CHICKEN BOXES

Lesson 3: Modelling an Array of Chicken Boxes

Australian Curriculum: Mathematics (Year 6)

ACMNA133: Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence.

Lesson abstract

Students consider patterns arising from the number of components in an array of boxes. Students first look at patterns in additional rows, and then combine these rules with the rules for the first row to calculate quantities of components for the entire array. Students are then challenged to find other patterns in the whole array.

Mathematical purpose (for students)

Models and data help us see how patterns grow. We can describe how patterns grow using words and numbers.

Mathematical purpose (for teachers)

Modelling a pattern shows how a pattern physically grows.

Collecting and organising data shows how a pattern grows numerically.

The way a pattern grows numerically can be described using recursive (step-by-step) rules and using relational rules. Relational rules generalise a pattern so you can efficiently calculate the number of objects at any given step in the pattern.

Suggested presentation One lesson of one hour

Vocabulary encountered

- array
- sequence of numbers
- vertex/vertices

Lesson materials

- reSolve PowerPoint *3a Chicken Array*
- Student Sheet 1 – Rules for New Rows (1 per student)
- Modelling materials as in Lesson 1 (at least 40 sticks and 25 joins per pair)

Modelling an array

In the previous two lessons we only considered a single row of chicken boxes. Suggest to students that one row of chicken boxes would not be an efficient use of space. A vertical array of boxes would be able to accommodate chickens more efficiently. We will be working with cubic boxes as in Lesson 1, i.e. with square fronts and panels, building extra rows on top.

Show the array on slide 2 of *3a Chicken Array* and look at how it is constructed from rows built on top of one-another. Remind students that we have already created a quick and easy way to work out the number of sticks required for the first row of boxes (review whole class statements from Lesson 1).

Issue the challenge: *We know how many sticks are required to build a row of four boxes. How many sticks might be required to build a 4 x 4 array of boxes? Or any square array of boxes?*

Ask students to make some predictions before modelling:

- *Will a 4 x 4 array of boxes require four times as many sticks as a row of four boxes? Why/why not?*
- *Will an additional row built on top of the first row use the same number of sticks as the original row? Why/why not?*
- *Will every additional row use the same number of sticks? Why/why not?*

Developing rules

Hand out construction materials and Student Sheet 1 – Rules for Additional Rows. Explain to students that at this stage the focus is on creating rules for additional rows. Later in this lesson you will look at creating rules for entire arrays.

Students construct models of the front of a 4 x 4 array to test their predictions. As they construct their models, they observe number patterns and fill out the two tables on the first page of Student Sheet 1.

Teacher note

- Students may want to build bigger arrays, but at this stage in the unit they should be moving towards quickly finding a relational rule and then using the rule to fill out the rest of the table. Use time wisely.

Groups record a natural language statement describing a rule that links the number of squares in an additional row to the number of sticks required, and to the number of joins required.

Possible student responses

- The first square in each additional row will need three new sticks, and each square after that will require just two new sticks. The rule will be the same for each additional row. Students can check that the 'additional' rule will apply to all three rows.
- They can describe the pattern in a similar way to the following: *In each of the additional rows, if we multiply the number of squares by two and then add one, we will always get the number of sticks needed.*
- The first square in each additional row needs two new joins, and each square after that will require one new join.

Once students have completed the two tables, come together to discuss findings.

Looking at the whole array

Challenge students to use their findings to work out the total number of sticks/joins for the array. Emphasise that there may be several different ways to describe the pattern and they are encouraged to find them. All approaches should be accompanied by written justification.

Possible student responses

- The number of sticks for one additional row is multiplied by the number of additional rows. The result can then be added to the number of sticks in the first row:
  - The first row has (4 x 3) + 1 = 13 sticks
  - Each additional row has (4 x 2) + 1 = 9 sticks
  - Three additional rows have 3 x 9 = 27 sticks
  - Therefore the total number of sticks = 27 + 13 = 40.
- Some students may divide the array into horizontal sticks and vertical sticks:
  - Each column has (4 + 1) horizontal sticks, making a total of 4 x (4 + 1) = 20 horizontal sticks for the whole array
  - Each row has 4 + 1 vertical sticks, making a total of 4 x (4 + 1) = 20 horizontal sticks for the whole array
  - Therefore the total number of sticks = 20 + 20 = 40
- Students could also visualise the array as a large 'L' shape, made of the four left and four bottom sticks. Creating each box then only requires adding two sticks (top and right side of each square).This results in the equation 4 + 4 + (2 x 16) = 40 sticks for all 16 boxes.

- To calculate the number of joins required:
  - The first row has (4 x 2) + 2 = 10 joins
  - Each additional row has (4 x 1) + 1 = 5 joins
  - Three additional rows have 3 x 5 = 15 joins
  - Therefore the total number of joins = 15 + 5 = 20

Teacher note

- It is important to allow students to have a go at the various steps before too much teacher support is provided. Maintaining an appropriately high level of challenge for all students is key.

Conclusion

Have students present their different strategies to the class. Prompt reasoning with the questions:

- *How did you break down the array to make your rules?*
- *How do your rules reflect that?*
- *Would you recommend other students try breaking down the array using your approach? Did you think it was an efficient strategy?*

As a class discuss the important findings from the lesson, including:

- Any similarities and differences between the rules that have been found.
- That there can be many different rules that all correctly describe the same pattern.

As a class, consider how the rules can be combined to make one rule for an array of any size. Remember that an array will always be a rectangle or square, so every row will have the same number of boxes.

Teacher Notes

- You can check student answers to this challenge using the following algebra:
  - For an array that has a width of W boxes and a height of H boxes, the number of sticks required will be equal to $2 \times W \times H + W + H$.
  - The number of joins required will be equal to $(W + 1) \times (H + 1)$.

Further activities

Patterns between Arrays: Any students who would like a challenge might also explore patterns that emerge when calculating all of the square arrays (e.g. 2 x 2, 3 x 3, 4 x 4, 5 x 5 etc). They will find that the *difference* between the number of sticks/sides in each subsequent array *increases* by four.

- Number of sides in 1 x 1 array = 4
- Number of sides in 2 x 2 array = 12 (12-4=8)
- Number of sides in 3 x 3 array = 24 (24-12=12)
- Number of sides in 4 x 4 array = 40 (40-24=16)
- Number of sides in 5 x 5 array = 60 (60-40=20)
- Number of sides in 6 x 6 array = 84 (84-60=24)

Rules for Additional Rows

Name: ______________

We already know rules for making the first row. Let's find some rules for making additional rows. Use a diagram or model to look for patterns.

Squares and Sticks		
Number of squares in additional row	Number of sticks for additional row	How did you work it out? What rules have you found?
1		
2		
3		
4		
6		
8		
10		
100		
1000		

Find more patterns in your model by looking at the number of joins used.

Squares and Joins		
Number of squares in additional row	Number of joins for additional row	How did you work it out? What rules have you found?
1		
2		
3		
4		
6		
8		
10		
100		
1000		

Mathematics by Inquiry is an initiative of, and funded by, the Australian Government Department of Education and Training

Australian Academy of Science