reSolve MATHS BY INQUIRY Area of a Circle

Y8

Summary of learning goals

- Students use their knowledge of the relationship between the circumference and diameter of a circle to see why the area of a circle is πr^2.
- Guiding questions encourage students to think about issues of accuracy and informally introduce the idea of the circle as a limiting shape. The lesson involves visualisation and explanation.

Australian Curriculum: Mathematics (Year 8)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Summary of lessons

Who is this sequence for?

- Students who know basic properties of circles, the formula for the circumference of a circle, and the formula for the area of a triangle. They should be familiar with π as the ratio of circumference to diameter of a circle.

Lesson 1: Four Demonstrations

- Students work in groups to understand one of four demonstrations of the formula $A = \pi r^2$ and explain to others why the formula works. In the process, they refine both their own understanding and their explanations.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Australian Academy of Science

Reflection on this sequence

Rationale

This sequence emphasises justification and mathematical communication. Students learn that results in mathematics can be proved, rather than tested only from numerical evidence, and that frequently there are many different ways to demonstrate a result.

reSolve mathematics is purposeful

- Students are introduced to important mathematical ideas, such as approximation and informal ideas of limits, and the importance of proof in mathematics. One of the demonstrations is based on an historical method that dates back to Archimedes.

reSolve tasks are inclusive and challenging

- The variety of demonstration methods allows all students to gain an appreciation of the formula for the area of a circle. Visualisation encourages different ways to approximate a circle with shapes of known area.

reSolve classrooms have a knowledge-building culture

- Students work together in groups, and explain to each other, in order to increase their own understanding through the comments of others. This develops mathematical communication and reasoning.

LESSON 1

re(Solve) MATHS BY INQUIRY

Area of a Circle

Four Demonstrations

Y8

About this lesson

Students work in groups to understand one of four demonstrations of the formula $A = \pi r^2$ and explain to others why the formula works. In the process, they refine both their own understanding and their explanations.

Australian Curriculum: Mathematics (Year 8)

ACMMG197: Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.

Mathematical purpose

- Students explore a demonstration of the formula that $A = \pi r^2$. They engage with mathematical reasoning and develop skills in communicating clearly.

Learning intention

- Explain why the area of a circle is πr^2.

Time

A lesson of approximately 1 hour.

Vocabulary

- area
- demonstration
- formula
- limit
- polygon
- sector

Resources

- reSolve PowerPoint *1a Four Demonstrations*
- Student Sheet 1 – The Corner Square
- Student Sheet 2 – Slices of Pie
- Student Sheet 3 – Archimedes' Polygons
- Student Sheet 4 – Unravelling the Circle

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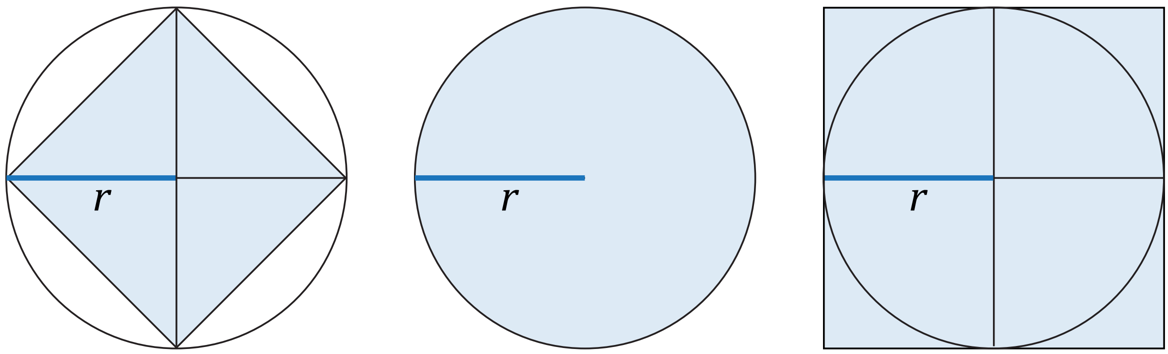
Introduction: Setting some limits on circle area

Resources: Use the reSolve PowerPoint *1a Four Demonstrations* to introduce the concept of a 'radius square'; that is, a square drawn on the radius of a circle.

Students are asked to draw diagrams to show that the area of a circle is greater than that of two radius squares but less than that of four radius squares.



$$2r^2 \quad < \quad \text{Area of circle} \quad < \quad 4r^2$$

When it has been shown that the area of the circle is between $2r^2$ and $4r^2$, students will probably guess (if they do not already know) that the exact factor is π.

Looking at four demonstrations

The main task involves students working in small groups to understand a given demonstration and to communicate to others how their particular demonstration shows that the formula for the area of a circle is $A = \pi r^2$. The aim is that students, in their small groups, discuss their allocated circle area demonstration to understand it sufficiently well to explain it to the class.

Suggested strategy

- Arrange students into groups of three or four and allocate a demonstration to each group.

  Resources: Provide one of the four Student Sheets to each group.

- **Consensus building:** Small groups discuss how their visualisation helps to explain why $A = \pi r^2$ and ensure that each member understands it fully.
- **Small trial planning:** Plan a presentation within the small group to explain the argument to other students.
- **Trial presentation:** Each small group presents its argument to another group, in order to convince them that $A = \pi r^2$.
- **Presentation refinement:** Small groups refine their presentations as needed.
- **Whole-class presentation:** One of each demonstration is presented to the whole class, with an opportunity for others to ask questions so that everyone understands each demonstration.

Summaries of the demonstrations

Demonstration 1: The corner square

This demonstration uses a numerical example to refine the result shown in the introduction (i.e. that the area of the circle is between two and four times the area of the radius square). Note that this does not prove that the area of a circle is $A = \pi r^2$ but, rather, gives an approximation.

- Students count squares to find the ratio of the area of the whole circle to the area of the 'corner square'.
- The observation that the area of the circle is approximately 3.1 times the area of the corner square suggests that the formula for the area of a circle is likely to be $A = \pi r^2$.

Demonstration 2: Slices of pie

This demonstration uses visualisation (which may be aided by actually cutting and rearranging), informal ideas of limits and generalisation.

- A circle is divided into equal sectors. The diagram shows 18 equal sectors. Students might wish to actually cut out the circle sectors and paste them into place.
- Sectors are rearranged to approximate a rectangle (of side lengths r and πr).
- The area of the circle is equal to the area of the 'almost rectangle', so for both $A = \pi r^2$.

Note that students will need to be comfortable with the formula $C = \pi d$ and be able to deduce that half the circumference of a circle is therefore πr.

Demonstration 3: Archimedes' polygons

This demonstration uses visualisation, informal ideas of limits and generalisation. It is the first part of an historical proof, first formulated by Archimedes around 250 BCE. The statement made by Archimedes is that the area of the circle is equal to the area of a right-angled triangle in which one of the sides of the right angle is the length of the radius of the circle and the other is the length of the circumference.

- A right-angled triangle with sides meeting at right angles of length r and C is drawn. The side of length C is divided equally into a given number of line segments. (In the diagram provided there are 12 segments.)
- A circle of radius r is drawn and divided into the same number of sectors as line segments constructed above, and with the vertex of each sector at the centre of the circle.
- As the number of sectors and line segments increases, the area of each sector becomes nearly the same as the area of the triangle constructed on each line segment; hence, the area is $A \approx \pi r^2$.

Note that Archimedes then used inscribed and circumscribed polygons, each divided into triangles, to show that the area of the circle cannot be less than the area of the right-angled triangle described above, and also that the area of the circle cannot be greater than the area of the right-angled triangle described above. Hence, the area of the circle must be equal to the area of the right-angled triangle or $\frac{1}{2}Cr = \pi r^2$.

Students will need to recognise that triangles with the same height and equal bases have the same area.

Demonstration 4: Unravelling the circle

This demonstration uses visualisation, informal ideas of limits and generalisation.

- The circle is visualised as being composed of a series of concentric rings.
- It is cut along a radius and the rings become near rectangles that can be laid one beside the other to form an 'almost triangle'.
- As the circle is divided into more and more rings, when they are cut, flattened and laid out, the shape formed will become closer to a right-angled triangle with base $2\pi r$ and height r. Hence, the area of the circle must be equal to the area of the right-angled triangle or πr^2.

The Corner Square

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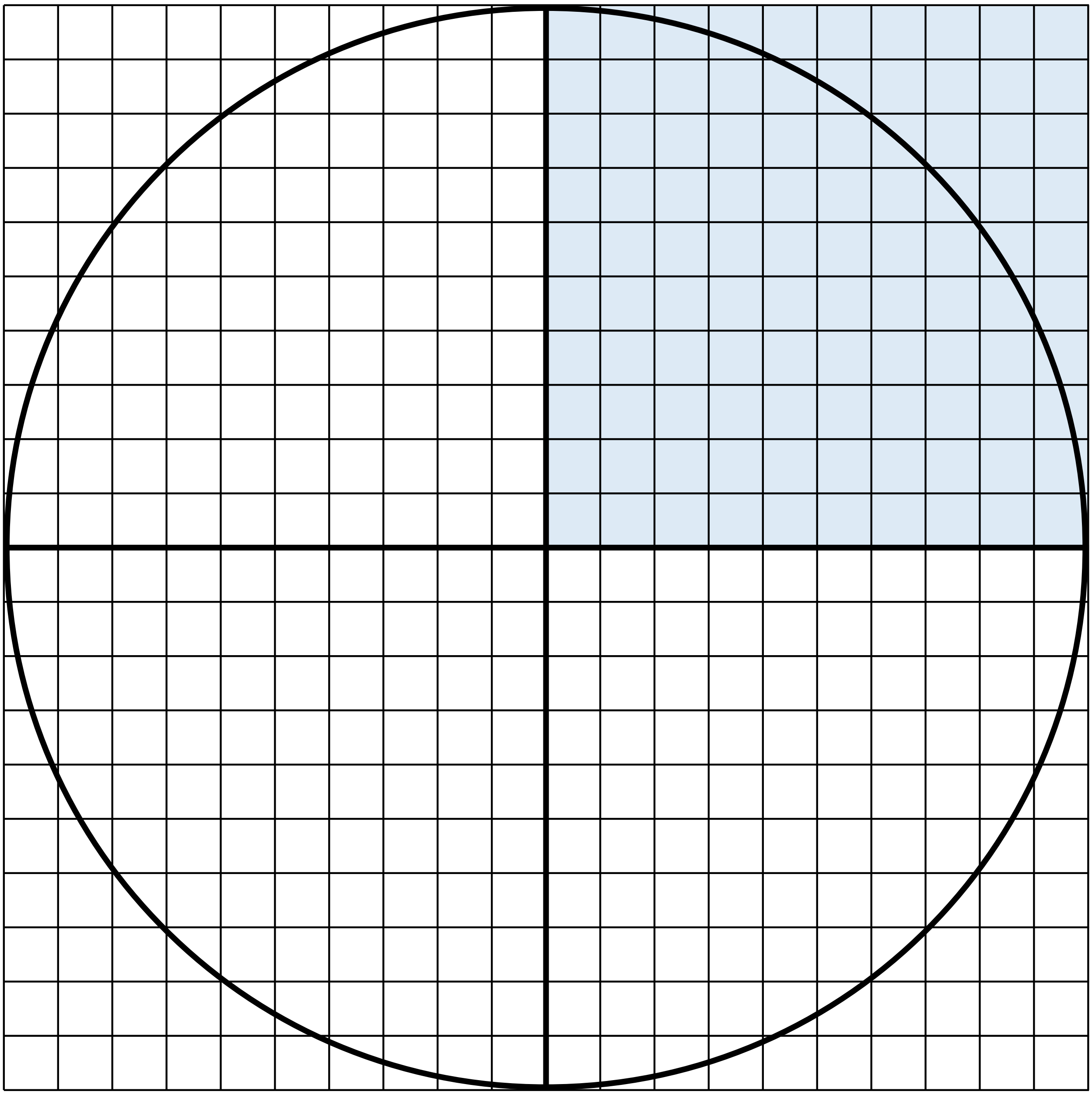
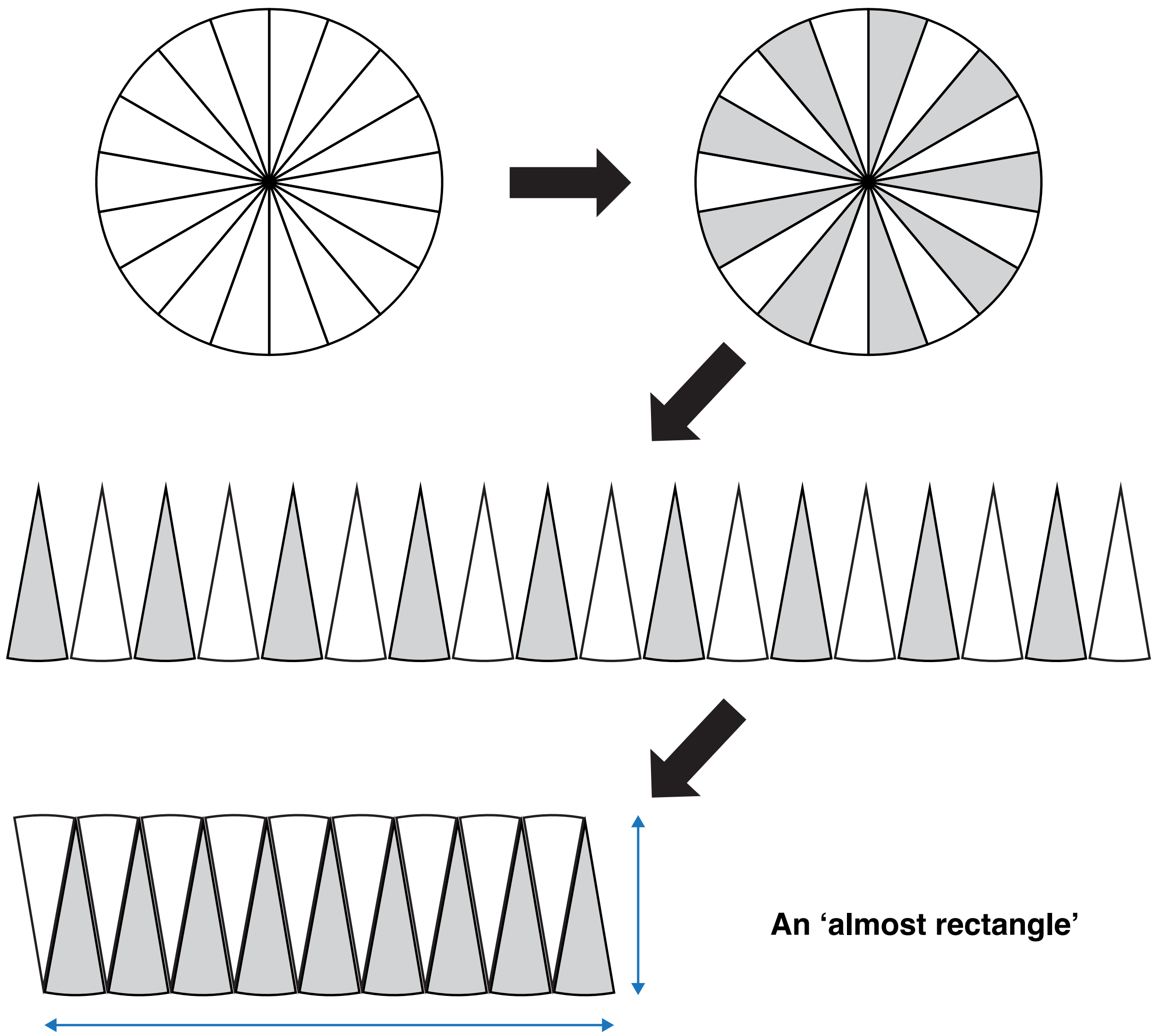


Guiding questions

- If the radius of the circle is r, what is the area of the shaded 'corner square'?
- How can you use the grid to work out how many corner squares fit into the circle?

Slices of Pie

Name: ______________________________



Guiding questions

- What happens to the shape of the 'almost rectangle' when you increase the number of slices of the circle?
- Why is the length of the 'almost rectangle' πr and the width r?

Archimedes' Polygons

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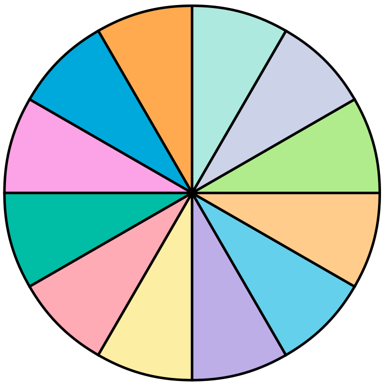

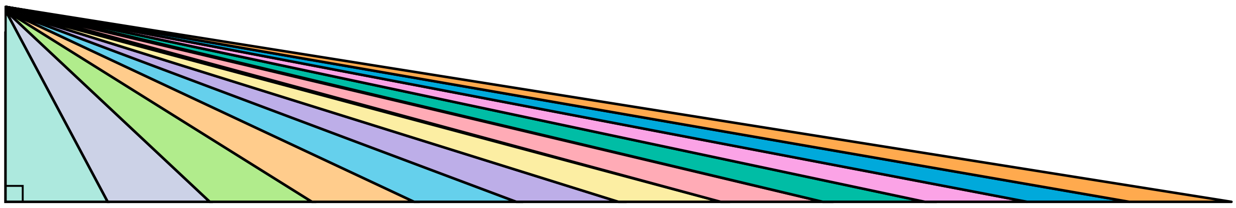

Guiding questions

- Why is the area of each small triangle in the large right-angled triangle the same?
- What happens to each sector of the circle when you increase the number of line segments along the base of the large triangle?

Unravelling the Circle

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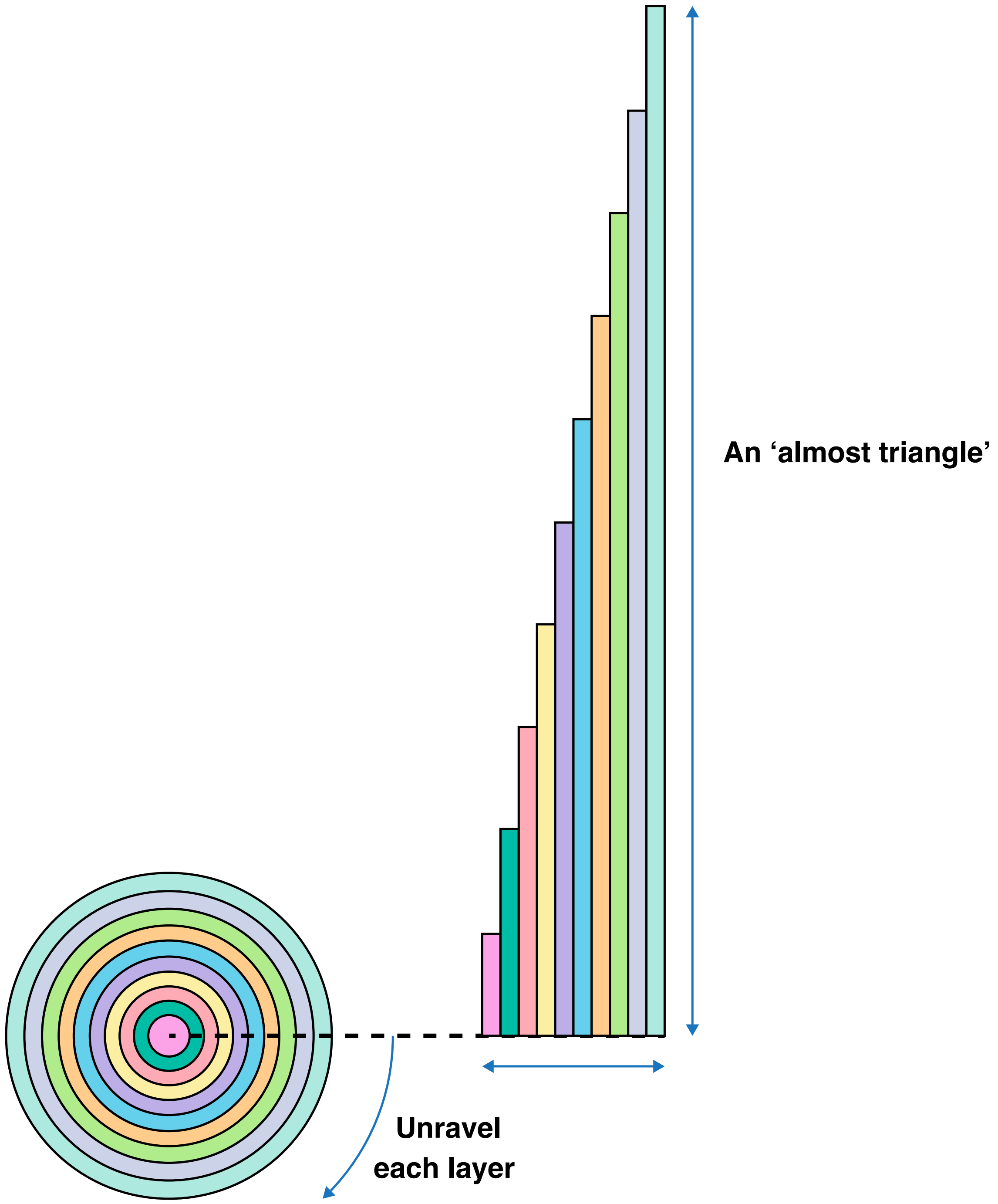

Guiding questions

- What happens as you decrease the width of the rings in the circle?
- Why is the base of the 'almost triangle' r and the height πd?