re(Solve) MATHS BY INQUIRY Partitioning

Y1

Summary of learning goals

- To explore the concept of equivalence. The equals sign is presented as a symbol indicating balanced value on both sides of an equation. Students explore equivalence by partitioning numbers into two parts.
- Opportunities to explore the commutative property and compensation also arise throughout the tasks as students work systematically to find all possible combinations.

Australian Curriculum: Mathematics (Year 1)

ACMNA015: Represent and solve simple addition and subtraction problems, using a range of strategies including counting on, partitioning and rearranging parts.

Summary of lessons

Who is this sequence for?

- Students towards the start of Year 1 who have one-to-one correspondence skills in counting and are developing a deeper sense of number.
- Students who are likely to use concrete materials to model the addition of numbers and use count all or count on strategies to find the total in a given collection.
- Students who are familiar with recording equations using symbols, including the use of the equals sign to indicate equivalent values on both sides of an equation.
- Students who would benefit from having some experience in making generalised statements about the structure of number and operations.

Lesson 1: Red Apples, Green Apples

Students explore the different ways that numbers can be partitioned into two parts. Working systematically, students are asked to find the partitions and show that they have all possibilities. This task affords an exploration of early algebraic thinking and scaffolds students towards seeing a pattern and making a generalisation regarding all possible combinations.

Lesson 2: 12 Ways to Get to 11

This task uses the picture book *12 Ways to Get to 11* to investigate how many two-number partitions are possible for any given number. Students apply their learning from Lesson 1: Red Apples, Green Apples to list all the ways that 11 can be made using two numbers. They then look at the number of possible partitions that can be made from some other numbers. Students investigate the fact that for any given number the possible combinations will be always one more than the number itself, and explain why this is the case.

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Reflection on this sequence

Rationale

Algebraic thinking requires a generalised understanding of the structure of our number system and operations. Understanding arithmetic through generalised algebra means moving from a search for answers to an exploration of relationships in number. One of the central algebraic relationships that students need to understand is that of equivalence and the meaning of the equals sign. This resource focuses on building students' understanding of equivalence through exploring the part–part–whole structure of numbers. Being able to recognise and rename numbers according to the smaller parts they comprise supports efficient addition and subtraction. Examples of this include:

- Near doubles: 8 can be thought of as 7 and 1.
  $8 + 7 = 7 + (7 + 1) = (7 + 7) + 1$
- Bridge to 10: 7 can be made into 5 and 2.
  $8 + 7 = 8 + (2 + 5) = (8 + 2) + 5$

reSolve mathematics is purposeful

- This task uses a context familiar to students to generalise an important algebraic understanding. The concept of equivalence is explored as students show they have found all the ways in which 10 can be partitioned into two smaller parts. They then see that the same method can be applied to all numbers. This develops an understanding of equivalence and part–part–whole that is foundational for students' additive thinking, specifically strategies for computation.
- Problem-solving skills, such as working systematically and looking for patterns, are also developed.

reSolve tasks are inclusive and challenging

- All students are provided with a common experience at the start of the task as they participate in a counter toss to generate some possible combinations. Students can continue to use counters to enable further exploration, or they can move to abstract thinking as they consider other possible values that are yet to be found. The challenge of the sequence lies in students justifying that they have found all possible combinations and then progressing towards a generalisation as to how many possible two-part partitions exist for any number.
- The use of 10 as a total is not central to this task. Access can be provided for some students via a smaller starting total, which still allows students to contribute to the class knowledge of how any number can be partitioned into smaller parts.

reSolve classrooms have a knowledge-building culture

- This sequence works to provide students with an emerging generalised understanding of algebra through the exploration of equivalence and partitioning.
- Classroom discussions provide a central part of this sequence. Through mathematical conversation and explorations, students build on their current understandings and are challenged by the contributions of others.

Acknowledgements

Merriam, E. & Karlin, B. (1996). *12 Ways to Get to 11*. Aladdin Paperbacks: New York.

LESSON 1

re(Solve) MATHS BY INQUIRY Partitioning

Red Apples, Green Apples

Y1

About this lesson

This task affords an exploration of early algebraic thinking around the concept of equivalence. Using the context of 10 apples in a bag, students explore how many apples might be green and how many might be red. The equals sign is introduced to show that the different combinations of apples in the bag all have the same total value. The task then asks students to order their work to find all possible ways 10 can be made with two whole numbers. This moves students towards seeing a pattern and making a generalisation for partitioning any number into two parts.

Australian Curriculum: Mathematics (Year 1)

ACMNA015: Represent and solve simple addition and subtraction problems, using a range of strategies including counting on, partitioning and rearranging parts.

Mathematical purpose

- To explore the concept of equivalence. The equals sign is presented as a symbol indicating balanced value on both sides of an equation. Students explore equivalence by partitioning a number into two parts.
- Opportunities to explore the commutative property and compensation also arise throughout the task as students work systematically to find all possible combinations.

Learning intention

- To investigate how many ways 10 can be made using two numbers.

Time

A lesson of approximately 1 hour.

Vocabulary

- equals
- equivalence
- partition

Resources

- Two-colour counters (10 per student, ideally with red and green sides). See the Teacher background information for alternatives.
- Number balance (optional)

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Teacher background information

Context

This task uses the context of a bag containing 10 red and green apples.

If two-colour counters are not available, red and green sticky dots could be placed on either side of a counter. Students could also draw 10 cubes/counters out of a bag that contains at least 10 red and 10 green cubes/counters.

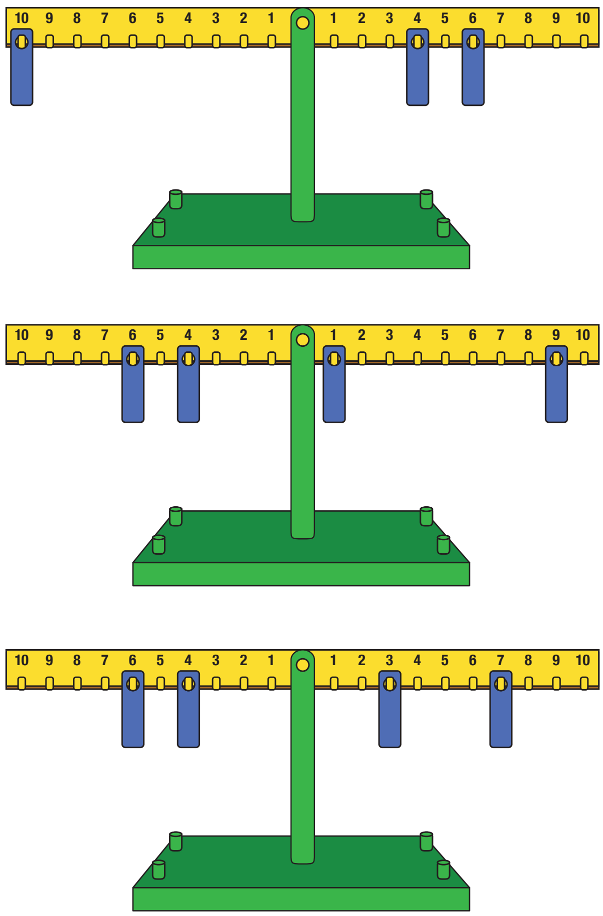
It is important that students associate two different colours of apples with the colours that are on the counters. If red and green counters are not available, make sure students are happy that one of the colours represents the red apples and the other colour represents the green apples. Alternatively, you might wish to choose a different context.

Other suitable contexts for this task might be:

- buying chocolate and vanilla cupcakes
- new students in the class, boys and girls
- bag of white and pink marshmallows.

Number balance

Using a number balance in the Exploring the commutative property stage helps illustrate the concept of **equivalence**.



10 = 6 + 4

6 + 4 = 1 + 9

6 + 4 = 3 + 7

Introduction and exploration

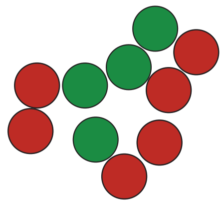
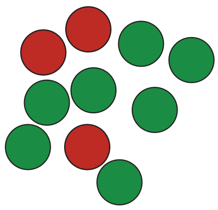
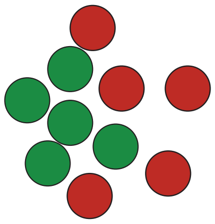
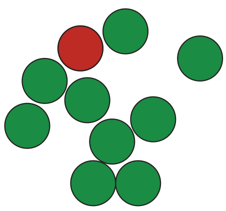
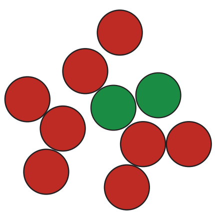
Introduction

Pose the question: *A shop is selling red and green apples. I buy a bag of 10 apples. How many red apples and how many green apples might I have bought?*

Resources: Provide each student with 10 two-colour counters and explain that the counters represent the apples.

Students toss the 10 counters and record how many there are of each colour. Have students record their results.

Possible student response:

	Red apples	Green apples
	6	4
	3	7
	5	5
	1	9
	8	2

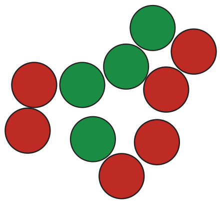
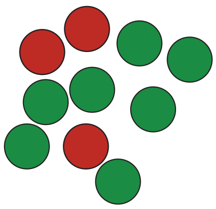
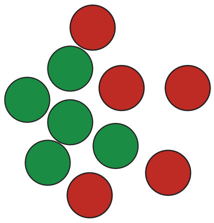
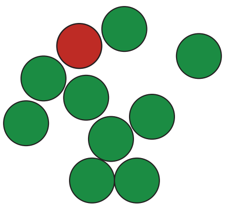
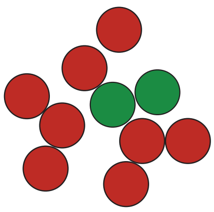
Enabling prompt:

- *I buy five (or six) apples. How many red apples and how many green apples might I have bought?*

Extending prompt:

- *Can you work out some different combinations without tossing the counters?*

Ask students to share some of the results that they have collected. Record the results randomly (i.e. do not sort them in any way). Look at how these can be recorded as an equation.

	Red apples	Green apples	
	6	4	$\Rightarrow 10 = 6 + 4$
	3	7	$\Rightarrow 10 = 3 + 7$
	5	5	$\Rightarrow 10 = 5 + 5$
	1	9	$\Rightarrow 10 = 1 + 9$
	8	2	$\Rightarrow 10 = 8 + 2$

This introduces the important concept of **equivalence**. The equals sign shows that all these combinations have the same value as each other.

We know that there is a total of 10 apples in the bag, so this is recorded first. After the equals sign we record the possible combinations.

Discussing our combinations

Pose the questions: *How many different combinations are possible? How do we know if we have found them all?*

Ask students to explore whether they have found all possible combinations. Students should record their results as an equation.

Questioning to direct the investigation

- *What is similar and what is different about the combinations you have found?*
  - ◊ Students will see that some combinations look similar when written down; for example, four red apples and six green apples is similar to six red apples and four green apples. This is an example of the commutative property. It is important for students to recognise that these combinations are different (i.e. they are **commutative** combinations).
  - ◊ The different combinations are also similar in that they all add to 10. This may seem obvious but it is important for students to realise that although the numbers are different, the *value* of the equation is the same. This is exploring the concept of equivalence and the meaning of the equals sign.
- *How can you order the combinations you have collected to help you see if you have found them all?*
  - ◊ Ordering the combinations will help show which possibilities are missing.

Extending prompt:

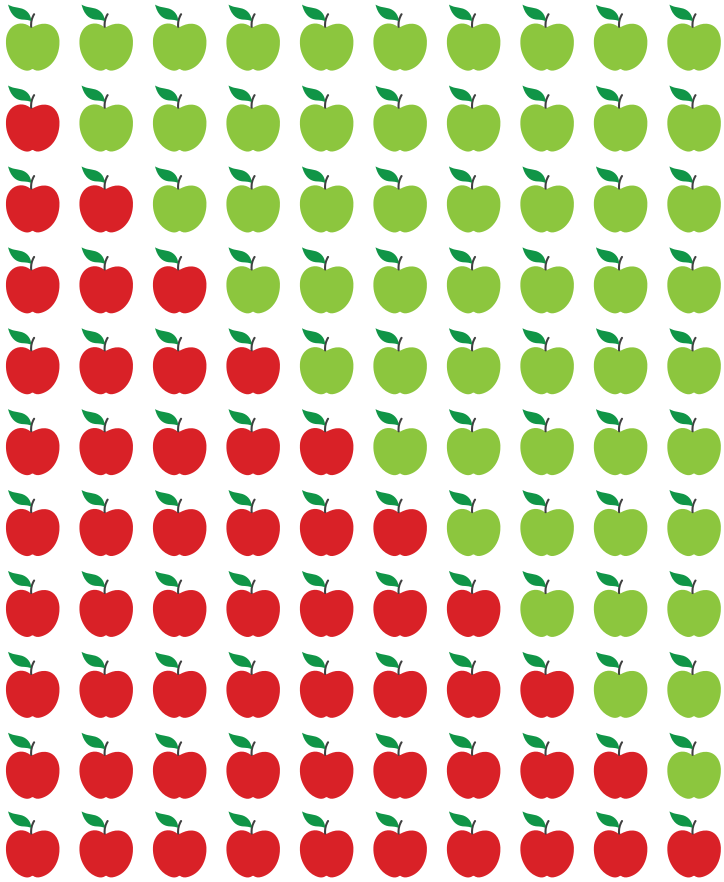
- *I buy a bag of red apples, green apples and pears. In the bag are 10 pieces of fruit. What is the same and what is* <u>*different*</u> *about my bag of fruit compared with my bag of apples?*
  - ◊ Students can explore equivalence between combinations of two or three numbers that add to 10. For example:
    - ▪ $2 + 3 + 5 = 1 + 6 + 3$
    - ▪ $4 + 6 = 2 + 3 + 5$

Exploring the commutative property

Select some students to present their work to the class. Look at the way the combinations have been ordered.

Teacher notes:

- Students might wish to organise the combinations in ascending or descending order. Ordering in this manner makes it easy to identify any missing combinations.
- The following example is ordered according to the ascending order of red apples.

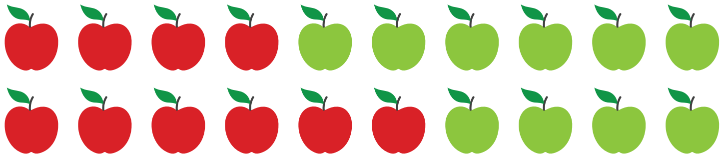


0 + 10
1 + 9
2 + 8
3 + 7
4 + 6
5 + 5
6 + 4
7 + 3
8 + 2
9 + 1
10 + 0

- Recording in this way highlights the concept of **compensation**, whereby a new combination is made by taking away a green apple and replacing it with a red apple.

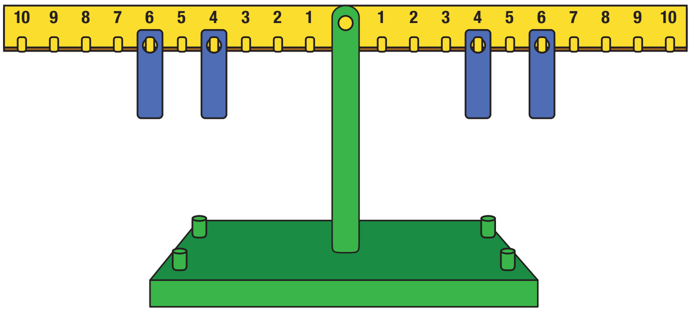
Let's now explore the commutative property of addition illustrated in the task.

Although the same numbers have been used, the combinations of apples are different.



$6 + 4 = 4 + 6$

This can also be shown on the number balance.
(See the Teacher background information for more uses of the number balance.)



$6 + 4 = 4 + 6$

Look at how the equations recorded are equivalent; that is, how they have the same value as each other.

Pose the questions: *What if I bought six apples? Or 12 apples? Or 17 apples?*
How could we find the combinations for any number of apples?

This encourages students to move towards a generalisation. The two-part partitions can be found by listing the combinations systematically. Students can explore the possible partitions for different numbers. Have students explain how all the two-number partitions can be found.

Where to next?

Lesson 2: 12 Ways to Get to 11 provides a deeper exploration into partitioning and focuses on creating generalised statements about the process of partitioning.

LESSON 2

re(Solve) MATHS BY INQUIRY **Partitioning**

12 Ways to Get to 11

Y1

About this lesson

This task uses the picture book *12 Ways to Get to 11* to investigate how many two-number partitions are possible for any given number. Students apply their learning from Lesson 1: Red Apples, Green Apples to list all the ways that 11 can be made using two numbers. They then look at the number of possible partitions that can be made from some other numbers. Students look at the fact that for any given number the possible combinations will be always one more than the number itself, and explain why this is the case.

Australian Curriculum: Mathematics (Year 1)

ACMNA015: Represent and solve simple addition and subtraction problems, using a range of strategies including counting on, partitioning and rearranging parts.

Mathematical purpose

- To build students' skills in generalisation.
- Students work systematically to find all possible combinations for partitioning a number into two smaller numbers. They then make a generalised statement about how all partitions can be found and why the number of possible partitions for a given number is always one more than the number itself.

Learning intention

- To find the different ways to combine two numbers to make 11.

Time

A lesson of approximately 1 hour.

Vocabulary

- partition

Resources

- The book *12 Ways to Get to 11* by Eve Merriam & Bernie Karlin
  - ◊ If this book is not available, alternatives are discussed in the Teacher notes on page 2.

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Partitioning eleven

Resources: Read the picture book *12 Ways to Get to 11*. The book partitions 11 in various ways, usually using more than two numbers to make 11. Discuss the different number combinations that are used in the book to make 11.

Teacher notes:

- If this book is not available, a similar context can be used, such as taking 11 marshmallows out of a bag and looking at the number of pink and white marshmallows.
  - ◊ I took 11 marshmallows out of a large bag of pink and white marshmallows. I got six pink and five white marshmallows.
  - ◊ I took out another 11 marshmallows. I got two pink and nine white marshmallows.
  - ◊ Continue this context with some other combinations that are not ordered.

Pose the question: *How many different ways can you make 11 using just two numbers?*

Exploring

Allow students to explore the different ways that they can make 11 by adding two smaller numbers. Encourage students to record their work using equations.

Enabling prompt:

- Have students toss 11 two-coloured counters, as in Lesson 1: Red Apples, Green Apples. Ask students to record the different combinations and see how many they can find.

Working systematically and drawing on the learning from Lesson 1: Red Apples, Green Apples, students should be able to generate 12 different ways to make 11:

$0 + 11 = 11$	$6 + 5 = 11$
$1 + 10 = 11$	$7 + 4 = 11$
$2 + 9 = 11$	$8 + 3 = 11$
$3 + 8 = 11$	$9 + 2 = 11$
$4 + 7 = 11$	$10 + 1 = 11$
$5 + 6 = 11$	$11 + 0 = 11$

Pose the questions:

- *How many different ways do you think it is possible to partition 8 into two smaller parts?*
- *What about 15? 21? 32?*
- *How could you work out the number of ways any number can be partitioned into two smaller parts?*

Rather than asking students to write all the two-number partitions for the given numbers, this question asks students to work out the total number of pairs there are for any given number. Although it is not too arduous to write out all possible combinations for 8, it is a lot of work to do so for 15, 21 and 32. Ask students to think about a strategy they could use to decide how many pairs there are without writing out all possible combinations.

If students see that the number of possible pairs is one more than the number itself, pose the question: *Why is it that the number of possible combinations is always one more than the number itself?*

Extending prompt:

- *Look at all the possible combinations for 10 and 11. What is different and what is the same about the combinations?*
  - ◊ The way that the numbers are arranged in ascending and descending order is the same. The main difference is that 10 has a double and 11 does not. This is because 10 is an even number.

Reflection

The focus of this reflection is to help students make a generalised statement as to why the pattern occurs.

Select some students to present their work and reasoning to the class.

As a class, discuss the generalisation: For any given number, the quantity of possible combinations will be always one more than the number itself.

$0+8=8$	$0+10=10$	$0+11=11$	The possible combinations will be always one more than the number itself.
$1+7=8$	$1+9=10$	$1+10=11$	This is because when zero is included, there is always one more number in the sequence. Each of these numbers is paired to make a combination.
$2+6=8$	$2+8=10$	$2+9=11$	For example, when listing the numbers from zero to 10, there are 11 numbers. Each of these numbers can be paired to make 10.
$3+5=8$	$3+7=10$	$3+8=11$	
$4+4=8$	$4+6=10$	$4+7=11$	
$5+3=8$	$5+5=10$	$5+6=11$	
$6+2=8$	$6+4=10$	$6+5=11$	
$7+1=8$	$7+3=10$	$7+4=11$	
$8+0=8$	$8+2=10$	$8+3=11$	
	$9+1=10$	$9+2=11$	
	$10+0=10$	$10+1=11$	
		$11+0=11$	