CHICKEN BOXES

Lesson 1: A Single Row of Bird Boxes

Australian Curriculum: Mathematics (Year 6)

ACMNA133: Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence.

Lesson abstract

Students model, develop and describe rules for the number of panels needed to build a row of chicken boxes for a poultry show. Students are guided to move from recursive thinking to the relational thinking of functions. Through class discussion, students see that there is more than one correct rule for describing a particular pattern.

Mathematical purpose (for students)

Models and data help us see how patterns grow. We can describe how patterns grow using words and numbers.

Mathematical purpose (for teachers)

Modelling a pattern shows how a pattern physically grows.

Collecting and organising data shows how a pattern grows numerically.

The way a pattern grows numerically can be described using recursive (step-by-step) rules and using relational rules. Relational rules generalise a pattern so you can efficiently calculate the number of objects at any given step in the pattern.

Suggested presentation Two lessons of approximately one hour each.

Vocabulary encountered

- pattern
- rule
- sequence

Lesson materials

- reSolve PowerPoint *1a Single Row*
- Supplies for constructing models (see Teacher background information for options)

We value your feedback after these lessons via our website.

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science

Teacher background information

In this sequence students construct models of chicken boxes and observe patterns in construction, then generalise from those patterns. These models are built using two parts: *sticks* and *joins*.

For sticks, toothpicks, paddle pop sticks or similar can be used. In this lesson a minimum of 31 sticks per pair of students is required. For the joins you may choose to use modelling clay, chickpeas soaked in 1 part Dettol 4 parts water (3x375g packets will create enough chickpeas for the entire sequence), or similar. Later in the sequence, up to 105 sticks and 50 joins per pair will be required.

Patterns in a row of bird boxes

Set the scene by discussing Pet and Agricultural Shows and in this case exhibiting chickens. Display slide 2 of *1a Single Row* and discuss how the birds are displayed and the practical problem of housing hundreds of birds for a one or two-day show.

To give additional context, show students the brief video (linked in slide 3 of *1a Single Row* or https://youtu.be/R2SmuV-SoLM) of students from Ballarat Grammar School with the chickens they prepare to exhibit at the Royal Melbourne Show. This video describes some mathematical activities (recording the weight of the chickens, their food and eggs) which are not covered in this unit, but could make an interesting activity if your school has chickens!

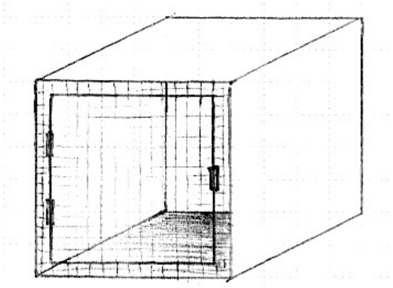
Organising bird boxes for a local show

The context for these lessons is planning the boxes that will be used to house chickens at a local show. At first we will be using a cubic design and working out how many parts will be needed to construct the boxes.

Present the context, using slide 4:

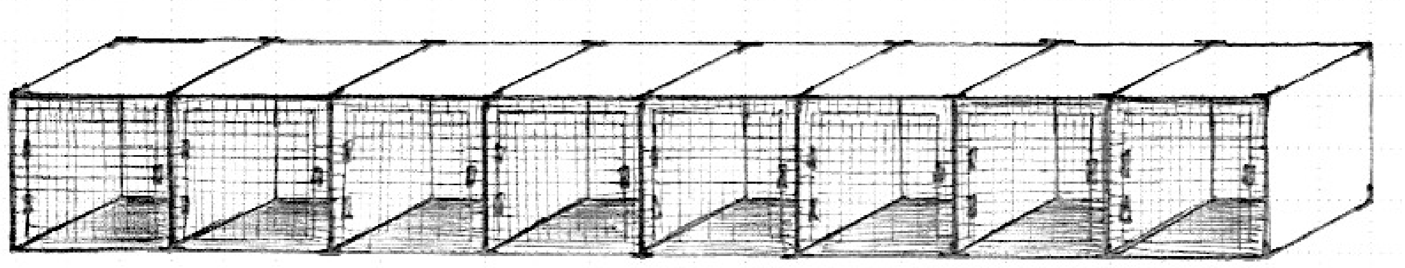
I have an idea for a new type of box to put chickens in for shows. This is my design.

It has five solid panels and one wire screen at the front (with a door in it). I have made it a cube so that all five solid panels are the same shape.



Discuss: *What are some advantages to this design? How could we make a row of these boxes?*

We could join the next box to the first one, using the same side panel for both boxes.



The local show would like to use my bird box design, but they don't know yet how many birds will be entered into the show. I need a quick way of working out how many panels and wire screens I am going to need to house all the chickens. Today we will be trying to find some rules for how many of each part we need.

Modelling patterns: number of bird boxes and number of parts

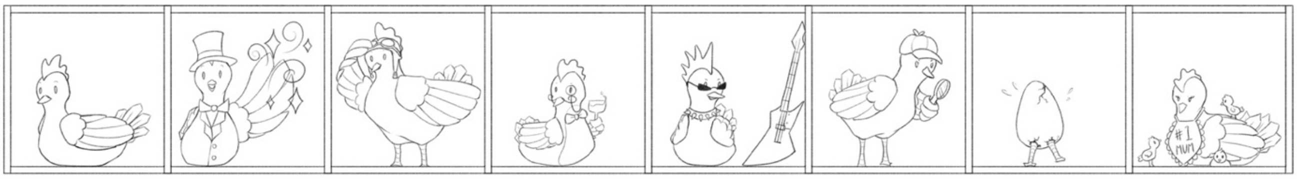
Show slide 5 with the 3D drawing of the row of boxes. Ask students: *what are the different parts of the boxes? Which parts are identical? How many wire screens are required for a row of boxes?* Encourage students to express their answer as a relational rule that relates the number of wire screens to the number of bird boxes.

Possible student response

- *The number of wire screens equals the number of bird boxes.*

Move on to the number of panels needed, i.e. top, base, back, left and right. This is a more difficult pattern because of the shared sides. Remind students that when we have a problem to solve we can often break it down into smaller problems. If we look at the back panel first, we can see we will need the same number as the wire screens. Then we can consider the other panels.

Suggest looking at the top, base, left and right panels from front-on (Slide 6) and draw the chicken boxes as squares in a row. Check that students understand that the drawing is the front-on view and that the sides of the squares represent panels.



Pose the questions: *How many sticks would be needed to make four boxes? 10 boxes? What about 15 boxes? Can you find a relationship between the number of boxes and the number of sticks needed?*

Have students make a 2D model of a row of boxes using sticks and joins. Ask students to create their own way to record the number of sticks used to make the boxes.

Allow students some time to explore and investigate patterns.

Possible student responses

- At this stage students will probably find a step by step rule like:
    - *The first bird box has four panels. For each box after that we add three panels.*
    - *The first square has four sticks. For each square after that we add three sticks.*

Teacher note

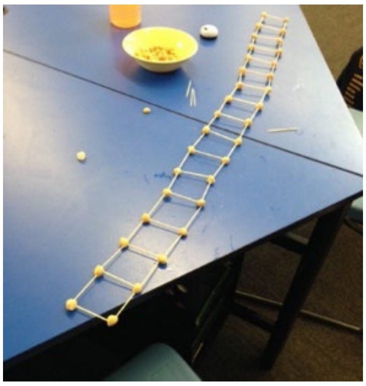
- Students may be thinking of the sticks as 'panels' or as sticks. Both of these interpretations are fine. The focus is on finding patterns and rules.

Ask students: *if we had 100 boxes, how could we go about calculating the number of sticks we needed? How can we calculate the number of sticks needed for any number of boxes?*

Discuss how it is easy to use a step-by-step rule to find a small number of boxes. Mathematicians call these step-by-step rules *recursive* rules. Finding the number of sticks for a large number of boxes using a recursive rule is inefficient. Instead, mathematicians use *relational* rules which directly relate the number of boxes to the number of sticks. To find relational rules, mathematicians collect and organise data so that they can look for patterns.

Expressing the pattern as a relational rule

Have students gather more information by making their row of bird boxes longer and recording data in a way that makes it easier to see patterns. Some students may create a table, which can be shared with the class. Otherwise, introduce the following table to the students.



Squares and Sticks		
Number of squares	Number of sticks	How did you work it out?
1		
2		
3		
4		
6		
8		
10		
100		
1000		

As students build more squares and add further data to their tables, encourage them to shift their thinking toward finding a relational rule so that they do not have to add three to the previous answer every time.

Enabling prompt

- *What changes with each new box you add? What stays the same?* Students write their observations in sentences.

Have students record their rule as a generalised statement using plain language. Some students may start to use symbols.

Possible student responses

- The total number of sticks for eight squares = (8 squares x 3 sticks) + 1 stick.
  A generalised statement would be: ***Number of sticks for any number of squares = (number of squares x 3 sticks) + 1 stick***.
- Number of sticks needed for eight squares = (8 squares x 4 sticks) – 7 sticks.
  After the first box each square shares one stick with the previous box, so I need to subtract one less than the total number of squares.
  A generalised statement would be: ***Number of sticks needed for any number of squares = (number of squares x 4 sticks) - (the number of squares - 1)***

Teacher notes

- There is more than one way of expressing the same rule. You may find students have thought about the arrangement of sticks in a different way and have found a different rule that is also correct.
- Some students may give a rule which is not fully relational. For example, to get the answer for 100 squares, some students may use their result from 10 squares and put 10 rows of 10 together. For the number of sticks, they calculate 10 x 31 and then adjust for the shared sides. These rules are harder to write down. There are many different possibilities.

Going deeper (optional)

This section can serve as consolidation and revision of the investigation.

Chicken boxes: back panels

Our original problem was to find a rule for working out how many panels will be needed to build the bird boxes for poultry at the show. Ask students what we still need to consider: the back panel. *How do you need to change your rule to also have a back panel for every box?*

Possible student response

- Every box has one back panel, so I can take my previous rule and add to it the number of boxes.
- A generalised statement would be: ***Total number of panels for any number of squares = (number of squares x 4 panels) + 1 panel***.

Another pattern: joins

Encourage students to apply what they have learned about relational rules to quickly find a rule for the number of joins required for any number of boxes.

Possible student response

- A generalised statement would be: ***Total number of joins for any number of squares = (number of squares x 2 joins) + 1 join***.

Summarise

Select some students to present their findings to the class. Have students relate their rule back to how they saw the pattern in the way the chicken boxes were built. Look at how the various strategies are different and how they are similar.

Teacher notes

- Using algebra we can show that the two generalisations outlined above are calculating the same total:
  - ***Number of sticks for any number of squares = (number of squares x 3 sticks) + 1 stick***
    - $n = 3x + 1$, where x represents the number of squares and n represents the total number of sticks.
  - ***Number of sticks for any number of squares = (number of squares x 4 sticks) - (the number of squares - 1)***
    - $n = 4x - (x - 1)$, where x represents the number of squares and n represents the total number of sticks. Simplifying this equation, we get:

      $n = 4x - n + 1$

      $n = 3x + 1$

Form whole class statements for each pattern. For example: *The number of sticks is always equal to the number of squares multiplied by three, plus one.*

Teacher note

- **Important:** Emphasise that we are talking about the *number* of squares and the *number* of sticks, rather than "multiplying the squares to get the sticks".

Discuss: *what makes this rule relational? What are the advantages of a rule which can go straight to the answer, instead of step-by-step?*

Further activities

Have the students look at the whole class statements on the whiteboard and suggest single symbols that we might be able to use to replace the words in each statement. For example, if the statement was:

> *If we multiply the number of bird boxes by four and then add one, we will always be able to find the total number of panels needed.*

Students will probably suggest 'x' for multiply, '+' for plus. Rub out the words and replace with the symbols they suggested.

They may suggest '4' for four and '1' for one. **Important**: Congratulate them on this. Sometimes we forget that our 'digits' are really 'symbols' we use to represent a number. Again, replace the words with symbols.

At this point they will be left with something like:

*If we **x** the number of bird boxes by 4 and then **+** 1, we will always be able to find the number of panels needed.*

Through discussion they may see that we can replace “we will always be able to find” with an ‘=’ symbol and end up with something like:

number of bird boxes x 4 + 1 = number of panels needed.

If the student has tried using pronumerals use the ones they have picked, for example using *b* to represent the number of bird boxes or *p* to represent the total number of panels.

b x 4 + 1 = p   or   4 x b + 1 = p   or   p = b x 4 + 1   or   p = 4 x b + 1

Show students how to substitute different values for *b* to find different values of *p*. Have students trial the same process and interpret their results.