QUARTER CARTONS

Australian Curriculum: Mathematics (Year 8)

ACMNA191: Factorise algebraic expressions by identifying numerical factors.
ACMNA192: Simplify algebraic expressions involving the four operations.
ACMMG198: Develop formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.
ACMMG202: Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

Lesson abstract

Students fold origami boxes from proportional paper rectangles, compare the dimensions of the boxes, and create an algebraic formula for the volume of a box folded from a paper rectangle of any dimensions.

Mathematical purpose (for students)

How does changing the size of a paper rectangle change the dimensions of origami folded from that rectangle?

Mathematical purpose (for teachers)

Students use algebra and proportional reasoning to justify why changing the size of an origami rectangle by a factor of 4 results in changing the size of a box folded from that rectangle by a factor of 8. They generalise their findings using algebra.

Suggested presentation One lesson of one hour

Vocabulary encountered

- dimensions
- proportional

Lesson materials

- Large and small paper rectangles
- Student Sheet – Folding Boxes
- *1a Folding a rectangular box* PowerPoint (for display) or video directions for folding an origami box, such as this video

Preparation: The small paper rectangles must be **one-quarter** of the size of the large rectangles. We recommend using A4 paper as the large rectangles and cutting A4 paper into quarters to make the small rectangles. This creates rectangles that are a convenient size for folding.

Each student will need at least 1 large rectangle and 1 small rectangle.

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education

Australian Academy of Science

Introduction

Explain to students that in this lesson you will be folding origami boxes using differently sized paper rectangles and investigating how changing the size of the rectangle affects the size of the box.

Give students Student Sheet – Folding Boxes and large paper rectangles. Students measure and record the dimensions of the rectangle on their sheet.

Using *1a Folding a rectangular box* PowerPoint or video directions such as this video on YouTube, have students fold their rectangle into an origami box.

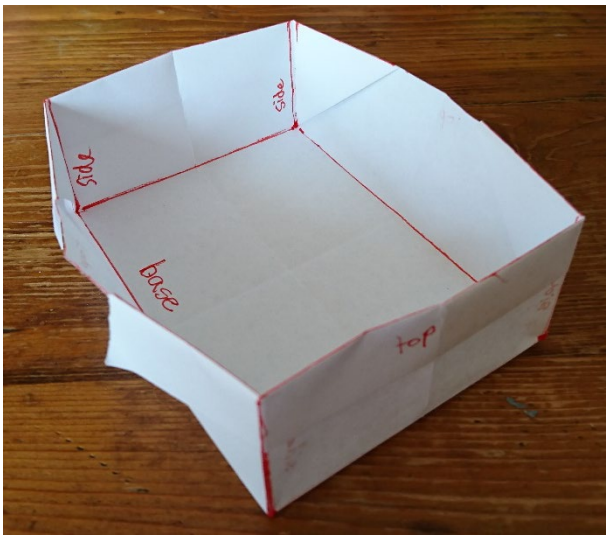
Teacher Notes

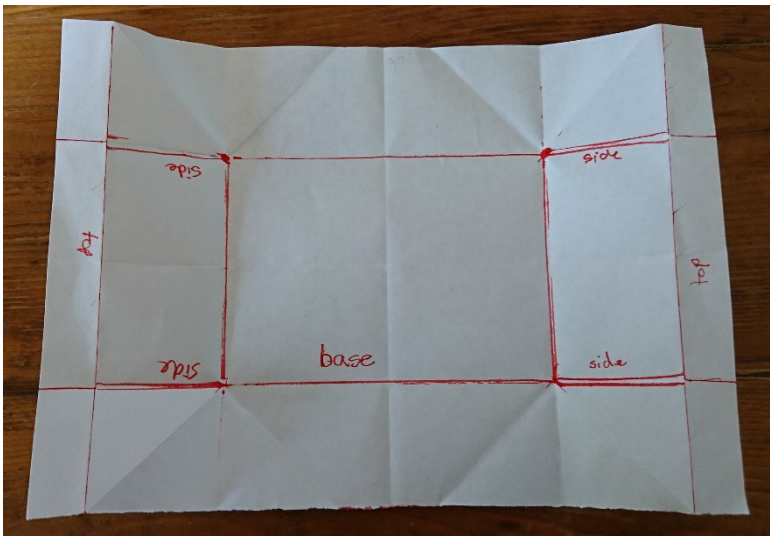
- Some of the steps are complex and it would be helpful to practice in advance.
- Some steps in the PowerPoint and in the video are given in a slightly different order, but the final box is identical.
- The PowerPoint requires some degree of familiarity with origami diagrams. If students are unfamiliar with origami diagrams the video will be more useful.

Exploration

Discuss: *what do you notice about your original rectangle and the box you have made? What do you wonder?*

Ask students to trace around the vertices and edges of their box, and then unfold their box. They can investigate the relationships between the dimensions of the rectangle and the dimensions of the box:





Challenge students to explain using fractions how the dimensions of the rectangle are transformed into the dimensions of the box.

Possible student responses

- The depth of the box is $\frac{1}{4}$ the length of the short side of the rectangle
- The width of the box is $\frac{1}{2}$ the length of the short side of the rectangle

Teacher notes

- Encourage students to prove relationships using physical manipulation of the rectangle, e.g. folding the sides of the paper rectangle into the middle to demonstrate the relationship between the width of the box and the length of the paper.

Give each student a small paper rectangle and ask: *how will a box folded from this rectangle compare to the box you have already folded? Can you predict the relationship between the two boxes?* Students record the dimensions of the rectangle and their predictions on their student sheet, then fold the rectangle into a box as before.

Discuss: *what do you notice? What do you wonder?*

Challenge students to create an algebraic formula to calculate the volume of a box folded from a rectangle with length a (short side) and width b (long side). They will need to focus on the ways that the length, width, and depth of the box are related to the dimensions of the rectangle.

Ask students to use their formula to explain why the small box has $\frac{1}{8}$ the volume of the large box.

Teacher notes

- Let a rectangle have length a (short side) and width b (long side). A box folded from this rectangle will have the following dimensions:

$$\text{The area of the base of the box:} \quad A = \frac{1}{2}a \times \frac{1}{2}b$$

$$\text{The depth of the box:} \quad d = \frac{1}{4}a$$

$$\text{The volume of the box:} \quad V = \left(\frac{1}{2}a \times \frac{1}{2}b\right) \times \frac{1}{4}a$$

$$V = \frac{a^2 b}{16}$$

- If we cut the large rectangle into quarters, we create new rectangles with length $\frac{a}{2}$ (the short side) and width $\frac{b}{2}$ (the long side). A box folded from this rectangle will have the following dimensions:

$$\text{The area of the base of the box:} \quad A = \frac{1}{2}\left(\frac{a}{2}\right) \times \frac{1}{2}\left(\frac{b}{2}\right)$$

$$A = \frac{a}{4} \times \frac{b}{4}$$

$$\text{The depth of the box:} \quad d = \frac{1}{4}\left(\frac{a}{2}\right)$$

$$d = \frac{a}{8}$$

$$\text{The volume of the box:} \quad V = \left(\frac{a}{4} \times \frac{b}{4}\right) \times \frac{a}{8}$$

$$V = \frac{a^2 b}{128}$$

$$V = \frac{1}{8}\left(\frac{a^2 b}{16}\right)$$

- This demonstrates that the volume of the small box is $\frac{1}{8}$ the volume of the large box.

Class discussion

Ask: *If we followed different directions to fold a different origami box using the same rectangles, will the smaller rectangle **always** make a box with one-eighth the size of the larger box?*

Further activities

Cut an A4 sheet of paper in half through the long side to make new A5-sized rectangles. Investigate how the boxes folded from these rectangles compare to the boxes folded from A4 rectangles.

Questions to prompt students' reasoning

- *If the original sheet of paper had length a (short side) and width b (long side), what are the dimensions of the new rectangle?*
    - The new rectangle has length $\frac{b}{2}$ (short side) and width a (long side).

- *Is the new A5 rectangle in proportion with the original A4 rectangle?*
    - Students will find that the ratio between the long side and the short side of both rectangles is the same. This is an important property of A4 (and all other A-series) sized pages: the ratio between the long side and the short side is $\sqrt{2}$, and this ratio is maintained when the page size is halved or doubled.
- *How does the volume of the box folded from the A5 rectangle compare to the volume of the box folded from the A4 page? Can you use algebra to show how the two boxes differ in volume?*
    - Let the large rectangle have length a (short side) and width b (long side). The volume of the box folded from this rectangle is $V = \frac{1}{16}a^2b$.
    - The small rectangle has length $\frac{b}{2}$ (short side) and width a (long side). The volume of the box folded from the A5 rectangle is $V = \frac{1}{64}ab^2$. This can be demonstrated by substituting the new values into the original formula, or by calculating the base and depth of the new box using the new dimensions.

Folding Boxes

Name: _____

Large rectangle

Side length (long):

Side length (short):

Area:

Observations:

Large box

Side length (long):

Side length (short):

Depth:

Area of base:

Volume:

Small rectangle

Side length (long):

Side length (short):

Area:

Predictions:

Observations:

Small box

Side length (long):

Side length (short):

Depth:

Area of base:

Volume:

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education

Australian Academy of Science