re(Solve) MATHS BY INQUIRY Equivalence

Y2

Summary of learning goals

- This sequence builds students' understanding of equivalence as balance. The equals sign is used to indicate the same value on both sides of an equation.
- Students develop their understanding of equivalence by looking at balancing scales with blocks of different weights.

Australian Curriculum: Mathematics (Year 2)

ACMNA035: Describe patterns with numbers and identify missing elements.

ACMNA036: Solve problems using number sentences for addition or subtraction.

ACMMG038: Compare masses of objects using balance scales.

Summary of lessons

Who is this sequence for?

This sequence is for students who:

- are familiar with symbolic recording for addition
- have worked with addition facts up to 10 + 10
- have experience using balance scales to compare masses of objects.

Lesson 1: Balancing Numbers

Students use weights to find two numbers on one side of the number balance that will balance with two numbers on the other side. They record their solutions as equations, using the equals sign as a symbol denoting balance.

Lesson 2: Balance Without Numbers

Students are shown pictures of differently weighted blocks balanced on scales. They then decide whether additional pictures of blocks will balance, using the first set of pictures as a reference. Students are encouraged to complete this activity without using numbers or assigning values to the blocks.

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Reflection on this sequence

Rationale

Equivalence is a central idea in algebra. Students need to appreciate that although two expressions might look different, they can have equivalent value. Equivalence between expressions is symbolised with the equals sign. For many students, the equals sign means something very different: it is an instruction to carry out the arithmetic to find the answer. This sequence challenges students' current conceptions of equivalence and the equals sign. They are asked to create equivalent expressions symbolised through balance and with the equals sign.

reSolve mathematics is purposeful

- Students explore the concept that, although two sides of an equation might look different, they can have the same value.
- The two tasks connect equivalent values to the concept of weight and balance.

reSolve tasks are inclusive and challenging

- The tasks draw on simple addition, allowing students to use their existing knowledge to develop their knowledge and understanding of equivalence.
- Enabling and extending prompts enable the teacher to differentiate the tasks to cater for the needs of all students.

reSolve classrooms have a knowledge-building culture

- The tasks challenge students' alternative conceptions of equivalence and the meaning of the equals sign.
- The first task requires students to persevere and find multiple answers. The use of the number balance allows them to experiment with different answers and to take risks in their learning.
- The second task asks students to work collaboratively to find answers. They are asked to reason mathematically and to listen to and make sense of the reasoning of others.

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LESSON 1

re(Solve) MATHS BY INQUIRY Equivalence

Balancing Numbers

Y2

About this lesson

Students use weights to find two numbers on one side of the number balance that will balance with two numbers on the other side. They record their solutions as equations, using the equals sign as a symbol denoting balance.

Australian Curriculum: Mathematics (Year 2)

ACMNA035: Describe patterns with numbers and identify missing elements.

ACMNA036: Solve problems using number sentences for addition or subtraction.

Mathematical purpose

- To build students' understanding of equivalence as balance.

Learning intention

- To find different ways of balancing the number balance.

Time

A lesson of approximately 1 hour.

Resources

- Number balances (digital version can be found online if necessary)

Vocabulary

- balance
- equals
- equation
- equivalence

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Introducing the number balance

Resources: Show the students a number balance.

Look at some numbers that balance, such as:

- 4 balances with 2 and 2.
- 3 and 1 balances with 4.
- 3 and 1 balances with 2 and 2.

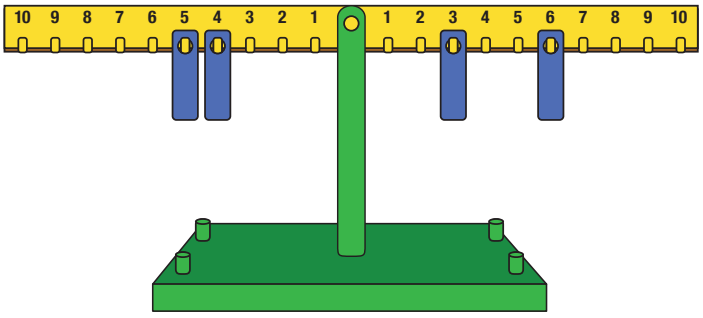
Discuss the meaning of the equals sign. It means balance and shows that there is the same value on both sides of the equation. When the number balance is balanced, an equals sign can be used to record the equation. This means that:

- $4 = 2 + 2$
- $3 + 1 = 4$
- $3 + 1 = 2 + 2$

Pose the question: *I put two weights on one side of the balance and I put two weights on the other side. The number balance is balanced. What numbers might I have put the weights on? How many combinations can you find?*

Balancing numbers

Provide students with a number balance and allow them time to find numbers that balance. Students should record the possible answers as equations. For example:



$5 + 4 = 3 + 6$

Students can complete this task simply by trial and error. Encourage them to think strategically about finding numbers that balance.

Questioning to prompt deeper inquiry:

- *Place two weights on one side of the balance. Can you find all the possible ways to create balance by placing two weights on the other side? How do you know you have found them all?*
- *What strategies did you use to help find numbers that balanced?*

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Enabling prompt:

- *I have 6 and 4 on one side of the balance. What numbers will balance on the other side?*

Students can look at other known ten facts. When they have some solutions, they can start to explore other numbers.

Extending prompt:

- *I put two weights on one side of the balance and I put three weights on the other side. The number balance balanced. What numbers might I have put the weights on?*

Students can use similar strategies to explore the way to balance two numbers with three numbers.

Reflection

Discuss the strategies used by the students. Look at how the strategies can be recorded on a number line and with an equation.

5 and 4 balance with 3 and 6 ⇒ $5 + 4 = 3 + 6$

If the 3 is moved to the 2, where should the 6 move to keep it balanced?

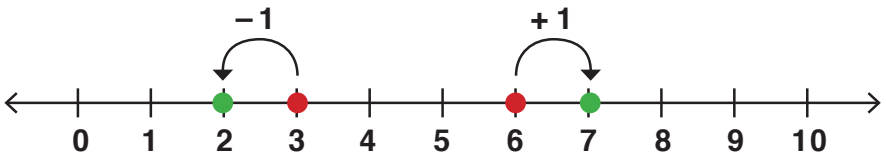
As the 3 decreased by 1, the 6 will need to increase by 1.

This can be shown as an equation and on a number line:

$5 + 4 = 3 + 6$

$5 + 4 = (3 - 1) + (6 + 1)$

$5 + 4 = 2 + 7$



If the 6 is moved to the 8, where should the 3 move to keep it balanced?

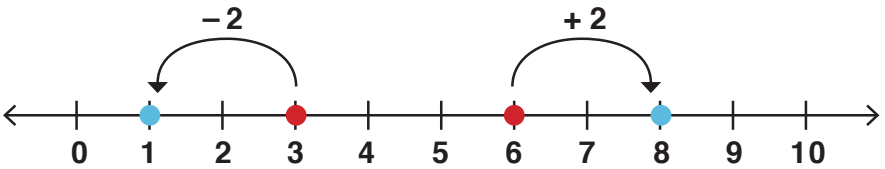
As the 6 increased by 2, the 3 will need to decrease by 2.

This can be shown as an equation and on a number line:

$5 + 4 = 3 + 6$

$5 + 4 = (3 - 1) + (6 + 1)$

$5 + 4 = 2 + 7$



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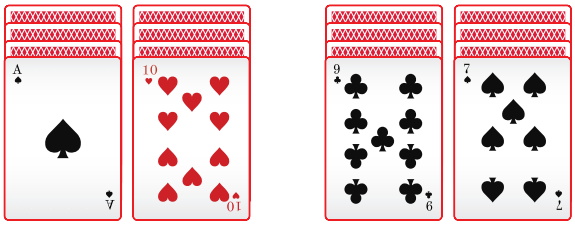
Further activities

Activity 1: Equivalence card game

This game is for two or three players. Each group of students needs a regular deck of cards. In the game, picture cards = 0 and ace = 1.



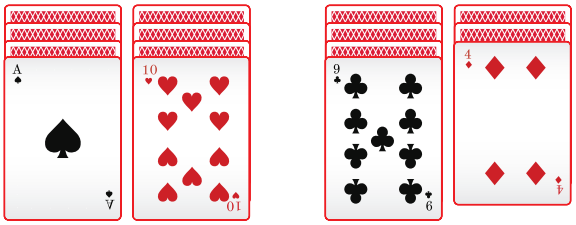
Each player needs four piles of cards with four cards in each pile, leaving a small gap in the middle, like a number balance with two numbers on either side. Turn the top card in each pile face up.

The aim of the game is to balance the sets of numbers.

Players take turns to make a move. To balance their cards, a player can discard one of the top cards and turn over the card underneath. Cards discarded are placed face down in the centre and are out of play.

A player can continue until they have no more cards left to flip over. They may also choose to stop at any point in the game.

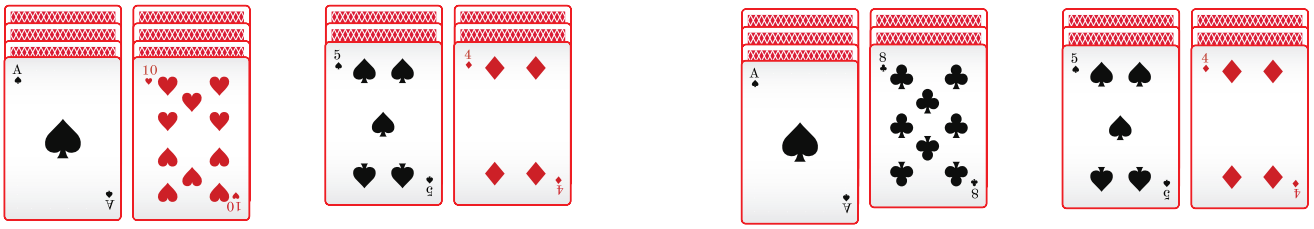
A player's score at the end of the game is the difference between the two pairs of cards. The player with the lowest score over three games is the winner.



For example, in this hand, the 7 was discarded and a 4 turned over.

This player can continue to try and balance the value of the cards on both sides. They can also choose to stop and they would end with a score of 2 points.

The player decides to continue for two more moves and balances their cards. They end up with a score of zero.



Activity 2

Ask students: *What numbers could go in the spaces?*

20 + ___ = ___ + 10

Other similar questions can be created for the students to solve.

Activity 3

Pose the question: *There is one weight on one side of the number balance and two weights on the other side. The number balance is balanced. What might the numbers be?*

Students can use this understanding to help them solve KenKen puzzles such as the one shown. There are a few websites that create KenKens. The generator should be set so that only addition is used in the puzzle.

4	4+		3+
7+		7+	
5+			7+
	3		

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LESSON 2

re(Solve) MATHS BY INQUIRY Equivalence

Balance Without Numbers

Y2

About this lesson

Students are shown pictures of differently weighted blocks balanced on scales. They then decide whether additional pictures of blocks will balance, using the first set of pictures as a reference. Students are encouraged to complete this activity without using numbers or assigning values to the blocks.

Australian Curriculum: Mathematics (Year 1)

ACMMG038: Compare masses of objects using balance scales.

Mathematical purpose

- To develop understandings of equivalence by looking at balancing scales without using numerical values.

Learning intention

- To use pictures of balanced scales to balance different scales.

Time

A lesson of approximately 1 hour.

Vocabulary

- balance
- combinations
- equivalence
- scales

Resources

- reSolve PowerPoint *2a Balance without Numbers* (for display)
- Student Sheet 1 – Three Blocks (one copy per student, also included as reSolve PDF *2b Three Blocks*)
- Student Sheet 2 – Five Blocks (one copy per student, also included as reSolve PDF *2c Five Blocks*)

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Balancing blocks

Resources: Display reSolve PowerPoint *2a Balance without Numbers*.

Show the students slide 2: *Each of the blocks in the picture has a different weight.*

Show the students slide 3: *Different combinations of blocks have been placed on scales so that the scales balance. Each scale shown is balanced.*

Discuss the balanced blocks in the picture. Avoid using numbers and assigning values to any of the blocks. Students might note which blocks appear to be heavier and lighter.

Which ones balance?

Resources: Provide students with copies of Student Sheet 1 – Three Blocks.

Pose the task: *Use the balanced scales* (on slide 3) *to help you work out which of these scales will balance*. Students might tick the pictures that will balance.

Allow the students time to discuss with a partner which of the pictures will balance. These questions have been designed so that they can be answered without assigning values to the blocks.

Ask students to draw in additional blocks on the pictures that don't balance, in order to make them balance.

Going deeper

Return to reSolve PowerPoint *2a Balance without Numbers* and show students slide 4: *Each of the blocks in the picture has a different weight. The orange cylinder, green sphere and blue cube haven't changed their weight. They are the same as in the previous pictures.*

Show the students slide 5: *Different combinations of blocks have been placed on scales so that the scales balance. Each scale shown is balanced.*

Resources: Provide students with copies of Student Sheet 2 – Five Blocks.

Ask them to complete the sheet as in the first task.

Reflection

Have some students share their work and explain how they decided if the scales would balance.

Key concepts for discussion:

- *Whatever is added/taken away from one side must be added/taken away from the other side. (The concept of compensation.)*
- *It doesn't matter what side of the scales that the blocks are on as long as both sides have the same weight. (The commutative property of addition.)*

Look at the blocks that they added to make the scales balance. Discuss the differences between students' work samples. Can the scales be made to balance using different blocks?

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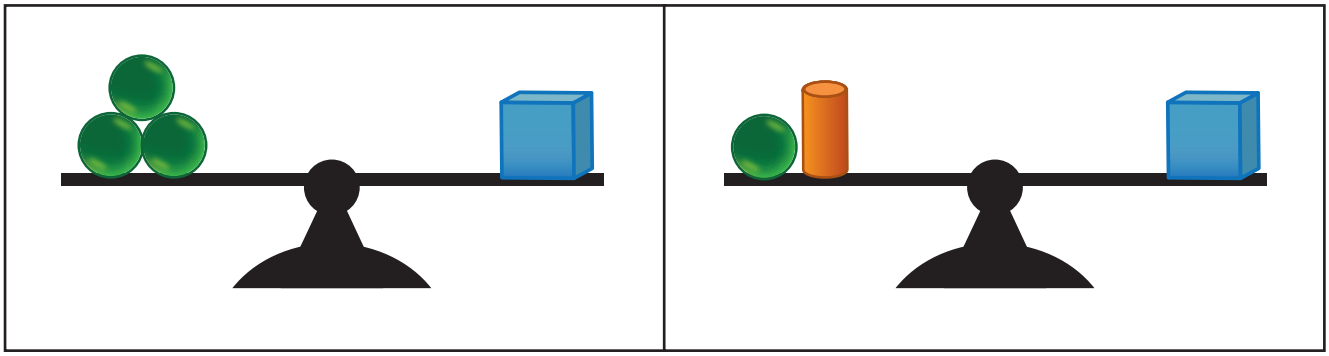
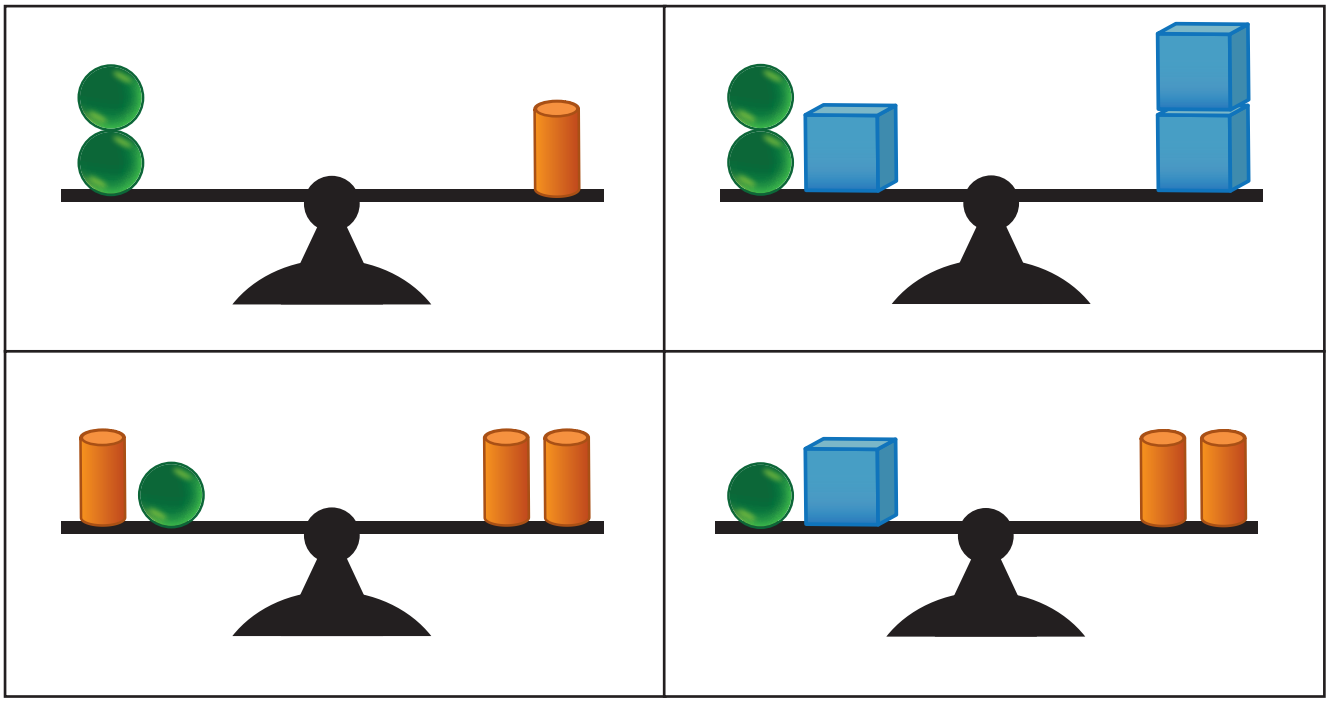
Three Blocks

Name: _____

The blocks in these two pictures balance.



Which of these will balance?



Which ones don't balance? Draw in blocks so that they will balance.

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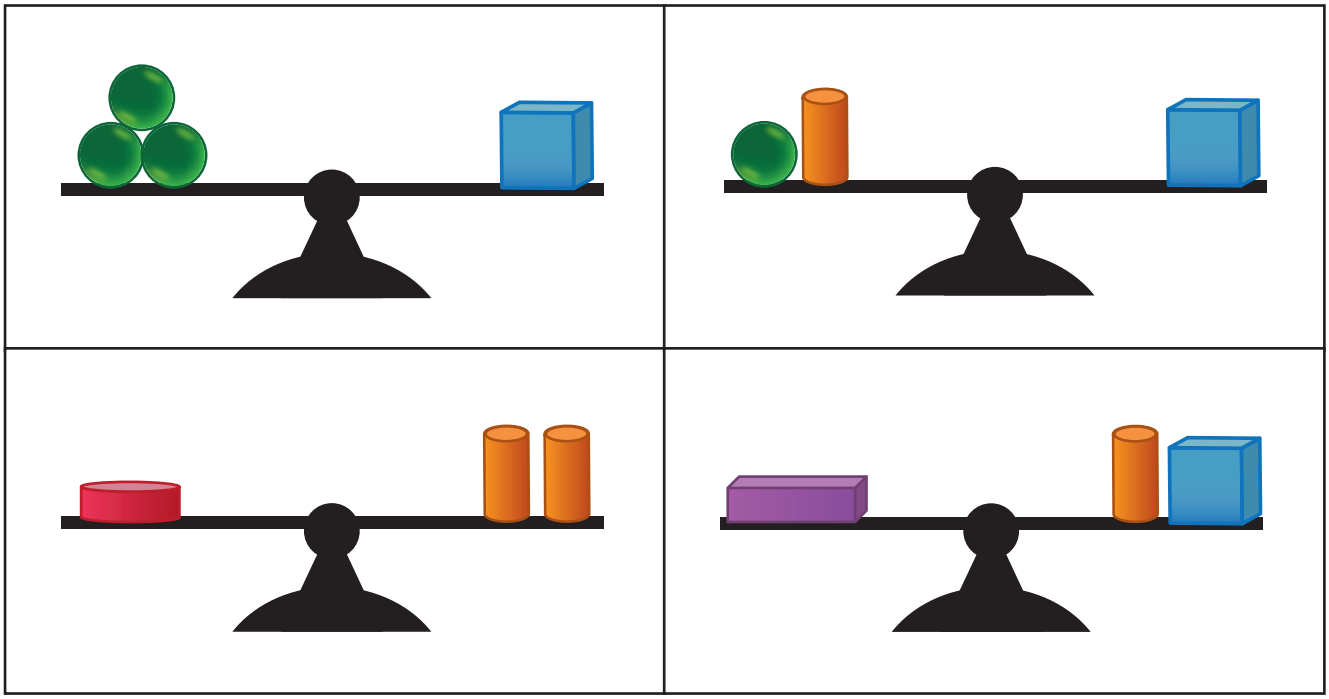
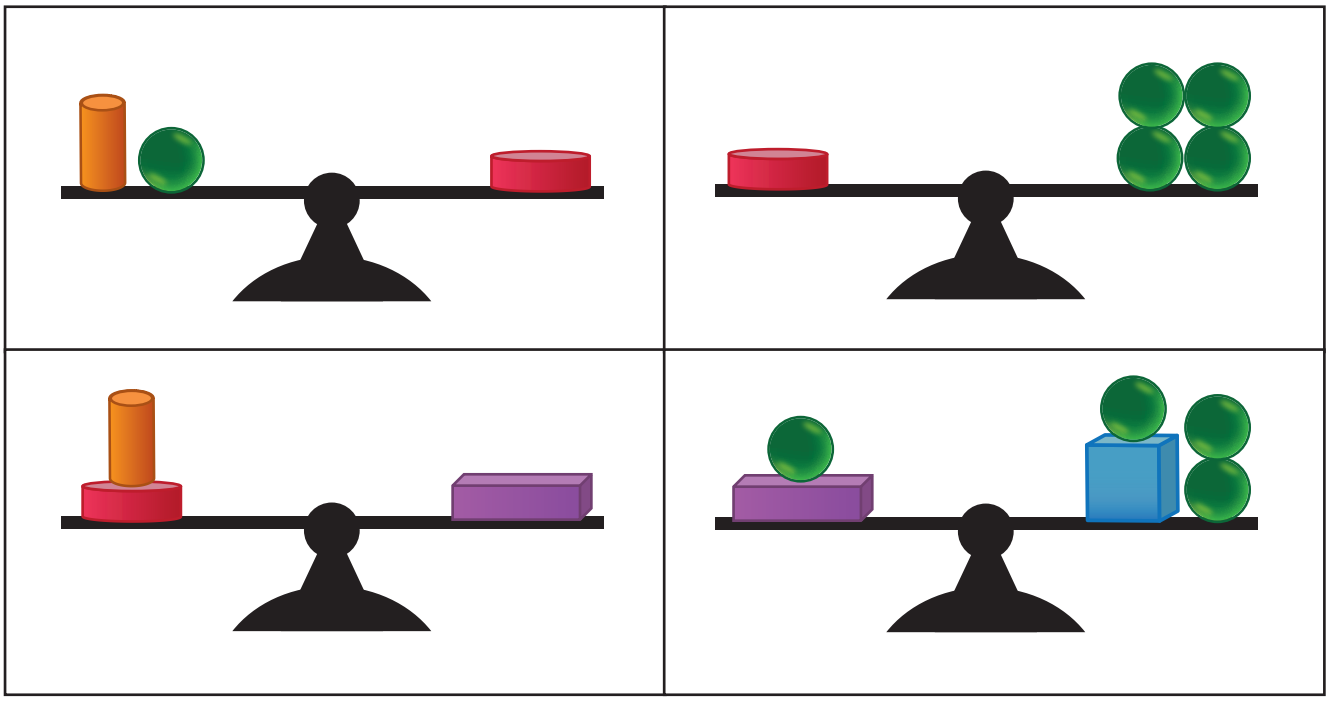
Five Blocks

Name: _____

The blocks in these four pictures balance.



Which of these will balance?



Which ones don't balance? Draw in blocks so that they will balance.

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