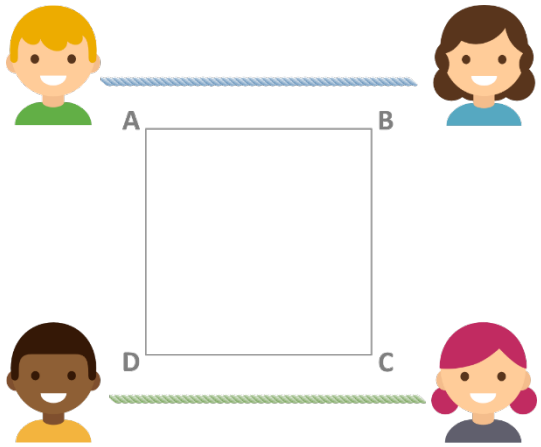
Introduction

Play the video on slide 2 of PowerPoint *1a Untangling Tangles* (alternate link). Ask students: *What do you notice? What do you wonder?* Spend some time unpacking the video.

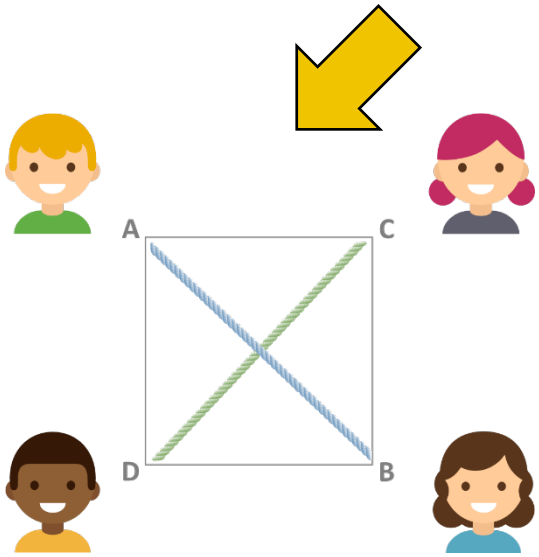
Have four volunteers stand in a square at the front of the class, with two ropes held between pairs (see slide 3). Explain that there are only two possible moves in this "dance":

1. Twist (shown on slides 4-5)
2. Rotate (shown on slides 6-7)

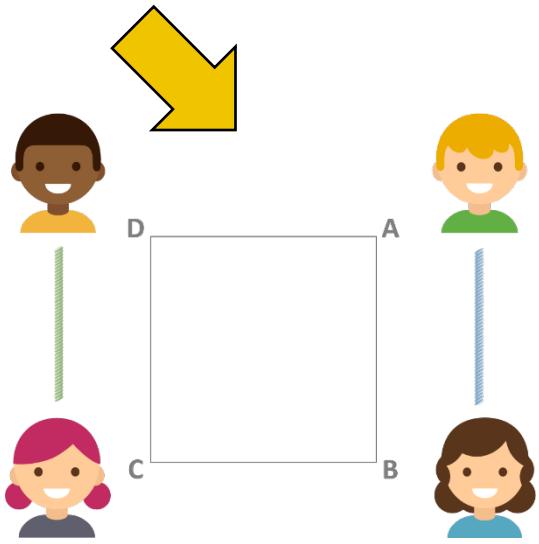
We can assign operations to each of these moves and values to the tangles that are formed (slide 8). A starting tangle has a value of 0, and the twist operation adds one. *What might the rotate operation do* (slide 9)*?*



Starting position
This tangle has a value of 0 ($t = 0$)



Twist (C under B, B over C)
T
$t = 0 \rightarrow 1$



Rotate (all rotate clockwise)
R
$t = 0 \rightarrow$???

Exploring operations

Ask the volunteers to demonstrate the following sets of operations and discuss each set as a class. Prompt each discussion with *what do you notice? What do you wonder?*

1. *Twist* followed by *twist* (TT), shown on slide 10. These operations produce a tangle with a value of 2.
2. *Rotate* followed by *rotate* (RR), shown on slide 11. Discuss: *is this the same tangle that we started with?* Even though the students and the ropes have moved, the value of the tangle is 0 because the tangle is still made of two parallel horizontal ropes.
3. *Rotate* followed by *twist* (RT), shown on slide 12. Again note that the tangle is the same before and after the twist, even though the holders of the blue rope have switched places.
4. Discuss the claim made on slide 13, which demonstrates that it is only the tangle that is significant and not the positions of the holders.
5. *Twist* followed by *rotate* (TR), shown on slide 14. *What might be the value of this tangle? How might we undo this tangle? What can we deduce about the rotate operation?*

Teacher Notes

- The tangle formed by the *twist* then *rotate* operations can be undone with a single twist (+1 operation). This twist creates a tangle with a value of zero. Hence, the TR tangle must have a value of -1, and so the *rotate* operation must have changed 1 to -1.

Ask students to hypothesise what the rotate operation might be (e.g. “rotate changes the sign”, “rotate subtracts two”).

Testing rotations

Set the challenge: *what does rotate do?*

Separate students into groups of five and supply each group with two ropes or pieces of string. Ask students to experiment by tangling and untangling with different operations and to record each set of operations they use.

Enabling Prompts

- *How do you undo the tangle made by one twist? Two twists? Three twists? Any number of twists?*
- *Tangle the rope using this set of operations: TTTRTTRT. What could be the value of the tangle you have made? How do you untangle this tangle?* This tangle has the value $\frac{2}{5}$, and can be untangled using this set of operations: *RTTTRTT*.

Once students have explained what *rotate* does, challenge students to identify a strategy to return any tangle to zero.

Reflection

Discuss findings as a class.