CHICKEN BOXES

Lesson 4: Modelling Chicken Boxes in 3D

Australian Curriculum: Mathematics

Australian Curriculum: Mathematics (Year 6)

ACMNA133: Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence.

Lesson abstract

Students are set the open task of finding the number of sticks required to make a three dimensional model of an array of chicken boxes of any size. They build a 4 x 4 array to test their ideas and generalise to create a formula.

Mathematical purpose (for students)

Models and data help us see how patterns grow. We can describe how patterns grow using words and numbers.

Mathematical purpose (for teachers)

Modelling a pattern shows how a pattern physically grows.

Collecting and organising data shows how a pattern grows numerically.

The way a pattern grows numerically can be described using recursive (step-by-step) rules and using relational rules. Relational rules generalise a pattern so you can efficiently calculate the number of objects at any given step in the pattern.

Suggested presentation Two lessons of one hour each

Vocabulary encountered

- generalise
- sequence of numbers
- vertex (vertices)

Lesson materials

- A minimum of 50 joins per group
- A minimum of 105 sticks per group
- reSolve PowerPoint *4a 3D Chicken Boxes*

We value your feedback after these lessons via our website.

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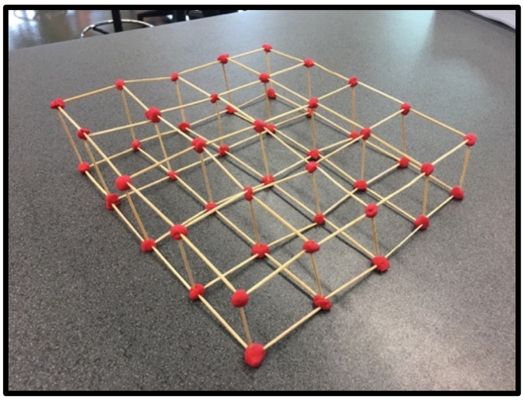
Modelling chicken boxes in three dimensions



Ask students: *what would a three-dimensional model of a chicken box array look like?*

Discuss what a "chicken box array" means: all rows have the same number of boxes, the array will only be 1 box deep, the array could be square or rectangular, etc.

After discussion show a picture of a completed array (slide 2 of *4a 3D Chicken Boxes*) or a completed model you have made yourself.

Ask: *about how many sticks do you think there are in this model? Why? What have we learned that we can apply to this model?*

Issue the challenge: *can you find a rule or method for the total number of sticks needed for this three-dimensional chicken box array? How about a rule or method for a three-dimensional array of any size?*

Exploration

Students experiment with constructing 3D arrays to gather information and start identifying patterns. Remind students of the strategies they have used in previous lessons (e.g. counting "the first square in the row" separately from the other squares, dividing the rows into "first row" and "additional rows"). A 4 x 4 array of boxes is a good place to begin.

Students who are confident working with the ideas of previous lessons can work independently and design their own methods for the task. Encourage them to look for rules that use the geometric structure of the array. Some possible approaches are shown below.

Possible student responses

- *If we multiply the number of boxes by eight and then add four, we will always be able to find the number of sticks required to make the first row of an array.*
- *If we multiply the number of boxes by five and then add three, we will always be able to find the number of sticks required to make additional rows of an array.*

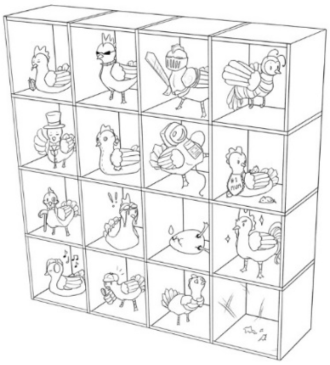
Enabling prompts

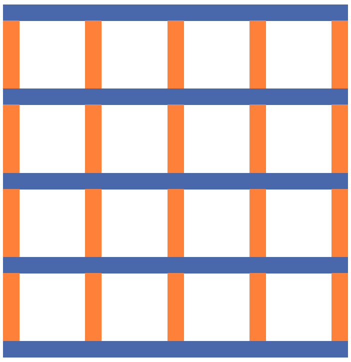
- If students are struggling with a lack of direction, suggest modelling a single chicken box and then adding boxes and rows up to a 4 x 4 array. Record the number of sticks added with every box/row.
- By adding the results of the rule for each row, students will determine that the number of sticks in a 3D 4 x 4 array of chicken boxes is 105.
    - First row: 4 x 8 + 4 = 36
    - Three additional rows: 3 x (4 x 5 + 3) = 69
    - Total: 36 + 69 = 105

Multiple Approaches

As with the previous lessons, encourage students to use and justify multiple methods to solve the problem.

For example, students might determine the number of sticks on the front and back of the 3D model, then the sticks between. Applying ideas from Lesson 3 to a 4 x 4 array:





Front:

- Horizontal sticks: five lines of four
- Vertical sticks: four rows of five
- Total front: (5 x 4) + (4 x 5) = 40

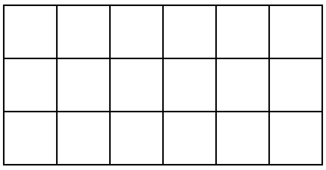
Back: same as front.

Sticks required to join the front and back faces: we see five rows of five sticks. 5 x 5 = 25.

Adding numbers of sticks for the front and back and joining together gives the total: 40 + 40 + 25 = 105.

Students should then write a rule (or a series of steps of calculation) for an array of any size. To do this they will need to work out what is general from their specific case. For example, instead of 'five' they will need to say 'the number of rows of boxes plus one'.

Generalising is an important step. Care is needed because the symmetry of the square array masks some of the complexity. In the case above, the number of horizontal sticks in the front face is the same as the vertical. For a rectangular array this will not be the case. For example, the front of a 6 x 3 array:



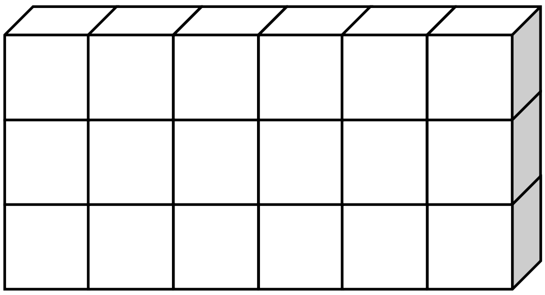
Horizontal sticks: 4 lines of sticks with 6 in each, or (3+1) x 6 = 24

Vertical sticks: 7 lines with 3 sticks in each, or 3 x (6+1) = 21

The most general form of the method outlined above is:

- Number of horizontal sticks in front = (number of rows plus 1) x (number of boxes per row)
- Number of vertical sticks in front = (number of rows) x (number of boxes per row plus 1)
- Number of sticks joining the front and back = (number of rows plus 1) x (number of boxes per row plus 1)
- Add the horizontal and vertical sticks for the front, multiply by 2 for the back, then add the joining sticks.

Once students have devised a method, they will need to test their method by constructing a different array with a new number of boxes per row and a new number of rows. For example, a 6 x 3 array:



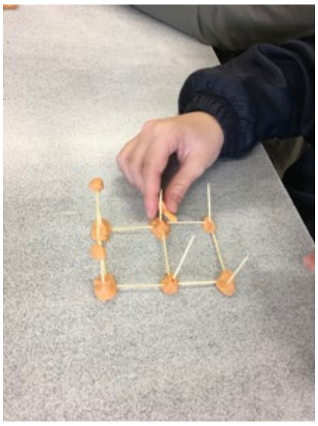
- *First row: 6 x 8 + 4 = 52 sticks*
- *Additional rows: 6 x 5 + 3 = 33 sticks*
- *First row plus 2 new rows: 52 + 2 x 33 = 118 sticks*

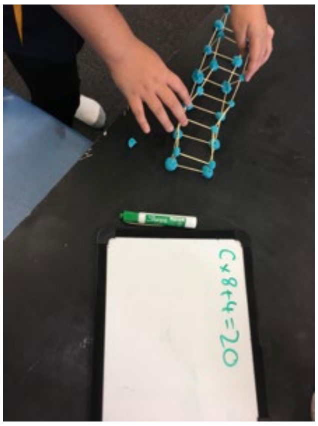
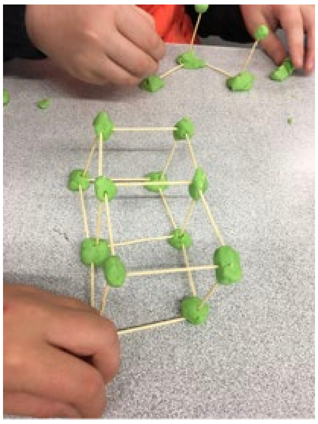
Teacher notes

- As with the previous lessons, whatever methods utilised by students must be justified. Encourage students to record their justification in written language and describe it verbally in their groups.
- If you need to test a student's answer for the number of sticks in an array, use the following rule which has been simplified from the example above:

  For a *W* x *H* array, the number of sticks = 5 x *W* x *H* + 3 x *W* + 3 x *H* + 1

  e.g. in a 7 x 4 array, number of sticks = 5 x 7 x 4 + 3 x 7 + 3 x 4 + 1 = 174



Conclusion

Select students who have used different strategies to present their rules and explanations to the class.

Construct a class display of different rules, accompanied by models or photos of models.

Revisit the learning intentions given at the start of the unit and have students discuss or write about what they learned:

- Models and data help us see how patterns grow.
- We can describe how patterns grow using words and numbers.

Further activities

More patterns

This lesson focused on the number of sticks. Students could also find a pattern for the number of joins, or for the number of panels required for these chicken boxes (with and without wire screens at the front).

Using symbols and pronumerals

Some students may like to use algebraic letters to write their rules. Encourage them to state clearly what the letters mean. Here we use *b* for the number of boxes and *s* for the number of sticks.

1. First row of boxes *b* x 8 + 4 = *s*
2. Subsequent rows within the array: *b* x 5 + 3 = *s*

To work out the total number of sticks in the 4 x 4 array:

1. First row: *b* x 8 + 4 = *s*, 4 x 8 + 4 = *s*, *s* = 36
2. Each subsequent row: *b* x 5 + 3 = *s*, 4 x 5 + 3 = *s*, *s* = 23
3. Three subsequent rows: 23 x 3 = 69
4. Total sticks in the 4 x 4 box array = 36 + 69 = 105 sticks